EXPERIMENTAL INVESTIGATION AND FINITE ELEMENT SIMULATION STUDIES ON BEHAVIOUR OF WALLS ON BEAMS CONSIDERING MATERIAL AND STRUCTURAL NONHOMOGENEITY

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
DOCTOR OF PHILOSOPHY

By
P. PURUSHOTHAMAN

to the

DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

MAY, 1973

CERTIFICATE

This is to certify that the work entitled " EXPERIMENTAL INVESTIGATION AND FINITE ELEMENT SIMULATION STUDIES ON BEHAVIOUR OF WALLS ON BEAMS CONSIDERING MATERIAL AND STRUCTURAL NONHOMOGENEITY" by P. Purusothaman has been carried out under my supervision and has not been submitted elsewhere for award of any degree.

Dr. J.K. Sridhar Rao
Department of Civil Engineering
Indian Institute of Technology
Kanpur - 16

ACKNOWLEDGEMENTS

The work reported in this thesis was carried out in the Department of Civil Engineering, Indian Institute of Technology, Kanpur, India, under the supervision of Dr. J.K. Sridhar Rao.

The author expresses his sincere gratitude to Dr.J.K. Sridhar Rao, for his guidance, help and encouragement throughout the period of this study.

The author has gained valuable knowledge from the technical discussions and the courses given by Prof. P.N. Murthy of the Aeronautical Engineering Department.

But for the excellent courses given by Prof. Y.C. Das this thesis could not have taken its present form. The author is thankful to Dr. A.V. Setlur and Dr. M.P. Kapoor for their useful courses. Dr. P. Dayaratnam gave timely help in the experimental investigations.

The help given by Prof. M. Anandakrishnan and Prof. G.D. Agrawal, in times of need, is gratefully acknowledged.

The author thanks the Director of Technical Education, Tamil Nadu State for granting leave for his study, and the Director, IIT, Kanpur for his financial assistance and encouragement.

The laboratory investigations form a part of the studies undertaken on behalf of the National Bureau of Standards, U.S.A., in the project on 'Composite Structural Systems in Buildings' with Dr. J.K. Sridhar Rao as the Principal Investigator, wherein the author worked as a Senior Research Associate. Financial assistance from the above project, made it possible for the author to pursue works leading towards the doctorate degree. The very excellent work done by Mr. V.K. Kapur, Research Assistant of the above project, in assisting in the laboratory tests is appreciated and acknowledged.

The author values the discussions he has had with Dr. N.G.R. Iyengar, Dr. S. Ramaseshan and Dr. S.S. Prabu of various departments in the institute. Fellow students, Messrs. C.P. Vendhan, D. Venugopal, A. Varadarajan, M. Hariharan and S. Dashina Moorthy were of great help in going through the earlier versions of this thesis.

But for the wonderful cooperation extended by the staff of the Structural Laboratory and the Computer Centre, this work could not have been undertaken.

Special thanks are due to Mr. S. Kumar, for his customary efficiency and responsibility in typing this thesis.

- - -

TABLE OF CONTENTS

SYNOPSIS

EXPERIMENTAL INVESTIGATION AND FINITE ELEMENT SIMULATION STUDIES ON BEHAVIOUR OF WALLS ON BEAMS CONSIDERING MATERIAL AND STRUCTURAL NONHOMOGENEITY

A Thesis

Submitted in Partial Fulfilment of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

to the

Department of Civil Engineering

Indian Institute of Technology, Kanpur

May, 1973

Thesis Supervisor : Dr. J.K. Sridhar Rao

In this thesis, the behaviour of brick masonry walls on reinforced concrete beams under vertical loads is studied experimentally and simulated in the computer using finite element techniques of analysis. Elastic, post-cracking and failure stages are covered. Openings are included. It is recognised that brickwork has orthotropic properties to start with, becomes anisotropic after cracking and introduces nonhomogeneity at post-cracking stages under progressive increase in loading. The brick walls with or without openings, interacting with reinforced concrete beams are essentially plane stress problems in nonhomogeneous media, especially after cracking, local compression failures and yielding of reinforcement in the beams.

Effects of nonhomogeneity on stresses and deformations have not yet been adequately understood and thus special attention is given in this thesis to nonhomogeneity in developing analytical models.

First, modern advances in the theory of composite materials are correlated with finite element predictions of elastic constants for concrete which is dealt with as a random, nonhomogeneous two-phase media consisting of aggregates embedded in a matrix of mortar. Moving from concrete, whose behaviour has been better understood, brickwork is simulated as inclusions of hard materials in a soft matrix, using finite element methods. The elastic constants obtained from these studies are introduced in the finite element analysis of walls on beams at the structural level (macro-level).

As analytical simulations need realistic data, extensive test programmes which have been undertaken are reported. The strength of bricks, mortar and brickwork have been studied in the laboratory and particular attention has been paid to statistical variations. The strength and behaviour of brick-walls on reinforced beams under tensile and compressive loading have been investigated. Height-span ratios, size and location of openings and mortar strength have been varied. Besides deflections and strains, progressive cracking, local compression failures and collapse loads have been recorded.

At the structural level also, the load-response characteristics of walls on beams have been simulated. A finite element programme has been developed to incorporate automatic generation of nodal coordinates and connections and random assignment of brickwork properties. Openings and reinforcement in the supporting beams have been included. Full interaction (no bond-slip) between brickwork and concrete beam is assumed, which is justified by experimental observations. Progressive cracking, local compression failures and yielding of steel have been traced. The stiffness of critical elements have been modified and incremental-iterative methods have been used to obtain load-response curves upto failure. Certain parameters have been identified as characterising the behaviour of brickwork and walls on beams and these are used for formulation of simple design rules.

From the above simulation studies, and laboratory tests the following observations have been made:

1. Computer simulation studies predict the elastic constants more realistically than the hypothetical models of composite materials.

2. The perturbation stresses due to local non-homogeneity are not secondary in magnitude and micro-cracking and vertical splitting of compression specimens may be attributed to these stresses.

3. Experiments show that the strength of brickwork is a fraction of the strength of the constituents due to the development of critical interaction stresses.

4. Computer simulation studies have shown that the parameter $E_b \nu_m / \nu_b E_m$ is an important parameter, which can be controlled to prevent cracking at service loads.

5. Experiments have shown that there is a safety reserve of 200 to 500 percent due to structural interaction provided there are no eccentric openings and the loading is through the top of the wall.

6. Based on critical compressive stresses in the supporting beams, computer simulation studies have shown a safety reserve of 200 to 400 percent.

7. The load-response characteristics obtained from computer simulation and laboratory tests agree quite well in the elastic range. Progressive cracking and local compression failures traced through computer simulation reflect the laboratory test results. However, the deformations in the post-cracking range are found to be lower than those obtained in the tests.

In conclusion it is felt that computer simulation of a few laboratory tests can be used for extensive parameter studies and development of design charts.

LIST OF TABLES

LIST OF TABLES (CONTD.)

LIST OF TABLES (CONTD.)

- - -

LIST OF FIGURES

LIST OF FIGURES (CONTD.)

LIST OF FIGURES (CONTD.)

- - -

NOTATION

A_s	-	area of steel reinforcement in supporting beam
$\underline{\underline{B}}$	-	matrix relating strains and nodal displacements
C	-	stress concentration factor at supports of walls on beams
C_V	-	coefficient of variation
$\underline{\underline{D}}$	-	material property matrix
$\underline{\underline{D}}_{ep}$	-	elasto-plastic matrix
E	-	Youngs Modulus
E_b	-	Youngs Modulus for brick
E_m	-	Youngs Modulus for mortar
E_s	-	Youngs Modulus for steel
F	-	stress reduction factor in design of walls on beams
f_b	-	basic allowable stress in brickwork
f_c	-	maximum allowable stress in brickwork
f_p	-	allowable stress in walls on beams
f_c'	-	cylinder strength of concrete in compression
f_t'	-	split tensile strength of concrete
G	-	shear modulus
g	-	volume fraction of particles in composite media
H	-	height of wall including supporting beam
I_b	-	moment of inertia of beam
I_s	-	moment of inertia of reinforcement rods

NOTATION (CONTD.)

$\underline{\underline{K}}$	-	structure stiffness matrix
$\underline{\underline{K}}_R$	-	stiffness matrix of reinforcement
$\underline{\underline{k}}$	-	element stiffness matrix
L	-	span length centre to centre of supports
m	-	mortar/matrix in composite media
$\underline{P}$	-	vector of known nodal forces
p	-	particles in composite media
$\underline{\underline{T}}_{\sigma}$	-	transformation matrix for stresses
$\underline{\underline{T}}_{\epsilon}$	-	transformation matrix for strains
$\underline{U}$	-	nodal displacement vector of structure
$\underline{u}$	-	nodal displacement vector of elements
u	-	displacement
V	-	body force potential
v	-	displacements
W	-	total load on beam
w	-	displacements
$\alpha_1 - \alpha_6$	-	constants
β	-	angle of crack in deep girders failing in shear
γ	-	shear strain
$\underline{\epsilon}$	-	strain vector of ϵ_x, ϵ_y, ϵ_{xy}
ϵ_x	-	strain in x direction
ϵ_y	-	strain in y direction

NOTATION (CONTD.)

ϵ_{xy}	-	shear strain in plane stress problems
$\underline{\sigma}$	-	stress vector of σ_x, σ_y, σ_{xy}
σ_x	-	stress in x direction
σ_y	-	stress in y direction
σ_{xy}, τ_{xy}	-	shear stress
σ_o	-	limiting compressive stress in uniaxial compression
Δ	-	area
ϕ	-	stress function
ρ	-	density of brickwork
ν	-	Poisson's ratio
ν_b	-	Poisson's ratio for bricks
ν_m	-	Poisson's ratio for mortar
μ	-	coefficient of friction

- - -

CHAPTER I

INTRODUCTION

Experimental and computational advances made in the past two decades have enabled structural engineers to design more rationally and economically, than ever before. While these developments have been systematically absorbed in the design of steel and concrete structures, the brickwork as a construction material and the masonry wall as a structural component are yet to benefit from these advances. Brickwork as a material preceded steel and concrete by a thousand years and there is more brick construction in India than that of steel and concrete. Behaviour of brick walls interacting with simply supported reinforced concrete beams has been chosen as the subject of study for this thesis.

1.1. General

Compared with steel, the behaviour of which has been extensively studied in the past sixty years, brick-work is essentially orthotropic in nature before cracking and becomes non-homogeneous and anisotropic after cracking or local crushing and these aspects must be explicitly considered. The constitutive laws and failure theories for brickwork have not been given adequate attention. At the structural level the nature and magnitude of interaction between brick panels and their supporting beams or surrounding frames deserve attention.

The safety factors associated with brickwork are in the order of six to twelve compared with values of two to four assigned to concrete.

It is now generally accepted that overloads, under strength of materials and unknowns in analysis and design contribute to the safety reserves assigned to a structural component. Overloads and under-strengths are subjects for statistical analysis requiring large amount of data and are not explicitly considered in this thesis. On the other hand unknowns in analysis and design are of direct importance to this study. These unknowns manifest themselves in the form of idealisations, assumptions and approximations which deserve detailed enumeration and critical re-examination in the light of modern developments in material science, continuum mechanics and numerical methods of analysis.

Considering the idealisations used for the study of walls on beams, at one extreme the self-weight of walls is included in the analysis but its interaction with the supporting beams is totally disregarded. At the other extreme is the theory of elasticity solution using traditional assumptions such as homogeneity, isotropy and linear, elastic stress-strain relations. Since closed form solutions are difficult to obtain, finite difference approximations have been used. Even then, computational difficulties have forced the designer to neglect cracking, local compression failures and the presence of

openings. Since real structures do have all these features, the resulting safety factors are rather high. The designer is perhaps justified at this stage in neglecting the unknown resistance of the wall altogether.

To circumvent the above mentioned difficulties, the research work in concrete structures has traditionally turned to laboratory testing of models and prototypes. While laboratory tests provide an overall picture of the load-response characteristics of simple material specimens or individual structural components, study of structural interaction of full scale buildings becomes extremely expensive and time consuming. Besides there is very little scope for enunciating the generalised principles of behaviour from isolated test data. Furthermore, a battery of empirical constants appear on the scene restricting the scope of the designer in using simple design principles for the individual structural component and the total structure. In fact laboratory testing has identified more problems for future research along with the answers originally sought.

More recent developments in computer simulation, particularly with the powerful finite element procedure, have provided a new tool for studying load-response characteristics of materials, structures and structural components. The limitations of computer simulation, which are bound to exist, are not yet apparent at this stage of development.

In the present investigation, the author has used experimental and computer simulation techniques for the study of materials and load-response characteristics of walls on beams.

1.2. Identification of the Problem

At the material level the following areas of study were identified:

a) Brickwork as an orthotropic material;

b) Elastic constants for brickwork from experiments and the theory of composite materials;

c) The complete stress-strain curve for brickwork;

d) Statistical variation of properties of brick, mortar and brickwork;

e) Failure theories for brickwork and concrete;

and f) Effect of non-homogeneity and randomness.

At the structural level the following aspects were chosen for detailed study:

a) Effect of non-homogeneity and anisotropy;

b) Effects of height/span ratio, mortar strengths, size and location of openings and tensile and compressive loading on brick walls.

c) Tracing of progressive cracking, von Mises failure in brickwork and concrete and yielding of steel;

d) Effects of progressive reduction in stiffness of the critical elements;

e) Effect of redistribution of stresses;

and f) Tracing of the load-response characteristics of brick walls on beams from zero-load to failure.

In Chapter 2, a critical review of the existing literature is presented. The theory of composite materials, various elastic models studied hitherto and simple structural analogies suggested for the design of walls on beams are reviewed besides a few ultimate strength concepts. There is a wealth of laboratory test data in previous research work often carried out at enormous cost and these are reviewed at some length. The scant attention given in codes of practice to walls on beams is mentioned in a relatively brief review.

Based on previous research and keeping in view the problems identified in this chapter, the scope of the author's investigations is identified in Chapter 3.

Extensive developments registered in the finite element methods of analysis in the past five years are suitably modified in Chapter 4 to deal with the problem on hand, providing a theoretical basis for the analysis of walls on beams and the results obtained from the computer programme developed by the author are detailed. The flow chart and

the programme are appended.

The laboratory investigations carried out by the author are detailed in Chapter 5.

The results obtained from computer simulation and laboratory tests are critically analysed in Chapter 6 so as to result in tentative recommendations for design. In addition, areas of further research are indicated.

In Chapter 7, after a brief summary, the conclusions drawn from this thesis work are recorded.

As for the contributions that can be claimed by the author the following may be mentioned:

a) The effects of non-homogeneity, anisotropy and randomness at material level on the behaviour of concrete and brickwork have been identified using finite element simulation.

b) The randomness in brickwork is traced to mortar strength and brick laying process, by controlling which the large safety factors associated with brick-work could be reduced. These conclusions were obtained from laboratory tests on basic brickwork.

c) The development of a computer programme using the CST element and tri-diagonalisation discussed by Zienkiewicz (B 19)*, with automatic generation of nodal connections

---

*Bracketed alpha-numerical characters indicate references.

and nodal coordinates, random assignment of material properties for the elements, introduction of reinforcement as linear elements, identification of local failures such as cracking and von-Mises failure, yielding of reinforcement, modification of the stiffnesses of these critical elements and finally obtaining stress redistribution using incremental-iterative methods/analysis.

d) The finite element solutions obtained truly reflect the behaviour of walls on beams observed in the laboratory, with respect to elastic deformations and local failures. However the post-cracking deformations were found to be lower than those obtained in tests.

e) It has been shown that beneficial interaction between walls and their supporting beams can be utilised in design, only when there are no eccentric openings, when the mortar strength is high enough to prevent local crushing failures at supports and when the loading is compressive in nature.

f) It has been shown that weak mortars and direct loading of the reinforced concrete beam through connecting floors, create significant cracking in the brick panels and the increase in load carrying capacity beyond that of the supporting beam is marginal in nature.

- - -

CHAPTER II

CRITICAL REVIEW OF EXISTING LITERATURE

2.1 Theoretical Studies

Early theoretical studies on interaction between brickwalls and their supporting beams were dependent on the classical theory of elasticity using stress functions. This was followed by Kantorovich methods and lattice analogy models. Simultaneously certain simplified analogies such as the beam, truss and vierendeel girder analogies, were improvised for immediate use. At the material level the theory of composite materials showed promise for prediction of equivalent elastic constants for composite media, which could presumably be used in the above analysis. These are briefly reviewed to show the nature of limitations that arose and how they were overcome by using innovative assumptions and idealisations. In this thesis the finite element method of analysis will be used and earlier developments will be reviewed in Chapter 4 and the author's modifications imposed on these developments will be traced simultaneously in that chapter.

2.1.1 Theory of Composite Materials

Popovics (A 11) has pointed out that computing the properties of a composite solid material, given the elastic constants and volume fraction of the components, is an important problem. Any model is however best suited for qualitative

examination of material behaviour. Of the various models proposed the laminated models are of special interest to brick-work. One of these models represents the internal structure where the strain is the same over any section of the composite material, while the stresses in the phases are proportional to their modulus of elasticity (Fig. 2.1). In this case,

$$E = (1-g)\,E_m + g\,E_p \tag{2.1}$$

where m, stands for matrix

p, stands for particle

and g, stands for volume fraction of particles.

The other simple model for the calculation of the modulus of elasticity represents the internal structure where the stress is same over any section of the composite material, while the deformations are inversely proportional to the modulus of elasticity of the phases. In this case

$$E = \frac{1}{(1-g)/E_m + g/E_p} \tag{2.2}$$

The equations (2.1) and (2.2) represent the upper and lower limits respectively, between which the modulus of elasticity of any two phases solid could be found, provided the Poisson's ratio of the two materials are the same.

Combination of the above two models represents a better fit between calculated and experimental values. Such a combination is shown in Fig. 2.2. Model I has the mathematical equivalent as follows:

$$\frac{1}{E} = \frac{A}{E_2} + \frac{(1-A)}{E_1} = A\left\{\frac{(1-g)}{E_m} + \frac{g}{E_p}\right\} + (1-A)\left\{\frac{1}{(1-g)\ E_m + g\ E_p}\right\} \tag{2.3}$$

where A = fractional volume of composite as shown in Fig. 2.2.

Model II has the mathematical equivalent as below:

$$\begin{aligned} E &= A\ E_2 + (1-A)\ E_1 \\ &= \frac{A}{(1-g)/E_m + g/E_p} + (1-A)\left[\ (1-g)\ E_m + g\ E_p\ \right] \end{aligned} \tag{2.4}$$

These equations are compared graphically in Fig. 2.3 for the exagerated case of $E_p/E_m = 10$, $E_m = 1$ and $A = 1/2$. Similar models are available for the prediction of $\bar{\nu}$, the Poisson's ratio and more sophisticated models have been evolved In the words of Calcote (A 2), " the state of the micromechanics of composite materials is not far advanced and must presently be considered to have more academic interest than practical value". The real difficulty lies in local microcracking, inelastic action and associated nonlinearities.

Recent efforts involving finite element techniques (A 5, A 6, A 8) and the studies made by the author yield more realistic pictures of load-response characteristics of composite materials.

2.1.2. Elasticity Solutions for Walls on Beams

The enormous reserves in strength available through interaction between slabs and beams and walls and beams or surrounding frames were discussed in the classical papers of Wood (C 21, C 22). He used relaxation methods to solve the plane stress problem, treating the self weight as a body force and loading the brickwall at the top. The basic biharmonic equation using the Airy's stress function is as below:

$$\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2\, \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0 \qquad (2.5)$$

wherefrom

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2} + \rho V$$

$$\sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2} + \rho V$$

and

$$\sigma_{xy} = -\frac{\partial^2 \phi}{\partial x\, \partial y} \qquad (2.6)$$

where ρ is density of brickwork and V is body force potential due to gravity loads. From known boundary conditions the

value of the stress function ϕ could be set up all round the boundary by a process of integration and the problem is then to determine the values of ϕ at internal points. This was achieved by means of the relaxation process of Southwell, using a fine mesh and working numerically in terms of the finite difference approximation to equation (2.5). It should be recorded that Wood (C 21) solved only plain brickwalls without their supporting beams.

Rosenhaupt (C 14) extended this approach to include the bottom beam and derived complicated interaction equations between the wall and beam. Besides the bi-harmonic equation for the wall, he used the familiar differential equation for the flexure of the beam,

$$M_b = E_b I_b \frac{d^2v}{dx^2} \tag{2.7}$$

Additional information needed to know the stresses in the composite structure are the boundary conditions between its components. These conditions shall be expressed in a form containing the Airy's stress function ϕ of the wall, so that only one redundancy at every interior or boundary point is unknown. The Poisson's ratio was assumed to be zero and self-weight was neglected. After describing in detail the effect of various wall materials and also the state of stress at several cross sections the following

conclusions were reached:

1) The shear stresses at the beam wall boundary induce the composite action of the wall structure. The tensile stresses concentrate in the foundation beam and the compressive stresses are distributed over the whole height of masonry.

2) At the supports high vertical stresses concentrate.

3) The vertical shear stresses are taken by the masonry part of the wall. The horizontal shear stresses between foundation beam and masonry concentrate near the supports.

4) The weakness inherent in the above analysis is that only shear forces are assumed to be transmitted at the beam wall interface. Besides, openings were not considered.

Coull (C 5) used a Kantorovich type of approach. The stresses in the walls were expressed as a power series in the horizontal direction, the coefficients being functions of height only. After satisfying the equilibrium and boundary conditions for both wall and beam, the remaining coefficients were determined from the minimization of the strain-energy of the system. The problem reduces to a fourth order differential equation subject to given boundary conditions, the influence of supporting beam appearing only in the latter. The analysis was simplified by dividing the applied load to symmetrical and antisymmetrical components. Gravitational loads were included in the analysis. In the present analysis

power series was used and the problem was essentially solved by one dimensional treatment. The results of this analysis compare favourably with that of Rosenhaupt (C 14) but significant differences in vertical stress transfer were noticed.

The next important development was due to Colbourne (C 4). The wall and beam are represented by a lattice analogy, in which it is assumed that the system remains elastic. Equilibrium equations were derived for the analogy, and were the same as obtained from finite difference equations at interior points of the wall. The analogy is then used to derive equilibrium equations for points affected by the beam and near the edge of the wall. The resulting equations were solved to obtain displacements from which stresses and stress resultants could be obtained. The Poisson's ratio was assumed to be zero.

It is clearly seen that these classical methods become highly complicated leaving no hopes for tracing of cracking, compression failures etc., and the introduction of openings. Even the Poisson's ratio is conveniently neglected to render the problem tractable, permitting little scope for the introduction of non-homogeneity and anisotropy.

But for the tremendous strides made in recent years in the finite element procedure the scope of this thesis would have become rather limited, as indicated above.

2.1.3. Simplified Analogies

The term analogy is used herein with a restricted scope. When a plane stress problem is treated as equivalent beam, truss or vierendeel girder it may be said that structural analogues have been used. While these analogies are certainly helpful to the designer, their scope of application is highly restricted and may inadvertently lead the designer to slip into dangerous errors.

The first simplification for the complicated stress state in a 'wall on beam' was suggested by Wood (C 21). Based on the observation that most of the compressive load was transferred to the supports through arch action, he suggested that the beams be designed for a reduced bending moment of WL/50 to WL/100 depending on the position of openings as against the customary WL/8. An equivalent moment arm of 2/3 depth subject to a maximum of 0.7 times the span was also suggested to apportion steel reinforcement. The limiting moment arm method was restricted to walls without openings. For smaller depths the beneficial arch action may be absent. Rosenhaupt (C 14) has also suggested the equivalent moment arm method using 0.6 times the height of wall.

Rosenhaupt and Mueller (C 16) have suggested truss analogies to deal with openings in walls on beams, which are sketched in Fig. 2.4. These analogies are more useful at the post cracking stages than as a measure of stress distribution

at the elastic stage. The truss analogy was supplemented by the vierendeel girder analogy by Rosenhaupt, Beresford and Blakey (C 15) and Rosenhaupt and Sokal (C 17) in dealing with prestressed walls on beams and continuous beam-wall systems with openings.

Carter and Smith (C 3) have suggested an equivalent strut concept in multistoreyed frames (Fig. 2.4) and a method of computing the diagonal stiffness of each panel after establishing the lengths of contact between masonry wall and frame is given, supplemented by the concept of equivalent effective widths of compression struts as in the case of the familiar effective width of T-beams.

The above analogies have a distinct role to play at the designer's desk. These analogies are the intermediate tools which he uses for progressing in his design, while he is waiting for more accurate methods of analysis or design charts to develop. As in the case of T-beams, these concepts, once generated, have a tendency to persist for ever by their very simplicity and familiarity. Even when more accurate methods of analysis emerge, the code generating bodies have a tendency to modify the associated empirical formulae, rather than suggest the use of realistic methods of analysis. The penalty for simplicity is usually paid in the form of increased safety factors.

2.1.4. Failure Prediction of Walls on Beams

Whereas one can quote literally a thousand references for ultimate strength in shear, bending, torsion of beams, ultimate strengths of columns and ultimate strength of concrete slabs, the ultimate strength of walls on beams seems to be a neglected area of research.

Prasada Rao and Mallick (C 12) discuss several simple methods of computing the ultimate strength of walls on beams, without openings.

The simplest approach is to use an equivalent lever arm of 0.75 to 0.85 times the depth and multiply the same by the area of reinforcement and yield strength of steel to obtain the ultimate moment. This approach is valid for tension failures only. Alternatively the familiar Whitney's theory could be used.

Walls on beams failing in shear have a crack pattern at ultimate loads as shown in Fig. 2.5. Neglecting the effect of the small amounts of tensile reinforcement in the supporting beams, the parameters chosen are the cohesion 'c' of the concrete and its angle of internal friction ϕ. The ultimate load carrying capacity is then given by

$$W_u = \frac{2 F_c}{\cos \beta \, (\tan \beta + \tan \phi)} \qquad (2.8)$$

where F_c is the cohesive force along the inclined plane. Assuming that the failure of concrete under combined stresses could be expressed by Mohr's theory of failure, the values of ϕ and c are given by the equations

$$\tan \phi = \frac{f'c - f't}{2\sqrt{f'c.f't}} \quad \text{and} \quad c = \frac{\sqrt{f'c.f't}}{2} \tag{2.9}$$

and β is as shown in Fig. 2.5.

This approach was originally used in concrete deep beams which was extended by Prasada Rao and Mallick as shown below. The transformed section would be as shown in Fig.2.5.c in which m is the ratio of elastic moduli of concrete and brick masonry. For this modified section it can be easily shown that

$$F_c = \frac{c\ b}{\sin \beta} \left| \frac{D-d_1}{m} + d_1 \right| \tag{2.10}$$

Substituting in equation (2.8), we find that

$$W_u = \frac{2\ c\ b \left| \frac{D-d_1}{m} + d_1 \right|}{\sin \beta .\ \cos \beta\ (\tan \beta + \tan \phi)} \tag{2.11}$$

The above method is applicable to two point loading of beams of depth/span ratio of 2/3 or more. The presence of openings etc., complicates the formulation of a general theory of failure and the ultimate capacity of walls on beams

cannot be deemed to have been satisfactorily solved.

2.2. Experimental Investigations

There are four avenues open to a structural engineer to obtain knowledge on the behaviour of structural components under loads. At one extreme is a closed form solution in mathematical symbols, which could then be used irrespective of the actual numerical values of geometry and loads on the physical system. This is ideally the best solution but the mathematics soon becomes intractable for real life systems with their infinite variations. At the other extreme is the prototype and model tests in the laboratory, which are generally costly and time-consuming with little lee-way for extrapolation. However, an overall picture of the behaviour is best obtained through this process, when nothing else is available to start with. In between are the mathematical and physical analogues and simulation studies. In the earlier pages previous work on the mathematical modelling and physical analogues were reviewed. The experimental investigations on brickwork and walls on beams by earlier investigators are briefly reviewed in the following pages.

2.2.1. Studies on Brickwork

It is generally agreed that compression, modulus of rupture and absorption tests on bricks are adequate to

quantify the behaviour of bricks. However, the same may not be said about the basic tests on mortar. There is no agreement among the codes of various countries on the size and nature of mortar specimens for tension, compression and bond tests. Furthermore, there is very little standardisation or information on the elastic constants E and ν for bricks and mortar. Most research workers have obtained the constitutive laws and strengths of brickwork from arbitrary tests of their own. These are reviewed herein. Forty years back it was found that the height/thickness ratio of compression specimens of brickwork is important. In order to obtain data on the effect of varying the strengths of brick and mortar very large number of tests have been conducted in Building Research Station, England (A 14) on brickwork piers, 9 in. square and 36 in. height. Some of the results obtained are shown in Fig. 2.6. The strengths have been plotted as proportions of the strength corresponding to a mortar of 2000 psi compressive strength. The relationships shown for three strengths of bricks are of interest in indicating that the effect of increasing the mortar strength, in terms of the resulting brickwork, is not proportional.

The Structural Clay Products Institute (SCPI) (C 20), has recommended that brickwork prism tests be conducted on specimens not less than 12 inches in height and shall have a height/thickness ratio of not less than 2 and not more than 5.

Hilsdorf (A 4) did an interesting study to analyse the failure mechanism of masonry piers. From the observation of brick failure mode, it was concluded that failure of masonry in compression is initiated by vertical cracking or splitting of the bricks. The tests were made on five layers of bricks. Numerous strain measurements at the surface of several bricks were made. Differences in lateral strains of mortars and bricks resulted in triaxial compression in mortar and biaxial tension in bricks. Flexural stresses were also observed in the above tests (Fig. 2.7). The resulting triaxial stress state in the mortar and bricks were analysed theoretically and a failure criterion was proposed. However, the failure predictions were not quite satisfactory. One observation that this triaxial state of compression, that a masonry mortar exhibits, is the main reason for the compressive strength of brickwork exceeding that of the mortar, is of interest. Later on it will be shown that finite element simulation of brickwork brings out all these results and some more, in a simple and elegant manner.

The Structural Clay Products Institute (C 20) recommends that E for brickwork be taken as 1000 f'm where f'm is the 28 day compressive strength of brick prisms of h/t ratio of 5. The shear modulus is predicted to be 400 f'm. These values are used in the brickwork simulation studies of this

thesis. However, no mention is made of the Poisson's ratio and most research workers have assumed this to be zero.

Benjamin and Williams (C 1) report an interesting brick couplet test where the mortar joints can be given orientations of 0, 45, 60, 75, 90, 105, 120, 135 and 150 degrees from a reference plane. In the latter half of the tests the load is compressive and otherwise tensile. The prediction of shear strength is reported to be reliable.

Smith and Carter (A 12) question the assumption that the shear strength be expressed as a combination of bond strength and internal friction between brick and mortar, usually expressed in the form

$$f_s' = f_{bs} + \mu f_c \tag{2.12}$$

where f_s' = ultimate shear strength of brickwork

f_{bs} = bond shear strength between mortar and brick

f_c = normal compressive stress

and μ = coefficient of friction.

Through a finite element simulation study followed by experimental investigations, they suggest that the observed shear failure is mixed up with local tension failures in mortar. This is of interest in identifying failures. Mention may be made of the diametral tests on brickwork cylindrical specimens and the appreciating stress-strain curve

(increasing tangent modulus) recorded by some investigators on brickwork.

2.2.2. Tests on Walls on Beams

Experimental investigations on walls on beams gained momentum from the tests of Wood (C 21). Tests on 9 in. brick-walls without supporting beams showed that even these unsupported walls can resist large vertical loads. When R.C. beams were used with brick and cavity walls, tension concentration in the supporting beam was noticed. Cavity walls with door and window openings were also tested. Wood warned however that this deep girder action did not apply to loading at beam level unless tensile connectors could be placed between wall and beam. He suggested that some light reinforcement may be necessary in the walls to take care of shrinkage and also continuity in the case of continuous walls. Ultimate loads could not be reached in these tests.

Improving over the scope of tests envisaged by Wood, Rosenhaupt and his associates (C 13, C 14, C 15, C 16, C 17) have studied the behaviour of continuous wall-beams, effects of various types of openings with and without stiffening concrete frames surrounding the openings and the effect of prestressing the brickwork, besides the effects of foundation settlement. There is a wealth of information recorded from these costly tests which are best read in the original. Such

of these findings which influenced the development of this thesis work are summarised herein. Reinforcement in the supporting beams was found to have secondary effect, except when flexural failure modes controlled as in the case of shallow walls or strong brickwork. On the other hand shear, bond and vertical compressive stresses in deep walls had a controlling influence. Inclusion of horizontal and vertical concrete ties in the beam-wall system had significant effect on the internal stress distribution. The introduction of openings created a truss action in the masonry and a vierendeel girder action at the openings. The openings definitely weakened the structure but the strength could be compensated by prestressing or the introduction of vertical concrete ties. While testing continuous composite walls it was found that positive moments existed even over the middle supports. The reaction distribution was found to be of great importance. The failure modes peculiar to such walls and beams are the crushing of masonry over the supports and effects of shear in the masonry diaphragm.

The next important series of tests were by Burhouse (C 2) wherein reinforced concrete beams and encased joists were used for the supporting beam. Previous investigators had found the separation of the foundation beam and the brickwall at the mid span in the case of simply supported walls on beams. Burhouse introduced a building-paper joint in this region even when preparing the test beams. Crushing of brickwork was

identified to be the predominant mode of failure and strong brickwork is advocated if the beneficial arching action in walls on beams is to be fully utilised. It was also found that composite action in walls on beams will not take place without the transfer of shear stress across the interface between the components, nor without transfer of shear stress across the mortar joints between courses of brickwork. Encased joists pick up more moment than reinforced concrete supporting beams. In shallow beams the progressive slipping of the wall on beam was significant, showing the need for adequate connection between the walls and the beam. It is interesting to note that Wood had advocated the use of tensile connectors between the brickwalls and their supporting beams (C 21).

Prasada Rao and Mallick (C 12) conducted tests on walls on beams paying particular attention to their ultimate strengths and associated theoretical modelling has already been reviewed. Large number of strain measurements in the vertical and horizontal directions yielded interesting information on the strain distribution in such walls on beams.

2.2.3. Studies on Infilled Frames

Even though infilled frames, shear walls and planar frame-wall systems are beyond the scope of this thesis, lateral loading on walls on beams will have to be considered at some stage and such of those investigations which have reference to

brick infills are briefly reviewed to complete the picture of structural interaction between walls and their supporting beams or surrounding frames. Hinkley (C 7) reports on the behaviour of prestressed brickwork supported by steel beams and subjected to lateral loading. The rigidity of the wall and the load shared by the wall were considered. The work of Thomas (A 14) in testing infilled frames with lateral loading using a specially built testing frame must be mentioned. Lack of homogeneity and isotropy in a brick infill complicated the study. The stiffness of the structure changed as the load increased. Tensile cracking of the infill led to marked redistributions in interface stresses, almost invariably moving the centres of pressures away from the corners. Indeed in many cases the concept of a single diagonal equivalent strut ceases to be valid after cracking of the infill. A major contribution to the composite stiffness and strengths will be through imposed modes of deformation of the surrounding frame. Mainstone (C 8) continued the experiments concentrating his attention on the bounding frame. Benjamin and Williams (C 1) tested brickwalls with and without bounding frames and concentrated on the loa- lateral deflection at the top in the elastic, inelastic and post-ultimate stages. Predictions based on average shearing strength and average shearing modulus were proposed. Without bounding frames the plain brickwall could not be relied upon to resist shearing loads. The errors caused by scale modelling

adopted in the tests were not significant. Mallick and Savern (B 11) used the finite element method to predict the points of separation between frame and infill. Slip between frame and infill were also considered. The computed and observed stiffness values had close agreement. Earthquake forces were also dealt with.

The most thorough experimental investigations on plane frames with infill including multistorey and multibay frames has been reported by Fiorato, Sozen and Gomble (C 6). One eighth scale models were used. The resistance mechanism and failure modes were studied in multi-bay, multi-storeyed frame-wall composites and related to geometric and material properties. It was observed that the interaction of the frame-wall results in a response which is significantly different from that of the frame acting alone. The frame-wall composite is considerably stiffer and stronger than the sum of the individual responses of the frame and wall. By ignoring the interaction of the frame and wall in design, a significant portion of the strength of the system is wasted. What is more important however is that the critical-points in the frame-wall composite are not the same as in the frame. Thus certain critical sections may be inadvertently neglected in the design of real-life frame-wall systems. Oakleston (C 10) conducted prototype tests on an actual building and had indicated the trends reported above as early as 1955. The load-response

characteristics of the three storeyed, single bay frame with and without brick infill is reproduced in Fig. 2.8, which speaks volumes on the nature and magnitudes of interaction neglected by designers. Further investigations of interest are those of Murthy and Hendry (C 9) and Sahlin and Hollers (C 18) which are however concerned with load bearing walls.

It is however noticed that reinforced brickwork, which is a logical follower of reinforced concrete has not been given adequate attention in the above studies. Simple reinforced brickwork beams and slabs have been given some attention, and the work done by Sridhar Rao and Parghi (C 11) is of significant interest for Indian engineers, especially in Gangetic Plains.

2.2.4 Shear Connectors in Walls on Beams

Loads coming directly on the supporting beams through the floors impose a tensile stress field in walls on beams, which may result in separation between walls and their supporting beams and it has been suggested that tensile connectors should be used (C 21). An interesting investigation by Ramesh et.al. (C 24) has shown that tensile connectors in the form of stirrups are helpful in increasing the load-carrying capacity of walls on beams, provided they are spaced closer towards supports and they have adequate bond length in the

maximum moment regions. However horizontal cracking between the wall and beam could not be prevented, even though the widening of such cracks could be successfully arrested. Furthermore the saving in steel affected through such incomplete interaction has been found to be in the order of 20 percent. Thus tensile loadings on walls on beams are found to be critical. In the prototype structural frame, the self weight of the wall is compressive in nature and a portion of load from top floor gets transmitted to the top of the wall as compressive load. The load transmitted from the floor directly to the supporting beam must be dealt with separately, as interaction between brick walls and their supporting beams are rather incomplete even with tensile connectors.

2.3 Review of Codes of Practice

It must be said first that there are no codes of practice which deal specifically with the problem of walls on beams or frame-wall composites. Gross and Dikkers (C 19) have drawn a comparison of the allowable stresses as given in various international codes on load bearing brickwork. In Table 2.1, average compressive stresses for load-bearing non-reinforced brickwalls are listed. These were computed for bricks having compressive strengths of 6000 psi and 1Pc : 1/2 L : $4\frac{1}{2}$ S mortar and are based on basic compressive strengths given in various codes and associated reduction

factors for eccentricity and slenderness ratio. A critical examination of the tabulated values indicate that there is very little agreement between the experts ofthe world in interpreting the behaviour of eccentrically loaded brickwork, which is much simpler than walls on beams. Wood and Simms (C 23) have suggested a tentative design method for 'Composite Action of Heavily Loaded Brick Panel Walls', which indicates a possible trend for design.

- - -

TABLE 2.1

ALLOWABLE COMPRESSIVE STRESSES FOR NON-REINFORCED BRICK MASONRY, PSI (6000 PSI BRICK AND 1 pc: 1/2L : 4 1/2S MORTAR)

Standard	Basic comp. stress psi	h/t = 10			h/t = 15			h/t = 20		
		e/t=0	e/t=1/6	e/t=1/3	e/t=0	e/t=1/6	e/t=1/3	e/t=0	e/t=1/6	e/t=1/3
Germany	428	428	214	107	180	0	0	73	0	0
Britain	344	289	175	83	217	124	52	165	83	28
Canada	292	292	219	163	257	178	111	225	137	61
Switzerland	568	511	340	80	455	284	11	398	222	0
U.S.A.	475	437	300	128	376	266	104	304	199	76

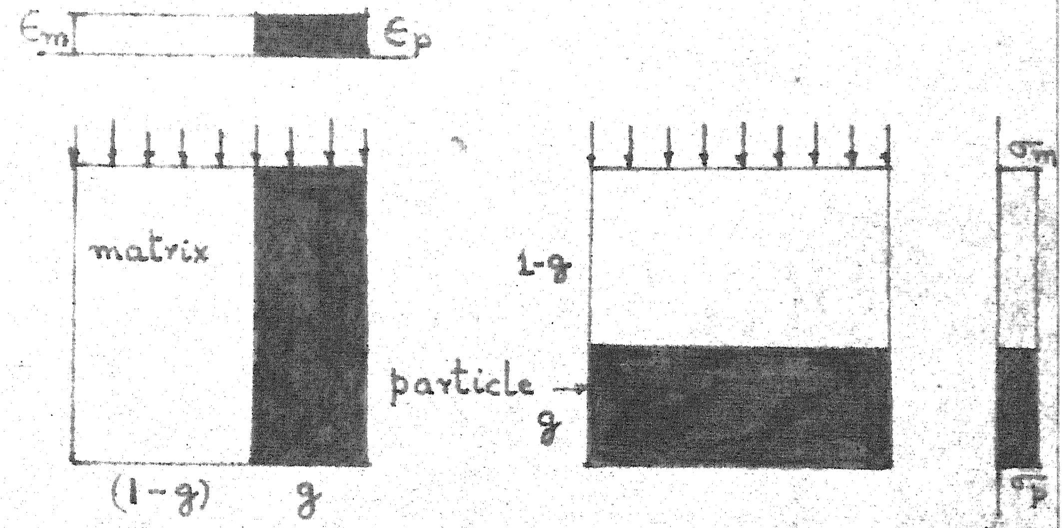


FIG. 2.1. ELEMENTARY MODELS - TWO PHASE MEDIA

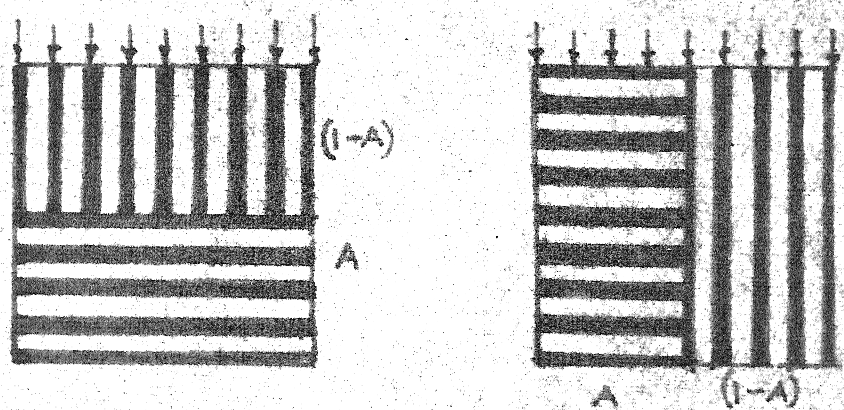


FIG. 2.2. COMBINATION MODELS - TWO PHASE MEDIA.

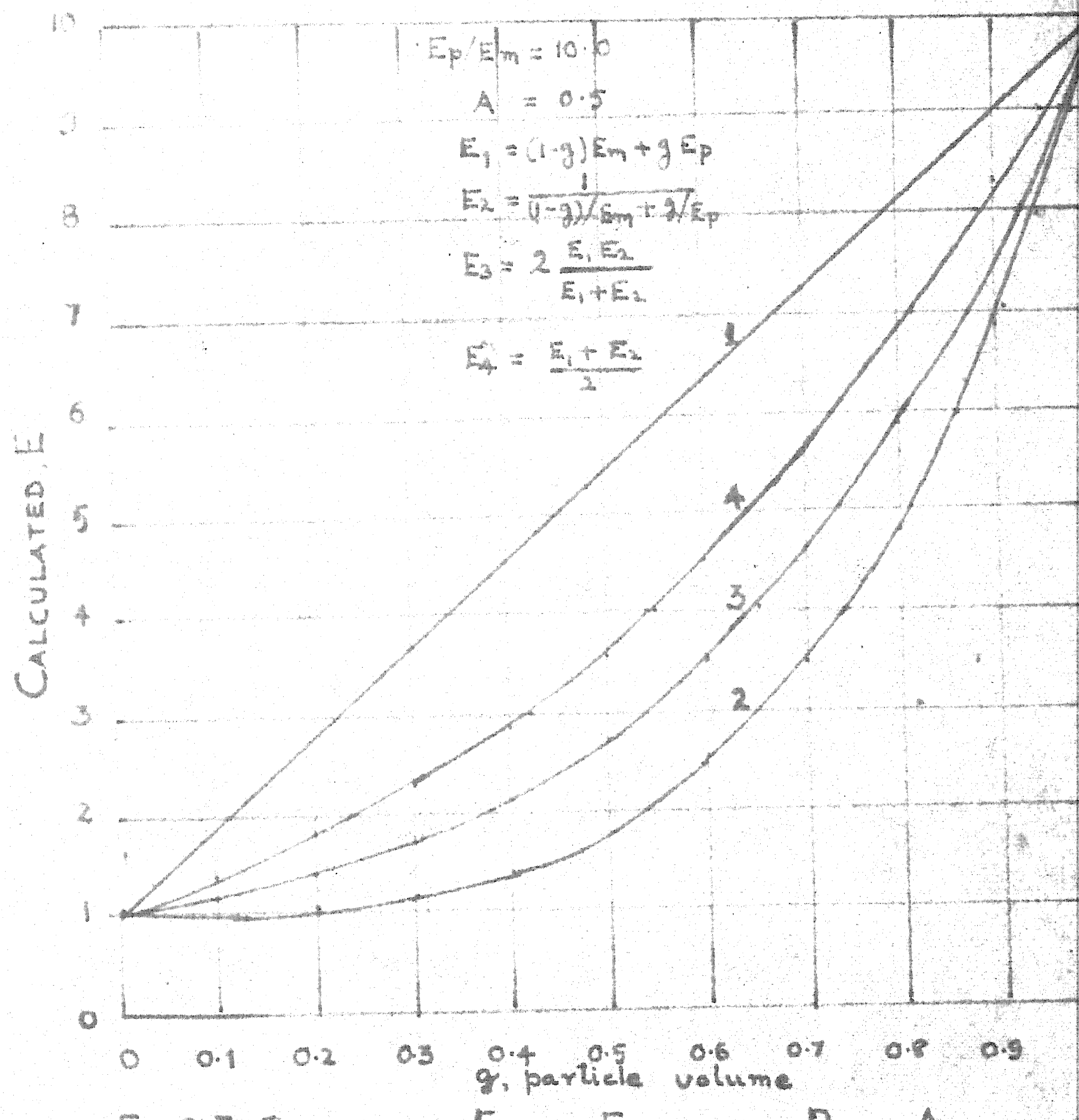


Fig. 2.3. Calculated E as a Function of Phase Arrangement and Fractional Volume of Particles

Ref

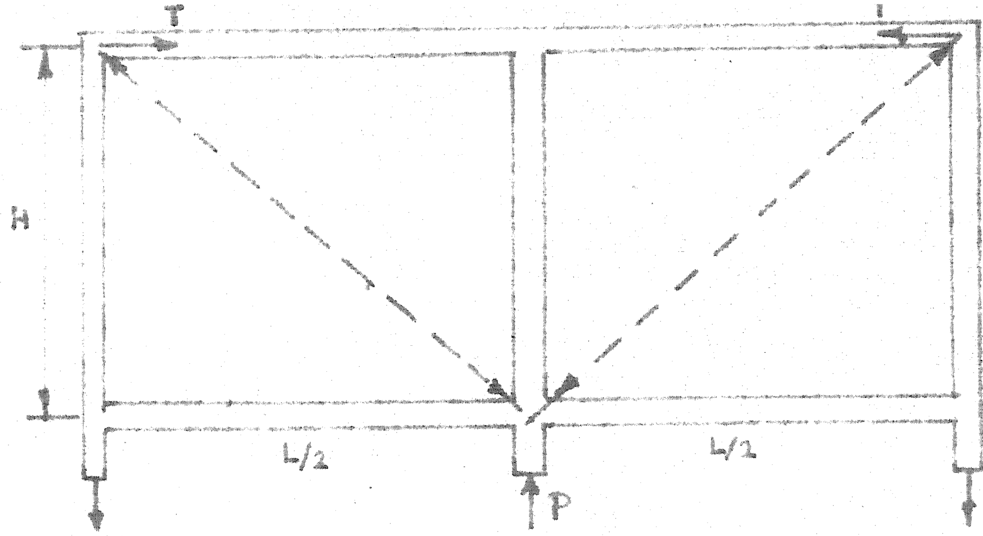


Fig. 2.4(a). Truss Analogy for Infilled Frames

$T = PL/4 \times \frac{1}{H}$

Fig. 2.4(b)

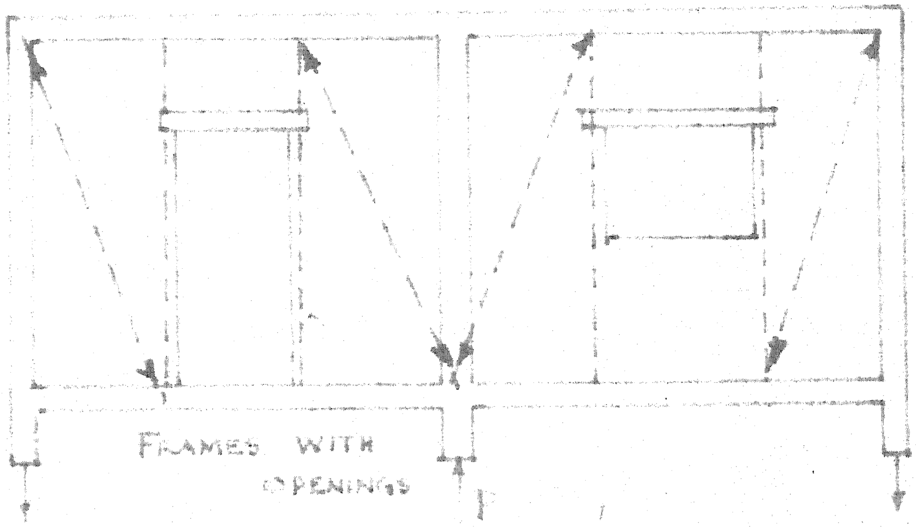
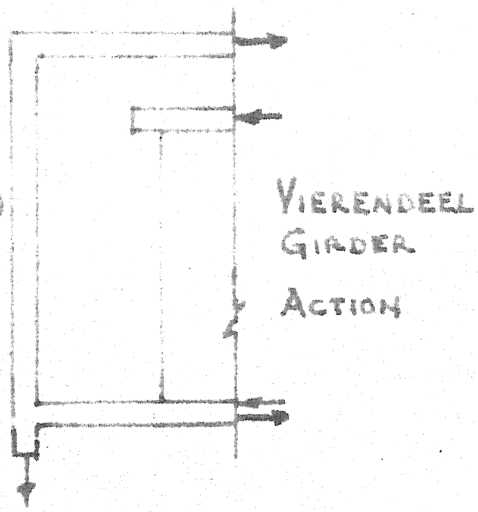


Fig. 2.4(c)



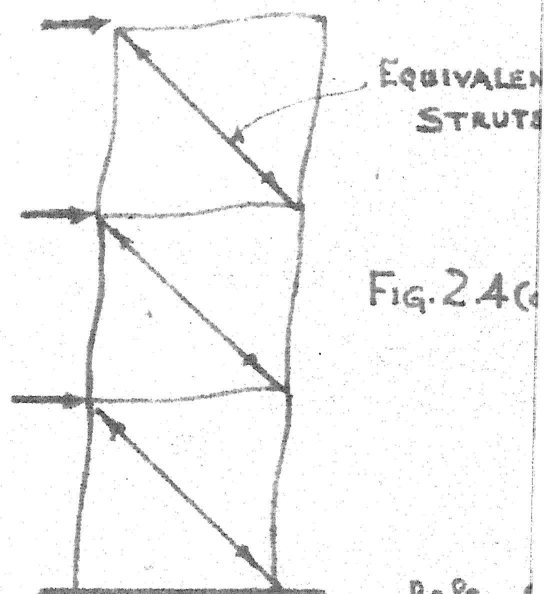


Fig. 2.4(

Refs.,

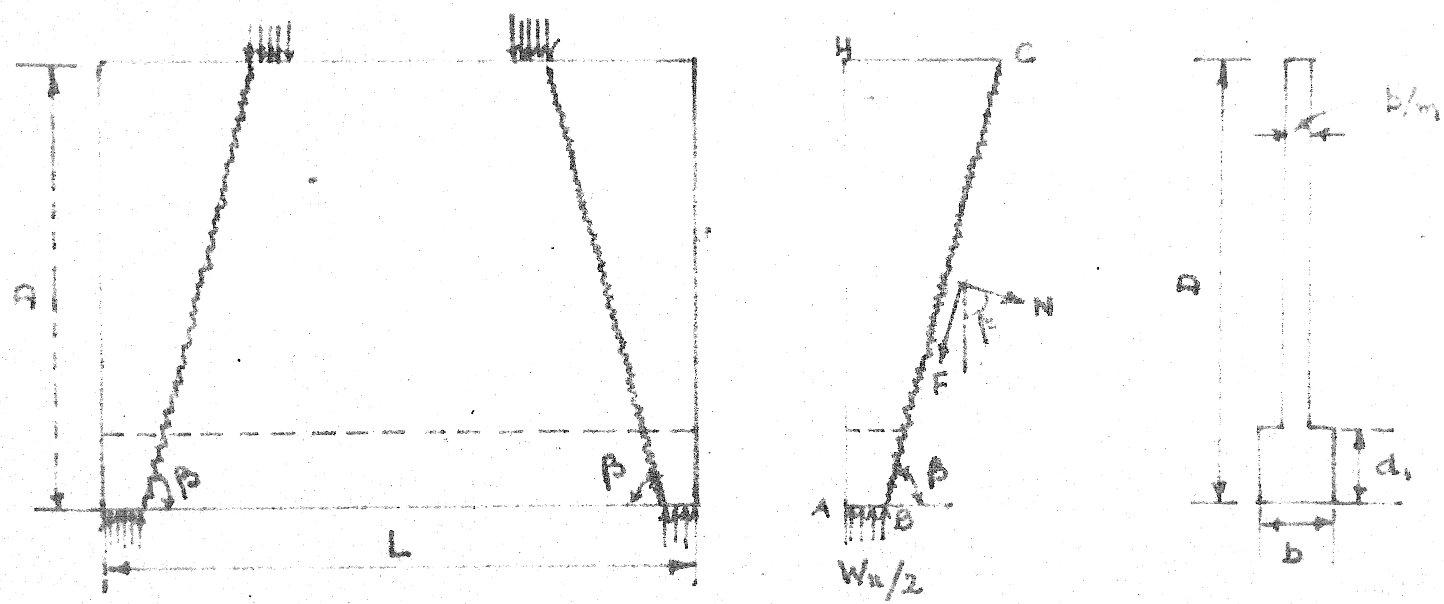


Fig. 2.5. Shear Failure of Brickwalls on R.C. Beams

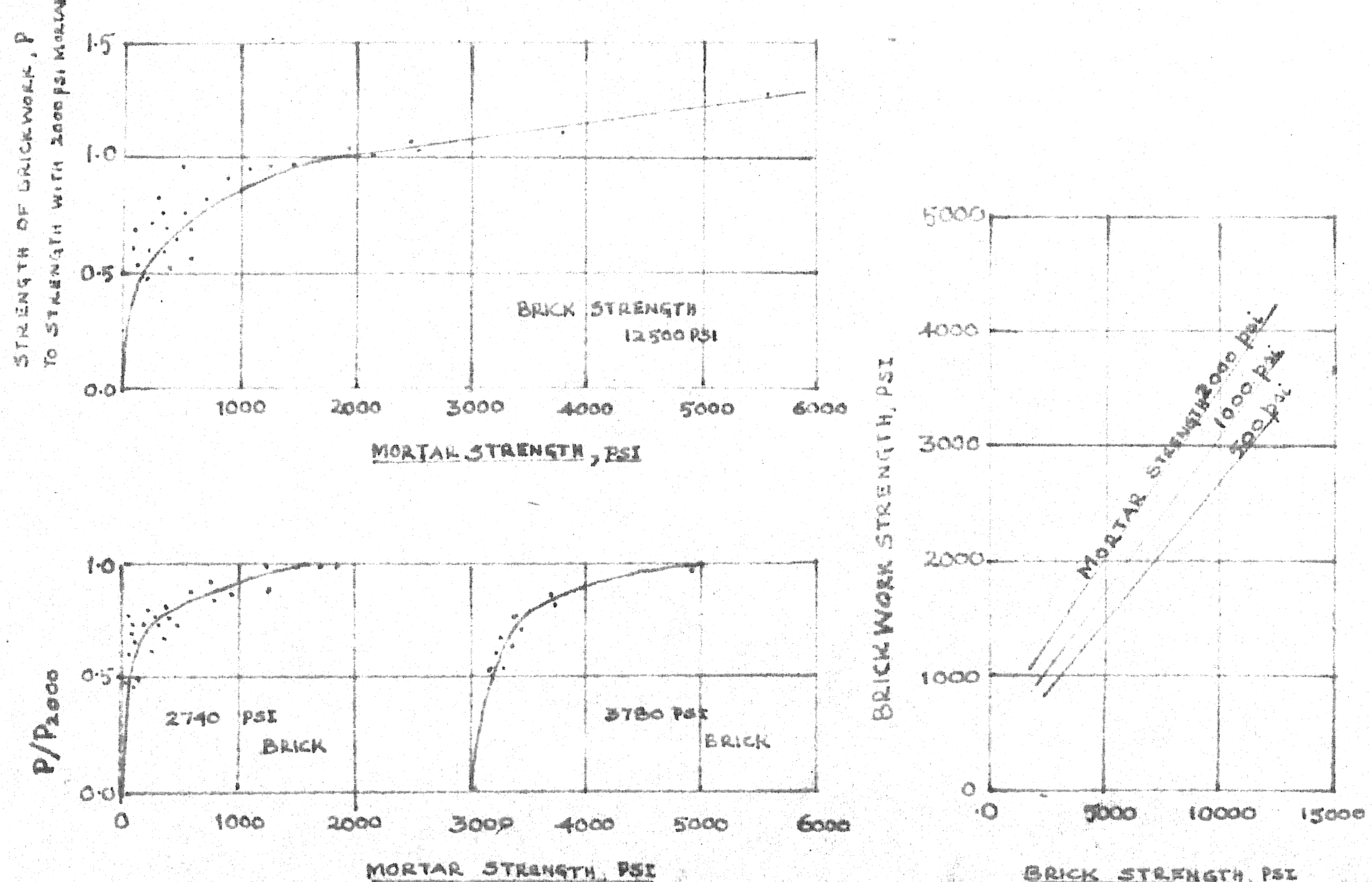


FIG. 2.6. STRENGTH OF BRICKWORK RELATED TO STRENGTH OF BRICK AND MORTAR

REF: A 11

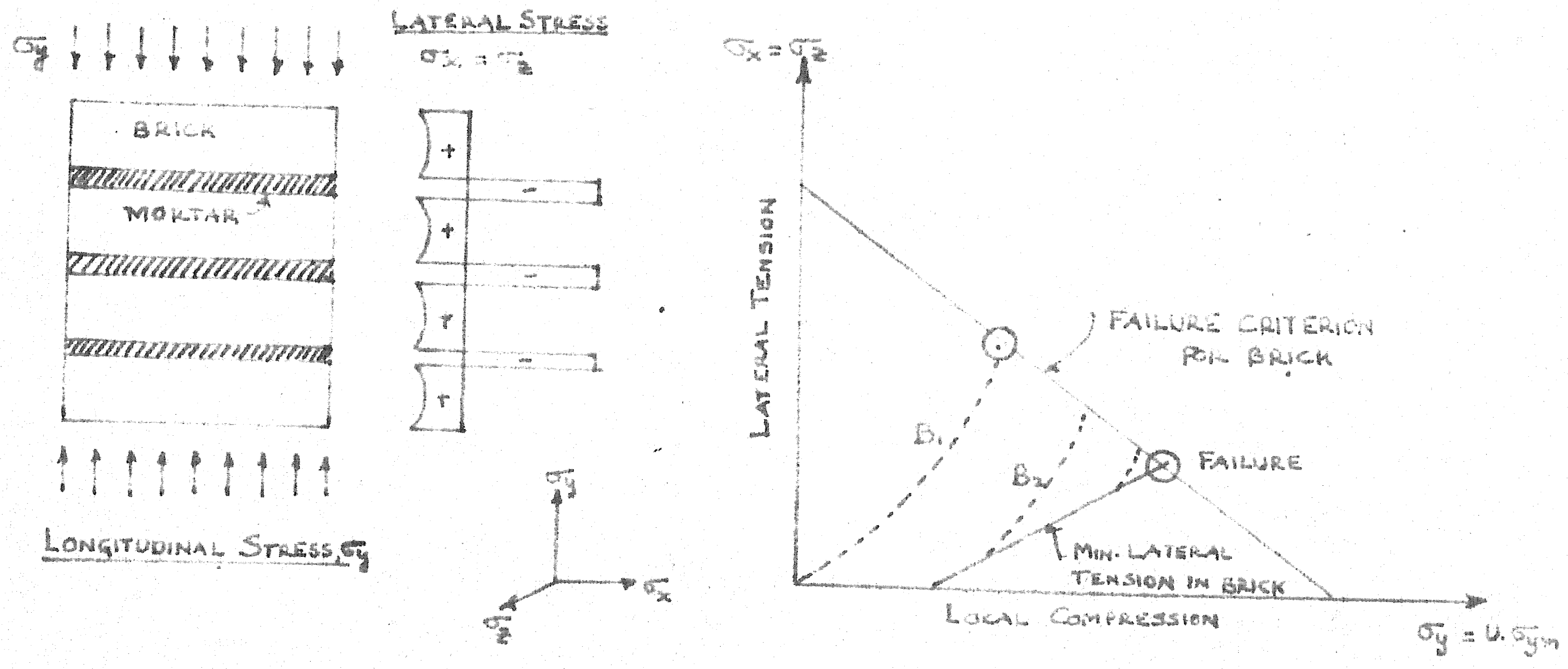


FIG. 2.7. IDEALISED STRESS DISTRIBUTION IN MASONRY UNITS.

Ref: A4

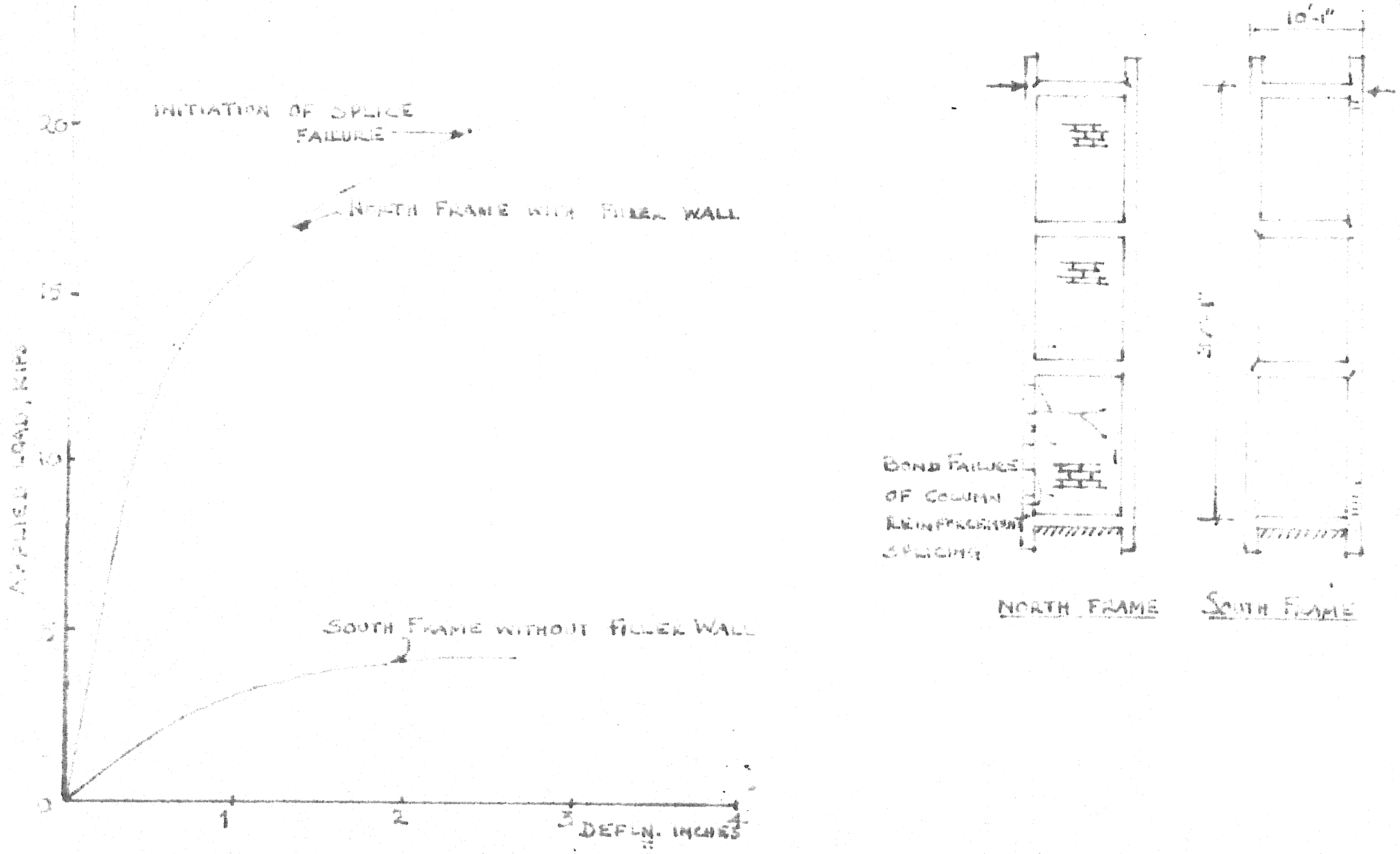


FIG. 2.8. EFFECT OF FILLER WALLS IN TESTS OF FULL-SIZE 3-STOREY FRAMES

Ref: C10

CHAPTER III

OBJECT AND SCOPE OF PRESENT INVESTIGATION

The tremendous strides made in the past two decades in the field of structural analysis and design has enabled engineers to progress from the study of individual structural components to the study of structural interaction between various members in composite construction. That there is considerable reserve in strength in the case of brick-walls interacting with their supporting beams has been established in several experimental studies undertaken in various parts of the world. Attempts to place the problem of walls on beams on a rational theoretical foundation has met with limited success, mainly due to assumptions of homogeneity and isotropy at the material level and difficulties met with in analysing plane stress problems in non-homogeneous media with various sizes and location of openings. Furthermore only the elastic, crack-free stage has been given attention, while the crack-limited state and the post-cracking behaviour of walls on beams are of significant interest. Even the ultimate strength of walls on beams has not been satisfactorily computed due to the uncorrelated manner in which test programmes have been initiated by various investigators.

At the material level the strength of the constituent materials have been reported in detail, whereas the elastic

constants E and ν have been given scant attention. At the structural level, the height-span ratios, size and location of openings, mortar strength, type of brickwork, cavity walls, amount of reinforcement in the supporting beams, effects of prestressing on the sides of openings, effects of settlement in continuous walls on beams, effects of lateral and vertical loads, effects of loads on top of the wall and effects of loads directly transmitted to the supporting beam and the influence of tensile connectors between walls and their supporting beams have all been studied through elaborate experiments and yet comprehensive design recommendations have not emerged from these studies. It is the author's contention that material and structural parameters of walls on beams have not been satisfactorily established mainly due to poor attention given to theoretical studies.

At the material level, brickwork is obviously orthotropic to start with and becomes anisotropic due to cracking and local compression failures under biaxial stress states. These local failures, under progressive loading, introduce zones of nonhomogeneity besides the obvious nonhomogeneity of openings in walls and the brickwall interacting with the reinforced concrete beam. Even yielding of steel reinforcement introduces nonhomogeneity in local zones. Thus anisotropy should have been given basic attention and nonhomogeneity should have been considered at various levels. It has been

shown that anisotropy can be dealt with on the basis of axes of symmetry or planes of symmetry, wherein the essential elastic constants increase to four in number, as against the basic constants E and ν for plane stress problems in isotropic media. Adequate test programmes should have been undertaken to evaluate these constants for the wide variety of bricks and mortars used all over the world. It has been shown earlier that vertical splitting of brick masonry under uniaxial, uniform, compressive loading has not been explained satisfactorily. Moreover the strength of brickwork has not been correlated with the strengths of bricks and mortar used therein. Neither has there been a satisfactory failure theory for brickwork under biaxial states of stress. It is thus evident that the behaviour of brickwork itself is rather complex and requires studies at a rather fundamental level.

As for nonhomogeneity, at various stages of loading, of walls on beams it is clear that the use of conventional theory of elasticity results in undue mathematical complications, whereas the relatively modern finite element methods of analysis could tackle the problem in a rather simple and elegant manner. Since the region under study is divided into subdomains and the stiffness of individual domains are evaluated separately, it is obvious that nonhomogeneity at structural level could be introduced by assigning appropriate elastic constants of either isotropic or anisotropic nature to

various elements before and after they become critical. Furthermore random numbers could be generated and random sets of material properties obtained from laboratory tests (E_1, ν_1, E_2, ν_2 and G_{12}) could be introduced for purposes of probabilistic methods of analysis.

Moreover the very same finite element methods of analysis can be used at micro-mechanical level to evaluate the elastic constants and stress-strain curves for composite materials such as concrete and brickwork. These studies may form the basis for obtaining realistic stresses and deformations in nonhomogeneous media and for the evaluation of the nature and magnitude of interaction stresses in such media.

Besides the initial crack-free elastic state and the final collapse state the intermediate inelastic transition region in the load-response spectrum, deserves special attention since modern designs are based on stress, deformation, cracking and/or collapse limited states. Thus it becomes necessary to simulate the idiosyncracies peculiar to walls on beams, such as local cracking, bond-slip, local compression failures, yielding of reinforcement in the supporting beams in the process of analysing the load-deformation characteristics of walls on beams. Once a satisfactory correlation is obtained between computer simulation studies and a few laboratory tests, extensive simulation studies could be undertaken for further parameter studies. The necessary background for

such simulation studies have already been established in the case of concrete deep girders by earlier research workers. Once the basic material parameters for brickwork are identified, the above process can be repeated for brickwalls interacting with reinforced concrete beams.

This behaviouristic approach coupled with the generation of theoretical finite element models of the problem helps in simulating the true behaviour and structural interaction of masonry walls and reinforced concrete beams to a fairly micro-level, rather than the gross hypothesis of deep beam theory or semi-macro approach of framework analogy model. The gap between micro behaviour of constituent materials (brick, mortar, concrete and steel) and macro behaviour of the structural system can be identified. It is felt that more advances in materials engineering is necessary to utilise the full scope of the finite element method in simulating the true behaviour of the composite structural systems.

While the scope of the above nature is attractive, the development of a satisfactory simulation programme is a rather formidable task subject to the memory capacity, time limitations and physical facilities available to the author. The author proceeded from the beginning assuming that unlimited capacity is available and as and when constraints were faced, the simulation procedure was sequentially limited to progressively

simpler problems. However maximum information has been generated at each successful simulation stage.

Finally it has been the aim of the author to generate simple design rules from simulation studies at material and structural levels for brickwalls with openings, interacting with their reinforced concrete supporting beams.

In this thesis, the load-response characteristics of brick walls interacting with reinforced concrete supporting beams are investigated. Nonhomogeneity, anisotropy and randomness in material properties, which are inherent in brick walls, are given importance. Elastic, post-cracking and ultimate stages of loading are covered. The performance of this ubiquitous structural component is first studied through laboratory tests and finite element methods are used to simulate the load-response characteristics observed in the laboratory.

Earlier investigators have pointed that proper attention should be given to the material constants in finite element applications. Of particular interest are the elastic constants and the compressive and the tensile strengths of materials under use. Besides laboratory tests, the author uses the theory of composite materials and computer simulation at micromechanical level for the evaluation of these constants. Furthermore, the stresses and deformations which arise due to local nonhomogeneity, anisotropy and random arrangement of the various phases of a composite media, such as concrete,

are studied in some detail to obtain information on the basic behaviour of composite media.

Several investigators have identified the height-span ratios, size and location of openings and effects of mortar used in brickwork as important variables in the study of walls on beams. In addition, the nature of loading, tensile and compressive, is given special attention in the present experimental investigations.

Besides the elastic response, progressive cracking, local compression failures and yielding of reinforcement have been traced in the laboratory investigations and also in the computer simulation studies. Modifications in the structural stiffnesses of critical elements are attempted using concepts of transverse isotropy and elasto-plastic stiffnesses, which are currently in use in various finite element applications.

Finally, incremental-iterative methods are used for stress redistribution from the critical elements so as to obtain realistic load-response curves. It is felt that this study could be the basis for future probabilistic methods of analysis and optimal design of walls on beams.

– – –

CHAPTER IV

FINITE ELEMENT METHODS OF ANALYSIS

The scope and limitations of conventional theory of elasticity in analysing the behaviour of walls on beams have already been reviewed. Problems of anisotropy and non-homogeneity which are inherent in brickwork could not be tackled in particular. The author's search for a satisfactory procedure which can accommodate these aspects, resulted in the discovery that the finite element procedure can accommodate these variables in a natural manner. Furthermore, the power of this method to deal with plane stress, plane strain, axisymmetric, plate, shell and three dimensional problems at the elastic, inelastic and near-ultimate stages of load-response in equilibrium, stability and propagation problems has clearly emerged from the large volume of literature already published (B 2, B 3, B 4, B 19). However, the finite element procedure is a product of computer era and requires the use of high-speed computers with large memory banks. In this chapter the scope of the finite element procedure in solving the problem of stresses and deformations in walls on beams will be gradually unfolded and results obtained from the application of this procedure at various levels of sophistication will be presented.

4.1. Introduction

The basic concept of finite element method (FEM) is

not new. In structural analysis the behaviour of each member such as column or beam is separately established first, and then these are combined to yield a solution to the whole frame. Similarly a continuum can be idealised as an assemblage of several discrete elements whose individual behaviour can be approximated and systematically combined to yield a solution for the whole system. In this process the amount of data handled increases in proportion with the sub-divisions made, and big computers are required to mainpulate these large volumes of data. The power of the finite element methods, to a large extent, is dependent on the capabilities of the computer used. " The method can be systematically programmed to accommodate such complex and difficult problems as non-homogeneous materials, nonlinear stress-strain behaviour and complicated boundary conditions" (B 4). Finite element applications in solid mechanics are usually solved using one of the following three approaches: the displacement formulation, the force formulation and the mixed formulation. Displacements, stresses and combined displacements and stresses respectively are assumed as primary unknown in the above methods. In this thesis the displacement approach is used throughout.

a) <u>Convergence</u>

Once a numerical procedure is in use, the question of convergence must be discussed. In the displacement method we start with an assumed displacement field. In general the

states of constant strain. Although all the three conditions must be met with to prove convergence in the general case, practical results for elements that satisfy the third requirement only appear to converge satisfactorily.

The constant strain triangular element (CST element) used throughout this study, has been known to yield satisfactory results in large number of practical applications (B 19) of similar kind as investigated in this thesis and an explicit study of convergence was not undertaken. Greater attention has been given to engineering application of the FEM procedure.

4.1.1. Essential Features of the Displacement Method

The development of the constant strain triangular element displacement model and the displacement method, which are already documented (A 2, B 4, B 19) are reviewed herein so that the modifications proposed by the author to accommodate the problem of walls on beams could be substantiated later in a brief manner.

a) The Displacement Method

Basically the method begins in the division of the continuum under study into sub-domains or 'elements'. On the basis of an assumed displacement field, stiffness of each element is determined in terms of the displacements at the nodes' of the elements. The stiffness characteristics of the

whole structure is then constructed by assembling the stiffness of the individual elements. Finally the following force-displacement relation under static loading is obtained:

$$\underline{P} = \underline{\underline{K}}\ \underline{U} \tag{4.1}$$

Where $\underline{P}$ = vector of known nodal forces

$\underline{\underline{K}}$ = known stiffness matrix of the structure

$\underline{U}$ = vector of unknown nodal displacements.

The displacement boundary conditions can easily be incorporated by either deleting the appropriate degrees of freedom or through the assignment of very large values for the associated diagonal terms in the stiffness matrix with suitable adjustments in the load vector. The latter process has been used in this thesis. Procedures for solving large matrices of banded nature are of primary importance in finite-element procedures. The tridiagonalisation procedure discussed by Zienkiewicz (B 19) has been used in the present work.

After solving for displacements $\underline{U}$, the strains $\underline{\epsilon}$ and stresses $\underline{\sigma}$ within each element can be computed from the relations:

$$\underline{\epsilon} = \underline{\underline{B}}\ \underline{u}$$

$$\tag{4.2}$$

and $$\underline{\sigma} = \underline{\underline{D}}\ \underline{\underline{B}}\ \underline{u}$$

where $\underline{u}$ = nodal displacement vector of each element.

and $\underline{\underline{D}}$ and $\underline{\underline{B}}$ are matrices to be defined later.

b) Selection of Displacement Field for CST Element

The simplest representation of displacement field within a triangular element in plane problems of elasticity is given by two linear polynomials (B 19, A 2) :

$$
\begin{aligned}
u &= \alpha_1 + \alpha_2 x + \alpha_3 y \\
v &= \alpha_4 + \alpha_5 x + \alpha_6 y
\end{aligned}
\tag{4.3}
$$

The six constants 'α_i' can be solved for in terms of nodal displacements (Fig. 4.1), from the equations,

$$
\begin{aligned}
u_i &= \alpha_1 + \alpha_2 x_i + \alpha_3 y_i \\
u_j &= \alpha_1 + \alpha_2 x_j + \alpha_3 y_j \\
u_m &= \alpha_1 + \alpha_2 x_m + \alpha_3 y_m \\
v_i &= \alpha_4 + \alpha_5 x_i + \alpha_6 y_i \\
v_j &= \alpha_4 + \alpha_5 x_j + \alpha_6 y_j \\
v_m &= \alpha_4 + \alpha_5 x_m + \alpha_6 y_m
\end{aligned}
\tag{4.4}
$$

Substituting the appropriate values for constants d_i, we obtain

$$
u = \frac{1}{2\Delta} \left| (a_i+b_i x+c_i y)u_i + (a_j+b_j x+c_j y)u_j + (a_m+b_m x+c_m y)u_m \right|
\tag{4.5a}
$$

$$v = \frac{1}{2\Delta} \left| (a_i+b_ix+c_iy)v_i + (a_j+b_jx+c_jy)v_j + (a_m+b_mx+c_my)v_m \right| \tag{4.5b}$$

in which

$$\begin{aligned} a_i &= x_j\, y_m - x_m\, y_j \\ b_i &= y_j - y_m \\ c_i &= x_m - x_j \end{aligned} \tag{4.5c}$$

with the other terms obtained through cyclic permutations of the subscripts in the order i, j, m and using

$$2\Delta = 2 \text{ (area of triangle i j m)} = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{vmatrix} \tag{4.5d}$$

If the coordinates are taken from the centroid of each element then

$$x_i + x_m + x_j = y_i + y_j + y_m = 0 \text{ and } a_i = \frac{2\Delta}{3} = a_j = a_m ,$$

which provides a simplification in computations.

c) <u>Strains</u>: The total strain at any point within the element can be defined by its three components

$$\underline{\epsilon} = (\epsilon_x, \epsilon_y, \epsilon_{xy})^T = \left(\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^T \tag{4.6}$$

Carrying out the differentiations on u and v obtained in Equations 4.5, we find that,

$$\underline{\epsilon} = \frac{1}{2\Delta} \underbrace{\begin{bmatrix} b_i & 0 & b_j & 0 & b_m & 0 \\ 0 & c_i & 0 & c_j & 0 & c_m \\ c_i & b_i & c_j & b_j & c_m & b_m \end{bmatrix}}_{\underline{\underline{B}}\ 3\text{x}6} \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_m \\ v_m \end{Bmatrix} \qquad (4.7)$$

wherein the $\underline{\underline{B}}$ referred to in Eqn. (4.2), now gets defined. Since $\underline{\underline{B}}$ is independent of the space variables, strains are constant throughout the element.

d) <u>The Material Property Matrix</u>

Besides equilibrium and compatibility the constitutive laws are to be satisfied in structural analysis. In the finite element procedure the constitutive relationship between stresses and strains can be conveniently represented through the material property matrix, $\underline{\underline{D}}$, which takes the familiar form for isotropic materials;

$$\underline{\underline{D}} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \qquad (4.8)$$

and stresses may be obtained from displacements using Eqn.(4.2).

The modification of this material property matrix to accommodate cracking, orthotropy etc., will be discussed later.

e) <u>Transformation Rules</u>

The following transformation rules are valid to rotate the stresses and strains from the global coordinate system to any other cartesian system (x', y') at an angle α relative to the original global system (Fig. 4.1);

$$\underline{\sigma}' = \underline{\underline{T}}_{\sigma}\,\underline{\sigma}$$

$$\underline{\epsilon}' = \underline{\underline{T}}_{\epsilon}\,\underline{\epsilon} \qquad (4.9)$$

where

$$\underline{\underline{T}}_{\sigma} = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2-s^2 \end{bmatrix} \qquad (4.10)$$

$$\underline{\underline{T}}_{\epsilon} = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & +2cs & c^2-s^2 \end{bmatrix} \qquad (4.11)$$

$c = \cos\alpha$

$s = \sin\alpha$

Of particular interest is the application of the above transformation rules to the material property matrix $\underline{\underline{D}}'_{\alpha}$ of say,

a cracked element from which the following global material property matrix $\underline{\underline{D}}_{\alpha}$ is obtained:

$$\underline{\underline{D}}_{\alpha} = \underline{\underline{T}}_{\sigma}^{-1} \underline{\underline{D}}_{\alpha}' \underline{\underline{T}}_{\epsilon} \tag{4.12}$$

It can be shown that,

$$\underline{\underline{T}}_{\sigma}^{-1} = \underline{\underline{T}}_{\epsilon}^{T} \tag{4.13}$$

Hence

$$\underline{\underline{D}}_{\alpha} = \underline{\underline{T}}_{\epsilon}^{T} \underline{\underline{D}}_{\alpha}' \underline{\underline{T}}_{\epsilon} \tag{4.14}$$

f) <u>The Element Stiffness Matrix</u>

Let $\underline{F}^{e}$ be the nodal force vector equilibrating the stresses in an element. Let $\underline{\delta}^{e}$ be the nodal displacement vector, consisting of the degrees of freedom u and v at each node (Fig. 4.1). For the triangular element the $\underline{F}^{e}$ and $\underline{\delta}^{e}$ vectors are as below:

$$\underline{F}^{e} = \begin{Bmatrix} F_{xi} \\ F_{yi} \\ F_{xj} \\ F_{yj} \\ F_{xm} \\ F_{ym} \end{Bmatrix} \quad \text{and} \quad \underline{\delta}^{e} = \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_m \\ v_m \end{Bmatrix} \tag{4.15}$$

To make the nodal forces statically equivalent to the actual stresses in an element, the simplest procedure (B 19) is to impose an arbitrary virtual displacement at each node and to equate the external and internal work done by the various forces and stresses during that displacement. Let such a virtual displacement be $d\underline{\delta}^e$ at the nodes. This results in strains within the element equal to

$$d\,\underline{\epsilon} = \underline{\underline{B}}\,(d\,\underline{\delta}^e) \tag{4.16}$$

The internal work done per unit volume by the stresses is

$$(d\,\underline{\epsilon})^T\,\underline{\sigma} = (d\,\underline{\delta}^e)^T\,\underline{\underline{B}}^T\,\underline{\sigma} \tag{4.17}$$

$$= (d\,\underline{\delta}^e)^T\,\underline{\underline{B}}^T\,\underline{\underline{D}}\,\underline{\underline{B}}\,\underline{\delta}^e$$

The external work done is simply evaluated as

$$(d\,\underline{\delta}^e)^T \cdot \underline{F}^e \tag{4.18}$$

Equating the external work done with the internal work obtained by integrating over the volume of the element.

$$(d\,\underline{\delta}^e)^T\,\underline{F}^e = (d\,\underline{\delta}^e)^T\,(\int_{Vol}\underline{\underline{B}}^T\,\underline{\underline{D}}\,\underline{\underline{B}}\,d\,Vol)\,\underline{\delta}^e \tag{4.19}$$

Since this relation is valid for any value of virtual displacement, the equality of multipliers is required and hence

$$\underline{F}^{e} = \left(\int_{Vol} \underline{\underline{B}}^{T} \underline{\underline{D}} \; \underline{\underline{B}} \, d(Vol)\right) \underline{\delta}^{e} \qquad (4.20)$$

which is typical of displacement formulation and the element stiffness matrix is recognised as

$$\underline{\underline{K}}^{e} = \int_{Vol} \underline{\underline{B}}^{T} \underline{\underline{D}} \; \underline{\underline{B}} \, d(Vol) \qquad (4.21)$$

In the case of CST element the matrices under the integral sign are all constants, thus considerably saving the time consumed in numerical integration procedure, particularly when the thickness of the element is kept constant.

g) Boundary Conditions

If displacements at the boundary are specified, it poses no problem in the displacement formulation. If distributed loading per unit area is specified, a loading term on the nodes of the element which has the boundary force will now have to be added. This can be obtained from virtual work considerations; but this is rarely done explicitly. Often by physical reasoning the boundary loading can be replaced by simple concentrated loads on the boundary nodes from statical considerations.

h) Assesment of Accuracy

If the exact solution is in fact that of a uniform stress field, the finite element solution using CST elements will coincide exactly with the closed form elasticity solution,

irrespective of element sub-division. In the case of varying stress fields, only an approximate solution is obtained and an averaging procedure at each internal node helps in evaluating nodal stresses. However, the errors persist at the external nodes, which can be reduced only through a finer mesh division.

The over-view presented as above brings out the salient features of the FEM/CST plane stress solution which characterises the behaviour of walls on beams of isotropic, homogeneous materials, with unlimited strength so that limitations of cracking, local crushing etc., do not influence the problem. For the initial response of the structure, the above procedure will be adequate; however the complete load-response characteristic of the composite walls on beams from zero load to failure requires additional modifications which are detailed as below.

4.1.2. Non-homogeneity and Anisotropy

Conventional linear theory of elasticity is based on the assumptions, that materials are isotropic and can be characterised by two essential elastic constants E and γ and that the regions under study are homogeneous. The brickwall interacting with the reinforced concrete beam is obviously non-homogeneous. Brickwork is obviously orthotropic. Thus the theory of elasticity solutions based on assumptions of homogeneity

and isotropy are of academic interest only, particularly when the nature of errors introduced by these assumptions still remain undefined. Furthermore materials are mostly non-homogeneous and anisotropic at the micro-mechanical level and the basic load-response characteristics of modern composite materials and materials such as concrete and brickwork will be understood better if these assumptions are released. In this thesis such a basic study has been given due attention.

a) Anisotropy

On the subject of anisotropy the work of Lekhnitskii (B 9) is of significant interest. It is well known that the generalised Hooke's Law for a continuous material, requires 36 elastic constants in the equation

$$\sigma_i = C_{ij}\, \epsilon_j \; ; \; i, j = 1, 2, 3 \,\ldots\ldots\, 6 \tag{4.22}$$

The reduction of the material property matrix C_{ij}, begins with the assumption of existence of a strain-energy density function (elastic potential)

$$U = U(\epsilon_j) \tag{4.23}$$

with the property that

$$\frac{\partial U}{\partial \epsilon_j} = \sigma_j \tag{4.24}$$

Such an elastic potential exists when the variation of the body under deformation occurs isothermally or adiabatically. It will further be assumed that the variations for deformation occur isothermally; then it can be shown that

$$C_{ij} = C_{ji} \tag{4.25}$$

which implies that the C matrix is symmetric with 21 elastic constants. If there are no changes when the x, y and z directions are reversed (Orthotropic symmetry) these constants reduce to nine. In plane stress problems we have the required material property matrix having four independent elastic constants

$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{44} \end{bmatrix} \tag{4.26}$$

Now, the physical constants obtained from laboratory tests, say on brickwork will be related to C_{ij}. If E_{ℓ} and $\gamma_{\ell t}$ are obtained from tests parallel to layers and E_t and $\gamma_{t\ell}$ are obtained from tests perpendicular to layers and assuming that one can successfully conduct pure shear tests obtaining $G_{\ell t} = G_{t\ell}$, there are apparently five measured values which can be related to the elements σ_{ij}, following the method used by Calcote (A 2).

If σ_1 be the only non-zero stress, then

$$\sigma_1 = c_{11} \; \epsilon_1 + c_{12} \, \epsilon_t$$

$$0 = c_{12} \; \epsilon_1 + c_{22} \, \epsilon_t$$

from which

$$\epsilon_1 = \frac{\sigma_1 \quad c_{22}}{c_{11} \, c_{22} - c_{12}^2} \qquad (4.27)$$

$$\epsilon_t = \frac{\sigma_1 \; c_{12}}{c_{11} \, c_{22} - c_{12}^2}$$

If the modulus E_1 and ν_{1t} are defined as below, one has

$$E_1 = \frac{\sigma_1}{\epsilon_1} = c_{11} - \frac{c_{12}^2}{c_{22}} \qquad (4.28)$$

$$\nu_{1t} = - \frac{\epsilon t}{\epsilon_1} = \frac{c_{12}}{c_{22}}$$

A repetition of this process with σ_t as the only non-zero stress shows that

$$E_t = \frac{\sigma_t}{E_t} = c_{22} - \frac{c_{12}^2}{c_{11}} \qquad (4.29)$$

and

$$\nu_{t1} = - \frac{\epsilon_1}{\epsilon_t} = \frac{c_{12}}{c_{11}}$$

It can be easily seen that

$$\frac{E_1}{E_t} = \frac{\nu_{1t}}{\nu_{t1}} \tag{4.30}$$

Finally assuming that τ_{1t} to be the only non-zero stress,

$$G_{1t} = \frac{\tau_{1t}}{\gamma_{1t}} = c_{44} \tag{4.31}$$

Thus any three elastic constants in Eqn. (4.30) and G_{1t} constitute an equivalent set of four independent elastic constants for layered orthotropic material.

A more basic understanding of the elastic constants for composite materials such as brickwork and concrete, particularly after cracking, is required and the treatment by Lekhnitskii (B 9) will be followed. " If the internal composition of a material possesses symmetry of any kind then symmetry can be observed in its elastic properties" . Planes of elastic symmetry have already been considered and nine elastic constants in the three dimensional problems and four in the plane stress problems have been defined. An axis of symmetry will now be defined. If there are sets of equivalent directions in a body which can be superimposed by a rotation through an angle $2\pi/n$ about a certain 'g' axis, then this axis is an axis of symmetry of order n. Assuming that the z axis coincides with the 'g' axis, Lekhnitskii shows that

for $n = 2, 3, 4$ and 6, the independent elastic constants are 13, 7, 7 and 5, respectively. Now considering a plane of isotropy which can be treated as possessing an axis of elastic symmetry 'g' of an infinitely large order ($n = \infty$), it can be shown that equations of generalised Hooke's law have the form

$$
\begin{aligned}
\epsilon_x &= a_{11}\,\sigma_x + a_{12}\,\sigma_y + a_{13}\,\sigma_z \\
\epsilon_y &= a_{12}\,\sigma_x + a_{11}\,\sigma_y + a_{13}\,\sigma_z \\
\epsilon_z &= a_{13}\,\sigma_x + a_{13}\,\sigma_y + a_{33}\,\sigma_z \\
\gamma_{vz} &= a_{44}\,\tau_{yz} \\
\gamma_{xz} &= a_{44}\,\tau_{xz} \\
\gamma_{xy} &= 2\,(a_{11} - a_{12})\,\tau_{xy}
\end{aligned}
\tag{4.32}
$$

Once again indicating five independent elastic constants. Thus the axis of elastic symmetry of the sixth order is also the axis of symmetry of rotation. Introducing 'technical constants', one can write the above equations in the following form:

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) - \frac{\nu'}{E'}\sigma_z$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) - \frac{\nu'}{E'}\sigma_z$$

$$\epsilon_z = -\frac{\nu'}{E'}(\sigma_x - \sigma_y) + \frac{1}{E'}\sigma_z \qquad (4.33)$$

$$\gamma_{yz} = \frac{1}{G'}\tau_{yz}$$

$$\gamma_{xz} = \frac{1}{G'}\tau_{xz}$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} = \frac{2(1+\nu)}{E}\tau_{xy}$$

Here E and E' are the Young's Moduli (for tension-compression) with respect to directions in the plane of isotropy and perpendicular to it; ν is the Poisson coefficient which characterises the transverse reduction in the plane of isotropy for tension in the same plane and ν' is the Poission coefficient which characterises the transverse reduction in the plane of isotropy for tension in a direction normal to it. G' and G are the shear moduli for planes normal and parallel to the plane of symmetry. Such a body is defined by Lekhnitskii to be transversely isotropic. Materials which are cracked are better understood using transverse isotropy and hence the above details. One notices in particular the identification of G and G' and ν and ν'. Now one proceeds to define

appropriate elastic constants for plane stress problems, using the above development due to Lekhnitskii (B 9).

For plane stress problems x, y directions are used customarily to represent horizontal and vertical directions and using E_1, ν_1 (G_1 is related) for the horizontal layers of constant thickness and E_2, G_2 and ν_2 for the perpendicular planes (Fig. 4.2) one has the following relations:

$$
\begin{aligned}
\epsilon_x &= \frac{\sigma_x}{E_1} - \frac{\nu_2\,\sigma_y}{E_2} - \frac{\nu_1}{E_1}\,\sigma_z \\
\epsilon_y &= -\frac{\nu_2\,\sigma_x}{E_2} + \frac{\sigma_y}{E_2} - \frac{\nu_2\,\sigma_z}{E_2} \quad (\text{using } \nu_1 E_2 = \nu_2 E_1) \\
\epsilon_z &= -\frac{\nu_1\,\sigma_x}{E_1} - \frac{\nu_2}{E_2}\,\sigma_y + \frac{\sigma_z}{E_1} \\
\gamma_{yz} &= \frac{1}{G_2}\,\tau_{yz} \\
\gamma_{xy} &= \frac{1}{G_2}\,\tau_{xy} \\
\gamma_{xz} &= \frac{1}{G_1}\,\tau_{xz} = \frac{2(1 + \nu_1)\,\tau_{xz}}{E_1}
\end{aligned}
\tag{4.34}
$$

For plane stress problems σ_z, σ_{xz}, σ_{yz} are all zero, and hence the reduced set with four elastic constants is given below:

$$\epsilon_x = \frac{\sigma_x}{E_1} - \frac{\nu_2}{E_2} \sigma_y$$

$$\epsilon_y = - \frac{\nu_2 \quad \sigma_x}{E_2} + \frac{\sigma_y}{E_2} \qquad (4.35)$$

$$\gamma_{xy} = \frac{1}{G_2} \tau_{xy}$$

The inversion of the above relation gives

$$\sigma_x = \frac{E_1}{1- \nu_1 \nu_2} (\epsilon_x + \nu_2 \epsilon_y)$$

$$\sigma_y = \frac{E_2}{1 - \nu_1 \nu_2} (\nu_1 \epsilon_x + \epsilon_y) \qquad (4.36)$$

$$\tau_{xy} = G_2 \gamma_{xy}$$

Using the relations $\frac{E_1}{E_2} = n$ and $\frac{G_2}{E_2} = m$, one can rewrite,

$$\sigma_x = \frac{E_2 \cdot n}{(1-n \nu_2^2)} \epsilon_x + \frac{E_2 \cdot n \ \nu_2}{(1-n \nu_2^2)} \epsilon_y$$

$$\sigma_y = \frac{E_2 \cdot n. \nu_2}{(1-n \ \nu_2^2)} \epsilon_x + \frac{E_2}{(1-n \ \nu_2^2)} \epsilon_y \qquad (4.37)$$

$$\tau_{xy} = \frac{E_2 \cdot m \ (1-n \nu_2^2)}{(1-n \nu_2^2)} \gamma_{xy}$$

from which one concludes that the matrix, $\underline{\underline{D}}$, which relates the stresses and strains in plane stress problems in transversely isotropic media is of the form (B 19)

$$\underline{\underline{D}} = \frac{E_2}{(1-n\,\nu_2^2)} \begin{bmatrix} n & n\,\nu_2 & 0 \\ n\,\nu_2 & 1 & 0 \\ 0 & 0 & n(1-n\,\nu_2^2) \end{bmatrix} \qquad (4.38)$$

b) Nonhomogeneity

While the subject of anisotropy has been reasonably tackled, the subject of nonhomogeneity has not been satisfactorily solved. Retaining the assumption of isotropy, attempts have been made to introduce non-homogeneity in the form of either $G(x,y)$ or $E(x,y)$ in the elasticity equations (B 5, B 6) keeping ν as constant and even then satisfactory solutions could not be obtained. In particular the importance of ν in the elasticity problems has been emphasized by Golecki (B 6). A stress function approach for straight boundaries between two different media is discussed by Du Ching-Hua (B 5) and finite difference approximation has been indicated, but the Poisson's ratio has been kept constant. A cursory examination of these papers indicate that the general problem of nonhomogeneity is a difficult problem to solve in closed form. Coupled with anisotropy the problem becomes too very complicated. However, the finite element procedure discussed earlier

indicates that each element can be assigned arbitrary materi[al] properties and treated as orthotropic or transversely isotropic. The only difficulty introduced will be the computation of element stiffness matrix individually thus requiring large amounts of computer time. By a proper choice of element boundaries to coincide with the boundaries of various materials, interaction studies in non-homogeneous media can be undertaken. Random material property assignment becomes feasible through random number generation routines to study randomly arranged poly-phase media. The power of the finite element method in this direction has been fully exploited in this thesis and study of concrete like materials and brickwork has been render[ed] rather simple.

4.1.3. Steel Reinforcement

The influence of reinforcement in the supporting beams have been studied experimentally by Rosenhaupt (C 11) and Burhouse (C 2). The load-response characteristics of walls on beams at the post-cracking stages and also the failure modes are influenced by the quantity and location of reinforcement in the supporting beams. Thus the presence of reinforcement must be recognized in the finite element procedure. There are several avenues open for this purpose N_{go} and Scordelis (B 13) were the first to consider the presence of reinforcement in concrete beams including bond slip characteristics. They use

linkage elements to account for bond slip and separation due to cracking. On the otherhand Mikkola and Schnobrich (A 9), in their study of concrete shells, consider the steel-concrete composite shell material as having anisotropic properties due to the presence of steel reinforcement. The reinforced concrete itself is treated as a new material with the steel 'smeared in concrete', the actual location of reinforcement not being explicitly accounted for. On the same lines Yuzugulu and Schnobrich (B 17) studied the behaviour of reinforced concrete deep girders. Bond slip is neglected in these studies. The work of Ono and Mills (B 14) is of particular interest for this thesis, since the reinforcement is considered as linear element in a simple manner. The steel reinforcement is idealised as an assembly of one-dimensional bar elements along the boundary of the triangular elements. Bond between the reinforcement and concrete is assumed to be perfect upto failure. Since CST elements are used, the bar elements should be used as truss members in order to achieve displacement compatibility on the boundary of concrete and steel elements. In reality the reinforcement does resist loads in the transverse direction, and the stiffness of the reinforcement in this sense cannot be neglected. Therefore, in this thesis the element stiffness matrix of the steel element $\underline{\underline{K}}_R$ is taken as,

$$\underline{\underline{K}}_R = \begin{bmatrix} A_s E_s/L & 0 & -A_s E_s/L & 0 \\ 0 & 12E_s I_s/L^3 & 0 & -12E_s I_s/L^3 \\ -A_s E_s/L & 0 & A_s E_s/L & 0 \\ 0 & -12E_s I_s/L^3 & 0 & 12E_s I_s/L^3 \end{bmatrix} \quad (4.39)$$

where A_s, E_s, I_s and L are the cross sectional area, the modulus of elasticity, the moment of inertia and the length of steel bar, respectively. The stiffness contribution from the bar elements is added directly to the structure stiffness matrix in its proper location. During progressive loading of the structure, when an element of steel yields, its stiffness contribution is eliminated and nodal forces equivalent to the yield stress can be inserted at the ends of the element. It is clear that this approach which simplifies the computer analysis, because of the omission of bond slip and separation, will result in a stiffer and stronger simulated structure and the results obtained in this study must be interpreted in this light.

4.1.4. Modification for Cracking

The load at which various elements crack can be found using the maximum stress theory and checking whether the maximum principal stress exceeds the tensile strength of materia

defined by appropriate codes of practice. For this purpose the computation of principal stresses and directions becomes important. As a consequence of cracking the following modifications are found to be necessary.

1) The stiffness of the cracked element requires modifications.

2) The excess stresses over and above the maximum strength of the material in each cracked element require redistribution.

3) The width of crack could be kept track by the introduction of new nodes in the finite element programme which is computationally quite difficult.

4) Stress concentration effects due to cracking are to be included.

and 5) Eventual closing, reopening of cracks and subsequent formation of cracks in other directions in the same element may have to be considered, particularly in the case of cyclic loading.

Cracking is an important phenomenon in bi-strength materials such as concrete and brickwork, and deserves attention if a realistic load-response spectrum is to be obtained for structures, which will be designed on the basis of crack limited state. Author's search in the literature revealed that several schools of thought exist, regarding this problem. Schnobrich and his associates (A 9, B 17) have considered the

first two of the modifications listed above in deep girders and shells with a good measure of success. Zienkiewicz (B 19) and Mills (B 14) have solved many problems modifying the stiffness only. Scordelis (B 13), Kaldjian (B 4), Schnobrick (B 17) and Mallick (B 11) have used link elements to trace separation and cracking. The difficulty is in knowing a priori where the link elements are to be introduced. While attempts have been made to study the stress concentration effects due to cracking and also crack propagation (B 20) the thesis of Snyasi Raju (B 15) shows the classical nature of study of stress concentrations which are not suitable for the study of randomly oriented discontinuous cracks in brittle materials such as brickwork. The work of Goodman, Taylor and Brekka (B 7) in dealing with jointed rock deserves special mention. Stiffness matrix of a joint element (rectangular) is derived in this work. Joint cohesion and friction are automatically included in this analysis. This element could be used instead of link elements at nodes to evaluate separation and cracking; however it increases computational effort and furthermore the locations of cracking are not known a priori. Considering the above state of the art in modelling the cracking phenomenon the following approach has been chosen by the author:

1) The cracking load and direction are identified using maximum stress theory;

2) The stiffness of the cracked element is modified using the transverse isotropy concept developed earlier and assigning near-zero values for E_2, ν_2 and G_2 perpendicular to crack in Eqn. (4.37) and retaining E_1, ν_1 and G_1 values parallel to crack as in the uncracked state. A rotational transformation of the form derived in Eqn. (4.12) is required where α is the direction of crack from the x-axis;

3) The excess stresses will be redistributed as shown below. When an element cracks the material property matrix defined as above, can be used to compute the resistance of the cracked element as

$$\underline{\sigma}_{cr} = \underline{\underline{D}}_{cr} \; \underline{\epsilon} \qquad (4.40)$$

The difference between the previously attained stress $\underline{\sigma}$ and the new value of stress $\underline{\sigma}_{cr}$ is called the 'pseudo' or 'initial' stress, which is given by

$$\underline{\sigma}_i = \underline{\sigma} - \underline{\sigma}_{cr} = (\underline{\underline{D}} - \underline{\underline{D}}_{cr}) \; \underline{\epsilon} \qquad (4.41)$$

4) The above 'pseudo' stresses are converted into equivalent forces on the system (B 18, B 17) using the relation

$$\underline{P} = \int_V \underline{\underline{B}}^T \; \underline{\sigma}_i \; dv \qquad (4.42)$$

and the finite element solution is recycled until the $\underline{\sigma}_i$ values are negligible.

4.1.5. Modification for Openings

The presence of openings has always posed special problems in the theory of elasticity solution, since one has to deal with multiply-connected regions wherein additional requirements are to be satisfied other than the usual compatibility conditions. In the finite element displacement formulation these requirements are avoided and openings can be easily dealt with by arranging the elements to coincide with the boundary of the openings. Earlier investigations have shown the need for fine mesh near the openings (B 19). Partitioning techniques and sequential numbering of the elements become complicated in this procedure. However, a method of analysis successfully used in an earlier investigation (B 16) has shown that the openings can be treated as part of the full walls on beams with 1/40th of the thickness of the rest of the portion. If the thickness is less than this, ill-conditioning of the stiffness matrix has been noticed. Thus the introduction of openings becomes a relatively simple task.

4.1.6. Modification for Compression Failure

While cracking can be easily identified using the maximum stress theory, the failure of materials under more complicated biaxial states of stress has not been properly defined. In particular the problem of local compression failures in walls on beams deserves attention. For this purpose, one needs a

realistic failure theory. Desai (B 4) has shown that von-Mises criteria, Tresca criteria and Mohr-Coulomb theories have been applied with various degrees of success. Recent experimental investigations by Kuper, Hilsdor and Rusch (A 7) have helped in the proper definition of failure envelope for concrete under biaxial states of stress. Mikkola and Schnobrich (A 9) have presented an yield criterion to approximate the above experimental envelope using von-Mises yield criteria (Fig. 4.3). Yuzugullu (B 11) has shown that this criterion has worked quite satisfactorily for concrete deep girders. It is proposed to use von-Mises yield criteria in the present investigation for concrete and brickwork. However, the von-Mises yield criteria is based on elastic-perfectly plastic stress-strain relationship. Thus one ends up with the important question whether concrete and brickwork can be idealised to be elastic-perfectly plastic without introducing major sources of error. The material, concrete is investigated first before the study of brickwork.

The stress-strain curve for concrete has been modelled theoretically by several research workers, of which the Sargin's model has shown success in laboratory research on concrete members as verified by Ghosh (A 4). Sargin enumerated the following characteristic conditions which a $\sigma - \epsilon$ curve in compression always satisfies:

1) The curve passes through the origin

$$\sigma(\epsilon) = 0 \quad \text{at} \quad \epsilon = 0$$

2) The slope of the curve at the origin is equal to the initial tangent modulus of elasticity (by definition)

$$\frac{d \; \sigma(\epsilon)}{d\epsilon} = E_c \quad \text{at} \quad \epsilon = 0$$

3) The $\sigma - \epsilon$ curve has a peak at $\epsilon = \epsilon_o$

$$\frac{d \; \sigma(\epsilon)}{d\epsilon} = 0 \text{ at } \epsilon = \epsilon_o.$$

4) The coordinates of the peak are $(\epsilon_o, k_3 f'_c)$

$$\sigma(\epsilon) = k_3 f'_c \quad \text{at } \epsilon = \epsilon_o$$

5) The curve passes through an experimentally determined point (ϵ_a, σ_a) which is beyond the peak

$$\sigma(\epsilon) = \sigma_a \quad \text{at} \quad \epsilon = \epsilon_a, \quad \epsilon_a > \epsilon_o$$

Sargin postulated that an analytical expression representing the $\sigma - \epsilon$ relationship of concrete in compression should contain at least five parameters to satisfy the above five conditions and that the following expression satisfies the five given conditions;

$$\frac{\sigma}{f'_c} = k_3 \frac{Ax + (D-1)\,x^2}{1+(A-2)x + Dx^2} \qquad (4.43)$$

wherein $x = \epsilon/\epsilon_o$, $A = E_c\,\epsilon_o/k_3\,f'_c$, k_3 is the ratio of maximum stress to cylinder strength, ϵ_o is the strain corresponding to maximum stress and D is a parameter mainly affecting the descending branch.

From the laboratory tests conducted by the author on the concrete used in the supporting beams, the following values were obtained:

$$f'_c = 370 \text{ kg/cm}^2$$

$$E_c = 370000 \text{ kg/cm}^2$$

$$\epsilon_o = 0.002$$

There is adequate evidence to select k_3 as 0.85, but the descending branch of the stress-strain curve could not be recorded by the author and hence the empirical expression suggested by Ghosh (A 4) was used;

$D = 0.65 - 0.05\,f'_c$ where f'_c = maximum concrete stress in ksi.

$$D = 0.65 - 0.05 \times 5.260$$
$$= 0.387$$

Hence the final stress-strain relation was found to be

$$\sigma = 307.1 \frac{(1175\,\epsilon - 153250\,\epsilon^2)}{(1.0 + 175\,\epsilon + 96750\,\epsilon^2)} \qquad (4.44)$$

and the same is plotted in Fig. (4.4). Equivalent elasto-plastic stress strain relationship as per the method used by Yuzugullu (B 17) is also indicated in the same figure. The values of σ_o^c and ϵ_o^c now become important.

Defining $F(\sigma)$ as the function of yield surface, given by von-Mises yield condition and the associated flow rule for the plasticity of the uncracked concrete under biaxial stress, the yield surface $F(\sigma)$ is defined by;

$$F(\sigma) = \sigma_x^2 + \sigma_x \sigma_y + \sigma_y^2 + 3\,\tau_{xy}^2 - \sigma_o^2 = 0 \qquad (4.45)$$

where σ_o is the yield stress under uniaxial compression.

It should be remembered that in a biaxial state of stress the determination of the yielding stage is complex, since various combinations of stresses are the possible causes for yielding to start. All such combinations are expressed by the yield surface, Eqn. (4.45). If $F(\sigma) < 0$, at a particular material point, yielding has not occurred and elastic behaviour prevails. If $F(\sigma) = 0$, yielding is impending and $F(\sigma) > 0$, yielding has occurred and plastic behaviour prevails. Thus von-Mises failure can be easily identified.

The modification required in the finite element procedure to account for local yielding are more involved than those discussed earlier for cracking. Besides the yield surface, von Mises suggested a basic constitutive relation defining plastic strain increments in relation to the yield surface, known as flow rule. If $d\,\underline{\epsilon}_p$ denotes the increment of plastic strain then

$$d\,\underline{\epsilon}_p = \lambda \frac{\partial F}{\partial \underline{\sigma}}$$

and for any component, n; $d\,\underline{\epsilon}_{n,p} = \frac{\partial F}{\partial \sigma_n}$ (4.46)

In this λ is a proportionality constant as yet undetermined.

Total Stress-Strain Relation (B 19)

During an infinitesimal increment of stress, changes of strain are assumed to be divisible into elastic and plastic parts. Thus,

$$d\,\underline{\epsilon} = d\,\underline{\epsilon}_e + d\,\underline{\epsilon}_p \qquad (4.47)$$

The elastic strain increments are related to stress increments by a symmetric matrix of constants $\underline{\underline{D}}$ as usual. Thus we have

$$d\,\underline{\epsilon} = \underline{\underline{D}}^{-1}\, d\,\underline{\sigma} + \frac{\partial F}{\partial \underline{\sigma}} \qquad (4.48)$$

When plastic yielding occurs stresses are on the yield surface given by Eqn. (4.45). This equation is modified to

include a strain hardening factor k, and rewritten as

$$F(\underline{\sigma}\ ,\ k) = 0 \qquad (4.45a)$$

Differentiating this one can write therefore

$$\frac{\partial F}{\partial \sigma_1} \cdot d\sigma_1 + \frac{\partial F}{\partial \sigma_2}\, d\sigma_2 - \frac{\partial F}{\partial k} dk = 0 \qquad (4.49)$$

or

$$\left\{\frac{\partial F}{\partial \underline{\sigma}}\right\}^T d\,\underline{\sigma} + A\lambda = 0$$

in which

$$A = \frac{\partial F}{\partial k} \cdot dk\, \frac{1}{\lambda} \qquad (4.50)$$

Eqns. 4.48 and 4.49 can be written in a single symmetric matrix form as

$$\left\{\begin{array}{c} d\epsilon_1 \\ d\epsilon_2 \\ \vdots \\ \hline 0 \end{array}\right\} = \left[\begin{array}{ccc|c} & & & \frac{\partial F}{\partial \sigma_1} \\ & |D|^{-1} & & \frac{\partial F}{\partial \sigma_2} \\ & & & \vdots \\ \hline \frac{\partial F}{\partial \sigma_1} & \frac{\partial F}{\partial \sigma_2} & --- & A \end{array}\right] \left\{\begin{array}{c} d\sigma_1 \\ d\sigma_2 \\ \vdots \\ \hline \lambda \end{array}\right\} \qquad (4.51)$$

The indeterminate constant λ can be eliminated (B 19) and this results in an explicit expansion which determines the stress changes in terms of imposed strain changes with

$$d\,\underline{\sigma} = \underline{\underline{D}}^{*}_{ep}\, d\,\underline{\epsilon} \tag{4.52}$$

where

$$\underline{\underline{D}}^{*}_{\epsilon_p} = \underline{\underline{D}} - \underline{\underline{D}}\left(\frac{\partial F}{\partial \underline{\sigma}}\right)\left(\frac{\partial F}{\partial \underline{\sigma}}\right)^{T}\underline{\underline{D}}\left[A + \left(\frac{\partial F}{\partial \underline{\sigma}}\right)^{T}\underline{\underline{D}}\left(\frac{\partial F}{\partial \underline{\sigma}}\right)\right]^{-1} \tag{4.53}$$

The elasto-plastic matrix $\underline{\underline{D}}^{*}_{\epsilon_p}$ takes the place of the elasticity matrix $\underline{\underline{D}}$ in incremental analysis. It is positive definite, symmetric and is valid whether or not 'A' takes the zero value. For ideal plasticity, the value of A is simply zero. This incremental stress-strain relation is only valid from the instant when the stresses reach the yield surfaces, $F(\sigma) = 0$. If $F(\sigma) < 0$ purely elastic behaviour continues. Furthermore this relation is valid only for infinitesimal strain increases. For finite steps it is possible that stresses depart somewhat from the yield surface. To guard against this, stresses should be reduced to the yield condition after each iteration. Using the von-Mises yield criteria (Eqn.4.45), the elasto-plastic matrix specialises to

$$\underline{\underline{D}}_{ep} = \underline{\underline{D}} - \underline{\underline{D}}\,\underline{\psi}\,\underline{\psi}^{T}\underline{\underline{D}} \,/\, (\underline{\psi}^{T}\underline{\underline{D}}\,\underline{\psi}) \tag{4.54}$$

where

$$\underline{\psi}^{T} = \frac{1}{\sigma_o}\left[(\sigma_x - \frac{\sigma_y}{2}),\ (\sigma_y - \frac{\sigma_x}{2}),\ 3\,\tau_{xy} \right]$$

σ_c has been already been identified (Fig. 4.4) and $(\underline{\Psi}^T \underline{\underline{D}} \underline{\Psi})$ is nothing but a scaling factor.

4.1.7 Incremental, Iterative Method of Analysis

While Zeinkiewicz (B 19) has indicated the broad outline of attack for obtaining the complete load-response characteristics of a plane stress problem such as walls on beams, the work of Yuzugullu and Schnobrich (B 17) deserves special mention in applying the said procedure for concrete frame-wall system. The following description closely follows the work of the above two authors.

Initially the structure is uncracked and elastic, and conventional FEM procedure provides the required answers. By loading the structure is increments and through the use of an iterative procedure it is possible to extend the elastic solution into the study of propagation of cracks, plasticity of concrete and yielding of reinforcement. If slips between various surfaces, separation and friction between layers could be included the method of analysis would be more versatile; however the limitations of the IBM 7044 system used by the author precluded their incorporation in this study. When an element cracks or plasticity of any component material occurs, the released 'pseudo stresses' or the 'initial stresses' should be redistributed to the surrounding elements. Within one load

increment the distribution of initial stresses can be achieved using two alternatives, namely the variable stiffness and constant stiffness methods. Both methods have been used in this study and a complete description of the procedure is to be found in the book of Zeinkiewicz (B 19). Figures 4.5 and 4.6 illustrate schematically the procedures involved.

Effect of Cracking

The effect of cracking on the load displacement diagram is non-linear but the cracking process itself is elastic, that is the structure remains elastic before/after cracking. In the real behaviour of structure, cracking process continues gradually and the stiffness of the structure changes slowly. However, the path of the mathematical model is stepped because the loads are assigned in discrete steps. Large increments may indicate cracking of a number of elements of a time. In reality these should occur sequentially. The reduction of load increment thus becomes important. The author's work envisages 20% of the first crack load as incremental load.

Plasticity

The redistribution of 'pseudo stresses' due to plasticity requires some explanation. The following steps are necessary and should be viewed in conjunction with Fig.4.7.

1. Apply the load increment and determine the elastic stress and strain increments ($\Delta\underline{\sigma}$, $\Delta\underline{\epsilon}$).

2. Add the increments to the existing values of stress and strain at the beginning of the increment. Thus current totals obtained are ($\underline{\sigma}_1$, $\underline{\epsilon}_1$).

3. Evaluate $F(\sigma)$ from the equation 4.45:

   a) If $F(\sigma) < 0$, the element is elastic, no pseudo stresses are released and proceed to the next element.

   b) If $F(\sigma) > 0$ and also at the beginning of the increment $F(\sigma) = 0$, set $\Delta\underline{\sigma}_1 = \Delta\underline{\sigma}$, $\Delta\underline{\epsilon}_1 = \Delta\underline{\epsilon}$ compute $\Delta\underline{\sigma}_2$ using $\Delta\underline{\epsilon}_1$ and the elasto-plastic matrix $\underline{\underline{D}}\,\epsilon_p$ (Eqn. 4.54).

$$\Delta\underline{\sigma}_2 = \underline{\underline{D}}_{e_p}\ \underline{\epsilon}_1 \qquad (4.55)$$

where $\underline{\underline{D}}_{ep}$ is based on the total stresses obtained in step 2. Therefore the 'pseudo stresses' which cannot be supported by the elastic-plastic concrete are computed as

$$\Delta\bar{\underline{\sigma}} = \Delta\underline{\sigma}_1 - \Delta\underline{\sigma}_2 \qquad (4.56)$$

4. Compute the pseudo loads $\bar{\underline{P}}$ using the relation

$$\bar{\underline{P}} = \int_V \underline{\underline{B}}^T \Delta\bar{\underline{\sigma}}\ dV \qquad (4.57)$$

and analyse the structure using the initial stiffness at the beginning of the increment which will give a new set of

increments $\Delta \underline{\sigma}'_1$ and $\Delta \underline{\bar{\epsilon}}_1$.

5. Repeat steps 2 through 4 until the pseudo stresses reach sufficiently small values.

The above process results in artificial shifting of the yield surface from its initial position which may be corrected for. Furthermore when plasticity occurs the state of stress does not uniquely define the state of strain; it is path dependent. Also the process is irreversible. In this thesis, monotonic increase in loads is envisaged and hence the real behaviour of structure which is generally loaded in cyclic manner, may not be reflected from this simulation. If within an increment only biaxial plasticity takes place in the elements, the initial stiffness of the structure is not altered and the pseudo loads are iterated by the use of initial stiffness at the beginning of the increment. If, however, within an increment cracking occurs in some elements and concrete plasticity occurs in the other elements, then the element stiffnesses of the cracked elements only are changed.

If reinforcement yields, the element stiffness is assigned zero values and equivalent yield loads are assigned to appropriate nodes of the associated bar element.

The validity of assuming concrete to be elastic-perfectly plastic lies in the von-Mises criterion being

conservative than the corresponding relationship obtained from tests (Fig. 4.3).

There has been no evidence in published literature regarding failure criteria for brickwork. In tension the maximum normal stress theory could be used with confidence. Author's attempts to obtain complete stress-strain curve for brickwork showed large scatter and sometimes erroneous readings in electrical strain gauges due to local flaws in brickwork. Further work seems to be necessary in this direction. Brickwork shows the essential features of concrete when loaded in compression, except for the descending branch beyond ϵ_3. It was decided to use the von-Mises criteria for brickwork also, even though Mohr-Coulomb criteria could have resulted in a more realistic response as inferred from the works of Desai (B 4). σ_o^b and ϵ_o^b were obtained as was done for concrete (Fig. 4.4).

The above theoretical study for the analysis of walls on beams using FEM and CST element is completely based on previous research by various investigators in the area of concrete structures. It is unfortunate that published literature on the behaviour of brickwalls on reinforced concrete beams are biased towards experimental investigations wherein the overall response is observed and the individual influences have not been isolated in any systematic manner. It is the contention of the author that computer simulation techniques

in conjunction with laboratory experiments will be able to identify the primary variables. It can be noticed from the above theoretical development that the material constants, anisotropy and non-homogeneity, besides the strength parameters, require greater attention.

4.2 Results of Simulation Studies on Concrete and Brickwork

Non-homogeneity, anisotropy and randomness at material and structural levels have been emphasized in this thesis. The CST/FEM procedure has been chosen for simulation of the load-response characteristics of two phase composite materials such as concrete and brickwork and also the behaviour of walls on beams. In this section results obtained from computer simulation at material level are presented and effects of various parameters are evaluated. Knowledge of materials have been traditionally from simple tension and compression tests on prisms or cylinders. A prism of 20 cms width, 27.5 cms height, and constant thickness of 10 cms was chosen for the study of 'concrete-like' materials. Concrete may be viewed as aggregates randomly embedded in a matrix of mortar. Three dimensional tetrahedral finite elements would have represented the concrete like material in a realistic manner; but three dimensional analysis requires very big computers with access time in terms of nano-seconds. An important idealisation has been made that across the thickness direction, the prism has uniform geometric

and material properties, thus approximating the problem under study as a plane stress problem. There are two avenues open to the analyst in characterising concrete; one is to take microphotographs of slices of concrete like materials and approximate the positions of aggregates and mortar in the finite element simulation to reflect the photographs. Such an approach has been adopted by Jaenssen and Sundstrom (A 5) and Kavanagh (A 6). Another method is to generate idealised arrangements using random number generation, for purposes of parameter studies. This approach was chosen by the author, and it was noticed that Miamoto et. al. (A 8) have used a similar approach for interpreting the mechanical behaviour of grinding wheels in terms of micro-structure. However, rectangular elements were used by these authors, whereas triangular elements can be effectively used to represent the boundaries between the two phases.

The random number generation routine available to the author, could generate decimal fraction on the basis of uniform probability and these fractions were converted to yield numbers 1 and 2, and the number '1' was associated with aggregate and the mortar was identified by the number '2'. The resulting volume fraction centered around 0.5. Other volume fractions such as 0.33 could have been easily obtained. The prism is subdivided into 176 elements (16 per row) with 108 nodes so as to yield reasonably satisfactory distribution of stresses under pure flexure. The error in stresses was found to be

10 percent, whereas deformations were quite accurate. This is to be expected in the CST/FEM displacement method of analysis. Ten randomly arranged prism samples were simulated in the computer, besides pure mortar and pure aggregate as extremes. These sample prisms are illustrated in Fig. (4.8), the shaded areas representing aggregate. These were subjected to uniform compressive loading and the resulting displacements, stresses, Youngs Modulus E, and Poisson's ratio were studied. Besides simple vertical compression loading, lateral compression, compression in laterally restrained specimens (platen restraint in testing equipment was aimed at), pure shear and pure flexure loading were simulated on the first random arrangement of materials and compared with the results obtained for identical loadings on pure aggregate and mortar. (Fig.4.9). The random arrangement of materials compared favourably with the micro-photographs of concrete presented by Nakagawa et.al (A 10). To study the effect of Poisson's ratio, ν on the resulting stress distribution the first random arrangement was assigned values 0.0, 0.1, 0.2, 0.3, 0.4 and 0.5 and studied under uniform vertical compression. The results obtained from the above simulations (30 in number) will be discussed in appropriate sections.

The behaviour of brickwork is of significant importance in this thesis. A three brick, two mortar layer system was simulated first and the interaction stresses were studied. Each

brick is represented by 48 triangular elements and each mortar layer by 16 such elements (Fig. 4.10). Since symmetry about centre line could be exploited in this case, and a vertical mortar joint is desirable to idealise the behaviour of brickwork in a realistic manner, such a system was simulated (Fig. 4.11) using 224 elements and 136 nodes. The resulting brickwork was studied under uniform vertical compression. The variation due to mortar modulus was of significant interest and five different moduli were assigned and studied. It was noticed that the parameter, $E_b \gamma_m / E_m \gamma_b$ was of significant interest and four more simulations were undertaken to study its effects. The thickness of mortar joints in the horizontal direction was also varied and 0.5 cm, 1.0 cm, 1.25 cm and 2 cm thicknesses were assigned and studied. The resulting stresses and deformations from nearly 20 such simulations are discussed in the appropriate sections.

To study the gradual reduction in the stiffness of a composite material system due to cracking a cube of material with inclusions and openings was also studied requiring three additional simulations (Fig. 4.12). The stress concentration due to circular and rectangular openings in homogeneous media were also studied separately, but these results could not be included in the simulation of walls on beams with openings and were hence discarded.

Most of these simulations could be run as short jobs of less than 8 minutes duration for the study of elastic behaviour. Progressive cracking, local yielding and stress redistributions were not undertaken at this level since elastic constants of composite materials were of primary interest which are required for the simulation of walls on beams.

4.2.1. Effects of Non-homogeneity in Concrete

Conventional theory of elasticity assumes homogeneity and isotropy and hence a uniform compressive loading on prisms results in the simple solution that vertical stresses and strains are uniform and lateral expansion is symmetric. Computer simulations on homogeneous, isotropic materials have yielded identical results. This may be contrasted with the stresses obtained on a randomly arranged non-homogeneous media (Fig. 4.13) for the same vertical compressive loading of 1 kg/sq.cm. The stress distribution in random non-homogeneous media has obvious perturbations over the solution for homogeneous materials. The maximum vertical stress is in the order of 2 times the average and minimum stress is in the order of 0.5 times the average., while the modular ratio as per given data works out to 2.28. However, the stiffer material namely aggregate doesn't always pick up higher stresses, the stress in any element being affected by those in the surrounding element The lateral tensile and compressive stresses across any verti

section alternate so that net value is zero. However, the maximum lateral stress is in the order of 28 percent which in the case of a bistrength material like concrete is adequate to cause microcracking in local regions. The shear stresses also alternate, their net value in any horizontal section being zero. As a consequence of local cracking, there is a progress reduction in the stiffness of the cracked elements and consequently non-linear load deformation curves are obtained for bistrength materials. This complicated stress distribution and local stress concentrations may be causes for microcracking which has been noticed by Sturman, Shah and Winter (A 13) in extensive laboratory investigations. In one of the simulations, the load was progressively increased and it was found that nearly 80 out of the total of 176 elements showed cracking, even though average vertical compressive stress was well within the compressive strength of a material such as concrete. The crack directions were found to be predominantly in the vertical direction thus providing adequate evidence for lateral splitting of prisms which are uniformly compressed

The perturbation stresses, being self-equilibrating in nature, generate flexural deformations which could not be predicted from the homogeneous theory of elasticity. This aspect is significantly brought out in the deformation diagram plotted in Fig. (4.14). In the testing equipment these lateral deflections are suppressed creating additional external

constraints which will result in apparently high strength of the material studied and this aspect perhaps justifies the reduction factor $K_3 = 0.85$, used in ultimate strength design of concrete structures. This overall flexure in non-homogeneous media varies from section to section, assuming that it can be properly defined, and the applied uniform compressive stress is really eccentric with respect to the centroid which 'wiggles' around the centre line.

This simulation study also indicates that the use of electrical strain gauges which monitor strains at local areas and the use of long electrical strain gauges in concrete which have a tendency to average out the strains are to be viewed with caution. The familiar scatter in stress-strain curves obtained from laboratory test may perhaps be explained through the above observations. Furthermore values of Poisson's ratio cannot be accurately obtained from strain gauge measurements at local spots.

4.2.2. Effects of Random Arrangement of Two Phase Materials

In an attempt to obtain significant numerical results from the large volume of data generated through the computer simulation of ten randomly arranged samples, the following procedure was adopted.

1) The Young's Moduli obtained from average vertical strains in computer simulation and average vertical compressiv

stresses were compared with those predicted by the series, parallel and combination models proposed in the theory of composite materials which were summarised in Chapter 2. The E and ν values are detailed in Table 4.1. compared in Tables 4.2 and 4.3 and plotted in Fig. 4.15.

2) The computer simulation and in particular the random arrangement shows promise for evaluation of elastic constants and volume fraction assignment.

3) The combination models in the theory of composite materials predict more closely the E values obtained from computer simulation.

4) For the same volume fraction (0.51) the random arrangement of materials is seen to have significant influence on the values obtained for the modulus E (195,600 and 189,900 kg.ms/sq.cm).

5) The values of ν, the Poisson's ratio, are more sensitive to computer simulation and may fall outside the bounds predicted in the theory of composite materials. For the same volume fraction (0.51) the values of ν are quite sensitive to the random arrangement of the materials (0.1714, 0.1451).

6) The accuracy of the FEM procedure is verified through corresponding analysis of homogeneous prisms of aggregate and mortar.

4.2.3. Effects of Anisotropy Due to Cracking of Concrete

The advantage of computer simulation studies in isolating the effects of individual variables has been utilised in this study. The first random arrangement was loaded progressively from 0 to 160 kg/sq.cm. at 10 kg/sq.cm. interval and the number of elements which cracked in tension (Aggregate $f_t' = 10$ kg/sq.cm., mortar $f_t' = 5$ kg/sq.cm) were traced and after each increment E_2, G_2 and ν_2 directed perpendicular to the crack were assigned zero values, utilising the transverse isotropy concept discussed earlier and the number of elements that cracked cumulatively are as per the following series,

Stress kg/sq. cm.	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	150
Cracked Elements	0	0	0	0	1	4	8	15	27	38	42	49	55	65	68	71	79

However, a plot of the load-average vertical deformation for this specimen (Fig. 4.15) shows no deviation from linear elastic behaviour in spite of the reductions imposed on the material property matrix $\underline{\underline{D}}$. This deserves explanation. First and foremost the cracking direction was found to be more or less 90 degrees from the horizontal and reduction of the stiffness in the horizontal direction did not affect the load-response significantly, since stiffness in the direction of loads has not been altered. More important observation is that unless the excess stresses in critical elements are redistributed to adjacent elements, the computer simulation will not

result in realistic load-response characteristics. This aspect has not been discussed in similar studied made by Nakagawa et.al. (A 10) and Ono and Mills (B 14). Furthermore microcracks at interelement boundaries observed by Sturman et.al. (A 13) have not been included in this simulation. It has been found that at higher loads the simulation indicated a non-linear load-deformation response even without redistribution; but this is of academic interest only.

4.2.4. Effects of Various Types of Loading in Plane Stress Media

It has been mentioned earlier that besides simple vertical compression, lateral compression, vertical compression with lateral restraints at top and bottom of the plane prism, pure shear and flexure were simulated in the computer. The loading details and boundary conditions are illustrated in Fig. 4.9. Besides the first random sample, pure aggregate and mortar prisms were subjected to identical loadings for comparative study. The results obtained are summarised in Table 4.4. From the resulting deformations E, γ and G values were compute yielding important results, which are enumerated as below.

1) The E and γ values obtained from horizontal and vertical compression of the same prism with random arrangement of materials (volume fraction 0.47/0.53) are not the same wherea the theory of composite materials will assign identical values

for both cases. This shows that besides volume fraction the random arrangement of materials is important in that it introduces anisotropy. Concrete-like materials may never be idealised as isotropic or even orthotropic. They are essentially anisotropic.

2) Restrained compression apparently increases the elastic moduli even for homogeneous materials and more so far the randomly arranged media.

3) Pure shear simulation in composite media yields higher values of G, indicating higher shear stiffness than can be approximated from the formula $G = E/2\ (1+\gamma)$ where E and γ can be obtained from the theory of composite materials or from the computer simulation itself. It is important to notice that shear tests are difficult to conduct in the laboratory, and computer simulation offers a good alternative.

4) The elastic modulus obtained from pure flexural loading simulation using the formula $E = ML^2/2\delta I$ where δ is the maximum deflection obtained from computer simulation on randomly arranged two phase media, is lower by 5 percent than the values obtained in pure compression; this may be viewed in the light of self-equilibrating stresses in random-non-homogeneous media and local flexural effects discussed earlier.

5) While changes in E are in the order of 1 to 5 percent, changes in γ values are in the order of 15 percent, showing

the sensitivity of the Poisson's ratio to random arrangement and various types of loading. Hence further studies on the influence of ν, were undertaken which are described in the next section.

6) To show that the above observations are not influenced by errors in finite element modelling, the above studies were repeated on plane aggregate and mortar prisms, admirably yielding back the values fed as data (Table 4.4), with marginal errors.

4.2.5. Effects of Variation in Poisson's Ratio

To observe the effects of variation in Poisson's ratio, values of 0.0, 0.1, 0.2, 0.3, 0.4 and 0.5 were assigned in the first random distribution for both materials. The deformations obtained in the top of prism are as below.

S.No.	ν	δ_v cms	δ_h cms
1	0.00	0.14414	0.01269
2	0.10	0.14425	0.01281
3	0.20	0.14427	0.01286
4	0.30	0.14421	0.01286
5	0.40	0.14406	0.01278
6	0.50	0.14379	0.01269

It is seen that variation in Poisson's ratio has little effect on the lateral and vertical deformations. On the other

hand stresses in a typical element which represents an aggregate in the centre of the concrete prism were sensitive to the Poisson's ratio.

ν	σ_x	σ_y	τ_{xy}	σ_1	σ_2	ϕ
0.00	0.037	-1.5440	-0.1300	0.049	-1.556	85.02°
0.10	0.046	-1.5518	-0.1343	0.057	-1.563	85.23°
0.20	0.056	-1.5590	-0.1292	0.067	-1.569	85.46°
0.30	0.068	-1.5670	-0.1230	0.077	-1.576	85.69°
0.40	0.081	-1.5750	-0.1180	0.089	-1.583	85.94°
0.50	0.096	-1.5820	-0.1117	0.103	-1.589	86.22°

wherein, σ_1, σ_2 are principal stresses and ϕ is the principal angle measured from vertical.

A similar tabulation shows the stresses in an element representing mortar, adjacent to the aggregate element discussed earlier.

ν	σ_x	σ_y	σ_{xy}	σ_1	σ_2	ϕ
0.00	0.0390	-0.790	-0.0143	0.0390	-0.790	-89.02°
0.10	0.0330	-0.799	-0.0148	0.0330	-0.799	-88.98°
0.20	0.0266	-0.807	-0.0155	0.0270	-0.808	-88.94°
0.30	0.0194	-0.817	-0.0162	0.0197	-0.816	-88.89°
0.40	0.0115	-0.827	-0.0171	0.0118	-0.827	-88.84°
0.50	0.0023	-0.838	-0.0179	0.0027	-0.838	-88.78°

It is seen that σ_x stresses are quite sensitive to the Poisson's ratio and in turn these lateral stresses are responsible for micro-cracking and hence assigning arbitrary values for this material constant, while simplifying computation, masks valuable information on micro-mechanical behaviour of materials and affects in particular the failure theories developed for materials such as concrete.

4.2.6. Study of Brickwork

Brickwork is such a familiar material that very few research workers find it exotic enough for advanced research. The International Conference on Masonry Structural Systems at the University of Texas, (Nov., 1967) was an eye-opener to many with regard to the gaps in our knowledge of this ubiquitous building material. Of particular interest is the prediction of elastic constants for brickwork, knowing the elastic constants for brick and mortar separately. The peculiar behaviour of brickwork in splitting laterally under compressive loading under small loads has been noticed in several investigations. Furthermore brickwork often shows higher compressive strengths than that of the mortar strengths. Wherefrom this excess strength is obtained, deserves attention. The stress-strain curves show increasing tangent modulus and are dependent on the loading parallel and perpendicular to bed joints. Finally the failure theories for brickwork have not been given adequate attention.

In an attempt to explain the above experimental observations, simulation studies on brickwork have been undertaken by the author.

A three layer brickwork was simulated in the computer using CST/FEM techniques (Fig. 4.10). In this simulation, mortar was assumed to be stronger than the brick and hence higher modulus ($E = 1000\ f_c'$) was assigned to the mortar than bricks (1750000 psi vs 610000 psi) and ν values were taken to be 0.25 and 0.1 respectively. In this case the mortar was found to be in biaxial tension and bricks were in triaxial compression as anticipated by the author. It has been pointed out that earlier research has indicated the converse to be true. It is the contention of the author that by adjusting the elastic constants (how this is to be done in practice is not easily defined), we can adjust these interaction stresses to any desired level, thus leading to an optimization problem, avoiding premature lateral splitting of brickwork under compressive loading. Since there were no vertical joints in the above simulation, this study was abandoned and further simulation studies were carried out using vertical mortar joint also in the middle layer. (Fig. 4.11). The symmetric loading was taken advantage of in the finite element modelling. The effects of material constants and thickness of horizontal mortar layers were taken to be the primary variables. Laboratory tests undertaken by the author revealed that mortar tests showed coefficient

of variation in the order of 35 percent, while the brick itself responded with 22 percent. Random mortar properties could have been assigned in this study as described earlier in the case of simulations for concrete-like materials; however this important aspect was relegated for future research.

4.2.7. Effects of Material Constants on Behaviour of Brickwork

The σ_x (lateral) and σ_y (vertical) stress distribution in brickwork due to applied uniform compression of 80 kg/sq.cm are plotted in Fig. 4.17. The elastic modulus E, were assigned on the basis of the 28 day compressive strengths of bricks and mortar and using the SCPI recommendation (20), that $E = 1000\ f'_c$. Attempts to measure Poisson's ratio using electrical strain gauges in bricks and mortar cubes were not fruitful and hence ν for brick was arbitrarily assumed as 0.1 and ν for mortar was assigned a higher value of 0.20. E for brick was found to be 345000 kg/sq. cm and E for the mortar used was found to be 100000 kg/sq.cm (1:3 mortar by volume). The lateral stress distribution shows that brick is in biaxial tension and mortar is in triaxial compression, explaining simultaneously the reason for observed increase in strength of brickwork beyond that of the mortar and the lateral splitting of brickwork loaded in uniaxial compression.

The vertical stress distribution shows that vertical mortar joints are sources of stress readjustment in the

in the component materials, stresses in mortar being reduced to 38% and stresses in brickwork being increased to 130% of average stresses. Maximum stresses were noticed at the corners of bricks at the junction of horizontal and vertical mortar joints, thus showing that corners of bricks are critical regions of failure in actual brickwork. The influence of elastic constants on the stresses in critical elements in brick and mortar is presented in Table 4.5. The need for strong mortars ($E = 1000 f'_c$) for reducing the lateral tensile, stresses in bricks is self-evident. Very weak mortars force the brick to share the major load and Fig. 4.17, shows local flexures at the vertical brick-morter joints which were noticed by Hilsdorf (A 4) in actual experiments. The uniformly compressed brickwork is a myth, which results directly from the assumptions of homogeneity and isotropy.

In the serial arrangement of bricks and mortar, if the value of $\sigma \gamma_b / E_b$ is exactly equal to $\sigma \gamma_m / E_m$, then lateral expansions are same and interacting stresses in the lateral direction are absent. But actual brickwork, with its vertical joints is a series-parallel combiration model and attempts were made to study the effects of the parameter $E_b \gamma_m / \gamma_b E_m$ Several combinations are possible and values of , 1.0, 1.0, 3.45 and 69.0 were tried as shown in Table 4.6. The lateral tensile stresses in bricks were considerably reduced if the ratio of $E_b \gamma_m / E_m \gamma_b$ is adjusted to be in the order of

unity. Thus the parameter $E_b \gamma_m / E_m \gamma_b$ is identified as a basic parameter for crack-free brickwork. It is, however, noticed that tensile stresses in the vertical mortar joint are not eliminated at local points, indicating the need for strong mortars. The need for strong mortars to reduce the unbalance in vertical stress distribution has also been brought out through the simulation.

4.2.8. Effects of Thickness of Mortar Joints

The effects of thickness of horizontal mortar joints on the resulting stress distribution are summarised in Table 4.7. It is clear that thick joints increase the lateral tensile stresses in bricks and also in the mortar in the vertical joint, whereas vertical stresses in critical elements remain unaffected. Compared with the effects of elastic constants of brick and mortar, the effects of thickness of mortar joints are not critical. It is, however, reassuring that thin joints which are economical do not result in detrimental effects.

4.2.9. Elastic Constants for Brickwork

Comparison between equivalent elastic constants for brickwork obtained from the theory of composite materials and computer simulation is made through results tabulated in Table 4.8. The theory of composite materials predicts bounds for the elastic constants, and the series model (being lower

bound) was chosen for comparison with the results obtained from computer simulation. The following aspects are brought out:

1) The prediction of Young's Moduli by the theory of composite materials agrees quite well with the computer simulation.

2) The simulation results are not however bounded by the series model since Poisson's ratio has not been kept constant as assumed in theoretical derivations.

3) For the same volume fraction (0.90) values of Poisson's ratio, ν as predicted by the theory of composite materials are in gross error when compared with the simulated values. This is due to the E values not being kept the same as required in the theory of composite materials. As the values of E for brick and mortar approach each other, the agreement in the values is quite satisfactory.

4) Once again the series model is no more the lower bound for the prediction of ν.

5) It is the contention of the author that the elastic constants for composite materials can be accurately predicted only when E and ν are simultaneously accounted for.

It is necessary to mention here that the elastic constant E, measured in the laboratory for a three layer brickwork was much lower than these predicted through computer simulation or the theory of composite materials. This is as it

should be since the mortar strength was first evaluated by compressing 7 cm. mortar cubes which were vibrated thoroughly as per standard specifications, and the value of E was estimated from the equation $E = 1000\ f_c'$. In the brick laying process it is obvious that such vibrations are not attempted, and whatever be the richness of cement mortar, unless it is compacted well its strength and stiffness may not be utilised in practice. The increasing tangent modulus of brickwork as observed in the laboratory and its low modulus will be discussed later on when experimental results are presented. In the meantime the experimental values were used in the simulation studies on walls on beams, discarding the computer simulation values of elastic constants.

4.2.10 <u>Miscellaneous Studies</u>

To isolate the influence of an aggregate in a medium of mortar and the influence of mortar inclusion in a medium of aggregate, a cube of 15 cms. size was studied with 5 cm. prismatic inclusions. Uniaxial compressive loading on the cube was simulated. To see whether the reduction of element thickness to 1/40 of the surrounding medium can effectively represent an opening, the above inclusion problem was modified to simulate an opening and was loaded identically as above. The simulations are detailed in Fig. 4.12. The following observations were made

from these miscellaneous studies which were fore-runners to the simulation of walls on beams in the computer.

1) There were stress concentration effects which are well known, the vertical stress concentration factor being 1.40, the horizontal tensile stress concentration factor above and below the soft inclusion in a hard media being 14 percent. As the applied stress was increased from 40 kg/sq.cm. to 80 kg/sq.cm., the corner elements surrounding the inclusion cracked and failed.

2) For a hard inclusion in soft media, the hard inclusion cracked even at the applied stress of 40 kg/sq. cm. and the corners were highly stressed in compression also. The soft media itself was more uniformly stressed than the first case. This study is meant to bring out the nature of the stresses in an aggregate which is surrounded by mortar. It shows that aggregates which are surrounded by mortar become critical than the other way.

3) In the case of simulation of an opening, the stress concentration factor becomes nearly 2.00, the lateral tensile stresses being in the order of 38 percent, in the critical elements. The stresses are in the nature of a soft inclusion in hard material. As the applied load is increased from 40.00 to 110.00 kg/sq.cm. nearly 62 elements cracked showing the detrimental effects of openings/cracks. Even though stress redistribution was not attempted, the reduction in stiffness

due to cracking has been accounted for an in this case the load deformation curve is as in Fig. 4.18, clearly showing the isolated effects of reduction in stiffness due to cracking. Without redistribution this is a hypothetical curve of peculiar nature and is of academic interest only.

The above studies at the material level were helpful in obtaining a 'feel' for computer simulation and considerably influenced the work on simulation of walls on beams which will be detailed in the following sections.

4.3. Results of Simulation Studies on Walls on Beams

The primary aim of this thesis is the development of an appropriate simulation procedure which will reflect the load-response characteristics of brickwalls interacting with their supporting beams, so that parameter studies and generation of design charts may be undertaken. Such a simulation procedure is to be attempted for the elastic state, post cracking and von-Mises failure states and finally the collapse limit state.

Non-homogeneity and anisotropy are to be given importance and randomness in properties of brickwork is also to be considered.

The height span ratios, size and location of openings. the nature of loading (tensile or compressive) and the contribution of reinforcement in the supporting beams in resisting loads have been given due attention.

Of these the elastic analysis is straight forward, and the stresses and deformations are easily obtained. Based on the stresses at mid-span section of the supporting beams, design rules can be generated in a simple manner.

The post cracking and von-Mises failure stages, require the introduction of reinforcement in the supporting beams and tracing of failures in tension and compression in brickwork and concrete and yielding of steel. The modification of the stiffness of critical elements is essential.

The ultimate load stage requires stress redistribution through iterative-incremental method of analysis.

The necessary theoretical basis has been established in section 4.1 of this chapter. The material constants which are the basic input parameters in the simulation have been discussed in section 4.2. Computational details are given in the form of flow charts and in the Fortran programme that is appended at the end of this thesis.

The presentation of stresses and displacements obtained at very large number of points is a major problem and graphical presentation has been chosen, numerical values being indicated at critical points only.

The structures simulated in the computer were identical with those studied in the laboratory. The laboratory test results will be presented in Chapter 5.

It has been pointed out earlier that material constants will be assigned as per laboratory test results obtained from tests on concrete cylinders, brickwork prisms and tension reinforcement.

The boundary condition assigned reflects the hinged supports used at both ends of walls on beams, in the laboratory tests.

The loading is uniformly distributed compression of top of the walls or uniform tensile loading at the bottom of the beam. Lateral loads were not simulated, even though a few additional data cards could have taken care of this.

4.3.1. Effects of Height-Span Ratios on Elastic Behaviour

Four height span ratios were simulated in the computer, namely H/L = 0.10, 0.35, 0.52 and 0.68. The ratio of 0.10 represents the plane R.C. beam without wall and the ratio of 0.90 tested in the laboratory could not be accommodated in the computer for want of adequate memory. The short table below gives an idea of the computational effort involved.

S.No.	H/L Ratio	No. of Elements	No. of Nodes	Time Required
1	0.10	184	120	6 min. 06 sec.
2	0.35	414	240	18 min. 39 sec.
3	0.50	598	336	24 min. 38 sec.
4	0.68	782	432	28 min. 15 sec.

The details of finite element subdivision of the wall of height span ratio 0.68 are given in Fig. 4.19. The nodal coordinates and connections have been generated automatically, avoiding large number of data cards. Non-homogeneity at structural level has been considered; but each element is deemed to be isotropic. A small load of 1000 kg/metre was applied as against the failure load of 20000 kg/metre. The lateral and vertical stress distributions are sketched in Fig. 4.20. It is seen that the hinged condition of both supports has a prestressing effect on the bottom beam. The vertical stresses concentrate near the supports. The shear stresses (which are not sketched) concentrate at a distance 'd' (the depth of beam of 20 cms.) away from supports. The brickwork is relatively free from stress concentrations and the bottom beam takes the brunt of loading. This is due to the nature of the modulus assigned for brickwork and the concrete which are as below

	E_1	γ_1	E_2	γ_2	G_2
Concrete Beam	221000 kg/cm^2	0.15	221000 kg/cm^2	0.15	105000 kg/cm^2
Brickwork	5200 kg/cm^2	0.10	8320 kg/cm^2	0.16	2364 kg/cm^2

These observations tally with those of Rosenhaupt (C 14) and Coull (C 5). It is clear that the lateral stresses in the bottom beam are of special interest and these are plotted for

the bottom beam of the wall with H/L ratio of 0.52 in Fig. 4.21. The finite element subdivision, the lateral stresses and deformations for the wall with H/L ratio of 0.35 are given in Figs. 4.22, 4.23 and 4.24 respectively. The plane section hypothesis seems to hold for the brick portion of the wall also, but the slopes of these sections in the supporting beam and the walls are different. The finite element sub-division, deformations and stresses in the plain beam (H/L = 0.1) are given in Fig. 4.25. The local errors in stresses inherent in the CST/FEM displacement method is clearly brought out. Wherever elements join in the manner as in the case of elements 22, 23, 68 and 69, there are errors in the stresses. Wherever elements join in the manner 67, 68, 69, 70, 113, 115, 116 and 116, the solution is quite acurate and the stresses have been plotted based on these junctions.

A simple comparison which suggests itself is the plot of maximum compressive stresses in the mid-span of the supporting beam with respect to the height-span ratio (Fig. 4.26). The stresses seem to level out at H/L ratios of 0.68 and more If the allowable stress in M200 concrete (used in this investigation) is 70 kg/sq.cm. as per relevent Indian Standards then the working loads can be computed as 3200, 5350, 7000 and 8250 kgms/metre run. Thus the interaction between walls and their supporting beams suggest an interaction factor of 8250/3200 = 2.58 for H/L ratio of 0.68, linear interpolation being

indicated in the plot in Fig. 4.26, for other height-span ratios. Thus it is concluded in this study that the bottom beam can be designed for a mid span moment of WL/20.5 as against WL/8 now being assigned neglecting the interaction. This may be contrasted with the suggested maximum of WL/100 as indicated by Wood and Simms (C 23). With the presence of openings and tensile loading this interaction factor is bound to reduce further. The displacement details are summarised in Fig. 4.27. The reduction of central deflection with increase in H/L ratio is as shown in Fig. 4.27(a). The deflection seem to level out beyond H/L ratio of 0.68, thus indicating direct transfer of load to the supports in deep walls on beams. The deflection obtained through computer simulation are compared with deflections obtained in the laboratory in Fig. 4.27(b). Considering that the elastic constants are the only values which are in our control in the computer simulation and that very low values have been assigned considering the porosity of mortar, the agreement is quite close. It has already been pointed out that the theory of composite materials indicate very high modulus. It is once again emphasized that elastic constants for brickwork deserve better attention and pre-occupation with the strength of bricks and mortars individually must be de-emphasized. The deflected shapes of the bottom of supporting beams for various H/L ratios is of interest (Fig. 4.27(c)). In particular the closeness of the curves for

H/L ratio of 0.52 and 0.68 is significant. The reduction in deflections in walls interacting with their supporting beams is indicative of the overall interaction that has taken place. Once again an interaction factor of 2.60 is indicated for H/L ratio of 0.68.

An important aspect which deserves attention is the symmetry of finite element subdivision. Fig. 4.19 shows that the finite element subdivision was not really symmetric about centre line. This generates a bias in the solution towards one direction and lateral shifts of 0.0001 cm. were noticed. Hence this kind of subdivision was abandoned and further studies were based on a new arrangement of the elements, this time using symmetry about centre line. These are reported in the next few sections.

4.3.2. Effects of Openings on Elastic Behaviour

It has been known from earlier experimental investigations (C 6, C 13, C 16, C 21) that the size and location of openings affects the load carrying capacities of walls on beams, but the precise nature of interaction has eluded definition. First and foremost an elastic analysis of deep beams with openings is to be attempted so that an overall perspective of the internal stresses and deformations is obtained and critical spots located. The finite element displacement formulation can deal with openings in a natural manner as indicated

earlier. In this thesis the presence of openings has been accounted for by reducing the thickness of elements which characterise the openings to an arbitrary amount of 1/40th the original thickness as suggested by Sethuratnam (B 16). Furthermore the symmetry of centrally located openings had to be taken into consideration so that the programme could be accommodated in the IBM 7044 computer system with 32K memory. The lintels used in the associated experiments were encased steel beams and the same were simulated with finer mesh size. The resulting finite element arrangement for the brickwall with central opening is as shown in Fig. 4.28. There were reinforcements (2 nos. 10 mm. bars) at top and bottom of the supporting beam which were also included in the study as bar elements spanning the appropriate nodes. The applied loading is uniform compressive at the top of the wall as in the associated laboratory tests. The half-span region of the wall has 672 elements and 374 nodes as against 782 elements and 432 nodes for the complete wall which has been discussed in the previous section. In fact the full wall was simulated first with hopes of accommodating eccentric openings. Since memory constraints prevented successful analysis of the wall with eccentric openings only central door and window openings were studied with the modified finite element arrangement now under discussion.

The elastic constants used for concrete and brickwork are as below:

	E_1	γ_1	E_2	γ_2	G_2
Concrete	221000 kg/cm^2	0.15	221000 kg/cm^2	0.15	10500
Brickwork	5200	0.10	8320	0.16	2364
Lintel	1160000	0.25	1160000	0.25	464200

The time taken for various load increments for the following runs in the IBM 7044 system is as below

Plain wall without opening (Three increments)	29 min. 09 sec.
Wall with window opening (Three increments)	29 min. 08 sec.
Wall with door opening (One increment)	12 min. 18 sec.

The elastic stress distribution in the case of door openings is given in Fig. 4.29. The stress distribution in the case of window openings is given in Fig. 4.30. The applied load was 1500 kg/metre run in both cases. The lateral stresses (σ_x) were critical and discussion is restricted to these stress distributions. The following observations are valid:

1) The bottom beam is critically stressed along with the lintel and brickwall is relatively free from stresses due

to the small values of moduli assigned for brickwork.

2) The stress distribution is similar in all respects to those obtained for the wall on beams without openings, which were discussed in the previous section.

3) The portion of beam below the door openings seem to have a constant moment region which is reasonable since load is transferred to either side of openings.

4) The hinged supports at both ends of the beam seem to have a prestressing effect as inferred from the stress distribution at supports.

5) The vertical and shear stresses concentrate near supports but the stresses are not critical and hence these were not plotted. The vertical stress concentration at the bottom edge of openings noticed in experiments is totally absent in the finite element solution.

6) The mid-span compressive stresses in the bottom beam are the most critical and design loads can be computed for M_{200} concrete (70 kg/sq.cm. allowable stress as per Indian Standards) on the basis of critical stresses. These are as below:

Nature of specimen	Max.Stress for 1000 kg/metre load	Permissible Design Load
Plain walls	15.12 kg/sq.cm.	6950 kg/metre
Walls with door opening	14.15 kg/sq.cm.	7400 kg./metre
Walls with window opening	13.20 kg/sq.cm.	7920 kg./metre

7) It is seen that the supporting beam of the plain-wall is worst stressed since bending moment is larger in these walls than in walls with openings. The window opening while it distributes the loads to the ends, also furnishes some compressive resistance from the brickwork at bottom of the opening.

8) The central deflection for plain walls, walls with door openings and walls with window openings are respectively 0.2170 mm, 0.2021 mm and 0.1951 mm once again confirming reduction in maximum bending moment at mid-span when there are central openings.

9) It is obvious that working stresses are favourable in the walls with openings and their effect on ultimate loads will be discussed in the next chapter.

4.3.3. Contribution of Reinforcement at Elastic Stage

It is well known that post-cracking behaviour of the concrete beams supporting a brickwall is controlled by the reinforcement. The advantage of the FEM in isolating the effects of individual variables was utilised to study the contribution of reinforcement in sharing loads at the elastic stage in a brief study. The deflection is an overall measure of the contribution to the stiffness of the total beam and the central deflections with and without reinforcement in a particular study were found to be 0.2805 and 0.4892 mm

respectively, showing the beneficial effects of reinforcement in reducing the deflections of reinforced concrete beams. The maximum stresses in the elements adjacent to reinforcement at mid-span were found to be 12.5 kg/sq.cm. and these increased to 13.5 kg/sq.cm. without reinforcement. The importance of reinforcement in reducing the deflections has thus been clearly established. Attempts to reduce this steel using WL/100 as against WL/8 now being used (neglecting interaction) may at best save a few kilograms of steel but may result in large deflections and interpretation of structural interaction in the manner suggested by Wood and Simns (C 23) must be viewed with caution.

4.3.4. Effects of Loading, Tensile and Compressive

Wood (C 21) had indicated in his studies on composite construction that the loading of the supporting beams directly through the connecting floors may lead to separation of brick-walls from their supporting beams unless tensile connectors were introduced. In the experimental studies undertaken by the author this aspect has been given major attention. In the finite element method since tensile or compressive loading could be simulated with equal facility a study of this aspect was undertaken on a wall with height span ratio of 0.52 (255 nodes and 448 elements as shown in Fig. 4.28) and uniform compressive loading of 6000 kg/metre was applied at top of

the wall and the same loading was repeated at bottom of the supporting beam in the next run. Of interest is the comparison of central deflections of the supporting beam, the values obtained being 0.709 mm and 0.942 mm respectively. Since separation between the brick walls and their supporting beams is of primary interest, the σ_y (vertical) stresses and shear stresses along the interface were compared and the results are as below.

Loading	Stresses kg/cm^2	Support	0.075 of span	0.175 of span	0.275 of span	0.375 of span	0.475 of span	Remarks
Compressive	σ_y	-7.5	-5.2	-4.04	-3.2	-2.4	-2.23	
	τ_{xy}	+1.6	-6.0	-5.18	-2.10	-1.60	-0.56	
Tensile	σ_y	-3.30	-1.28	-0.10	+0.73	+1.51	+1.72	
	τ_{xy}	+1.86	-5.71	-5.13	-2.04	-1.57	-0.32	

It is seen from the above tabular statement that the nature of loading (compressive or tensile) affects the vertical stress at interfaces, but have insignificant effect on the shear stress. The applied load of 6000 kg/metre is equivalent to 4 kg/sq.cm. stress, on the top of the beam. It is seen that if a constant tension of 4 kg/sq.cm. is imposed

on the solution for compressive loading, the solution for tensile loading is easily obtained for σ_y stresses. This stands to reason from conventional theory of elasticity, since shifting a load is equivalent to adding a uniform stress field to the original solution. The tensile stresses at the interfaces are adequate to cause separation which has been noticed in the tensile loading experiments conducted by the author.

4.3.5. <u>Effects of Local Cracking and von Mises Failures</u>

In earlier sections, attention has been deliberately focussed on the elastic-response. The 'wall-on-beam' loaded in its plane can be treated as a plane stress problem in the theory of elasticity yielding reliable solution when the specimen is uncracked and other local failures are absent. In the earlier sections non-homogeneity and anisotropy have been included which is an improvement over the conventional solutions which are based on the assumptions of homogeneity and isotropy. The stresses and deformations obtained from the above solutions have been discussed at some length and working loads have been assigned based on the concept of allowable stresses. However, designs can be based on deflection limited states, crack limited states or collapse limited states. In these section the behaviour of the supporting beam (with 192 elements, 119 nodes, Fig. 4.28) under monotonically

increasing loads is described, particular attention being given to the identification of various local failures (cracking, local yielding, and yielding of reinforcement in compression or tension). Before assigning the next increment of load, the stiffness matrices of critical elements have been modified as described in Section 4.1. Stress redistribution has not been attempted at this stage. The aim of this intermediate step is to check whether the location of the critical elements agree with experimental observations.

Modelling of cracks, local compression failures and yielding of steel have been achieved by various investigators in several ways and there is a certain amount of speculation in these modelling procedures. One is not sure at this stage of development (1973) which procedure is realistic and economical in terms of computational effort. In this thesis all local failures are accounted for through appropriate changes in the stiffness matrix of critical elements. In this way anisotropy and non-homogeneity at local levels have been given primary importance, which are the important aspects motivating this particular research work.

The supporting concrete beam without the brickwall has been modelled so that several incremental loadings can be obtained within short computing runs and the validity of results could be checked against the observed behaviour pattern. With ten load increments the time required for this

simple problem is in the order of 30 minutes. If the brick wall had been included the time required would have increased to several hours. It is further assumed that the finite element method (once it works for a small region with a few nodes and elements) can be relied upon to yield results for bigger size problems without introducing numerical errors. This assumption is really based on the earlier elastic studies on various sizes of walls which showed satisfactory performance and the residuals in the load vector were computed and found to be negligible.

Two sequences of loading were attempted. The first loading sequence (loads are in kg/metre run) is as follows:

0, 3000, 3600, 4200, 4800, 5400, 6000, 6600,
7200, 7800 and 8400.

The elements that cracked or yielded are listed in Table 4.9, wherein the maximum displacements are also recorded. The second loading sequence is as follows:

0, 7500, 9000, 10500, 12000, 13500, 15000, 16500,
18000, 21000 etc.

The load-deflection curves are plotted in Fig.4.31 and the cracking sequence and von-Mises failures are sketched in Fig. 4.32.

The load-deflection curves are representative of the overall performance of the structure under study. The

curves obtained reflect truly the trends noticed in the laboratory tests. The total number of elements that have cracked or failed in compression are also indicated in brackets. It can be seen that changes due to cracking alone have not introduced significant non-linearity in the load deformation curves.

On the other hand von-Mises failures have a tendency to introduce significant non-linearity through reduction in the stiffness of associated elements ($\underset{=}{D}_{ep}$ was discussed in Section 4.1).

If the author had not tested these supporting beams in the laboratory and found that the actual failure load is in the order of 5000 kg/metre whereas this simulation shows failure at 14000 kg/metre (beyond which instability in computer solution occurs), the research work would have been terminated at this stage as was done by Nilson (A 1), Nakagawa (A 10), Ono and Mills (B 14) and a few other research workers. It is the contention of the author that these high ultimate loads are due to the omission of stress redistribution as soon as each element cracks or fails in compression. The excess stresses which could not be supported by critical elements are to be converted into pseudo loads and assigned to the load vector and additional deformations must be obtained through an iterative procedure. These aspects are described in the next section.

As for the sequential failure pattern (Fig.4.32(b)), the cracks first form at the mid-span section and then propagate upwards besides forming additional cracks at a section which is at a distance 'd' away from mid-span, where d is the depth of beam. These findings truly reflect the known laboratory performance of R.C. Beams. If bond-slip between the reinforcement and the surrounding concrete elements had been introduced the spreading of cracked region at mid-span noticed in this simulation could have been avoided and opening of cracks at mid-span would have resulted. This study indicates that more attention should be given towards an appropriate definition of crack width in such computer simulation. Fig. 4.32(c) shows that one of the elements near the support has 'yielded' in compression, which is quite reasonable. Thus the identification of local failures as built-in in the computer programme by the author, can be deemed to be realistic.

Under the second loading sequence simulated in the computer, the loading was carried through (in spite of the fact that redistribution is not included) so as to see whether compressive failure at the top of the beam could be obtained. The failure zones in Fig. 4.32(d) clearly indicate that compression at mid span and at supports were critical. Furthermore the top of the beam over the supports showed cracking which was also noticed in the laboratory tests. This is as a consequence of the hinged supports at both ends developing

horizontal reactions at the bottom of the beam.

Convinced that the computer simulation procedure could reflect the progressive local failures of a concrete beam, in a realistic manner further work on stress redistribution was undertaken.

4.3.6. Redistribution Studies

The incremental-iterative method for stress redistribution has been described earlier. For this purpose major modifications in the conventional finite element programme were found to be necessary. The stresses in each element and the nature of failure, if any, in each element had to be indexed and stored separately. If there were no local failures in any of the elements, under an incremental loading, the resulting stresses in the system were up-dated. Even if a single element failed during a load increment, an iterative procedure was initiated to modify the displacements to accommodate the released 'pseudo-loads' which could not be supported by the critical elements. Both the initial and variable stiffnesses were used in the iterative procedure. The iteration process continued until the pseudo-loads did not cause any further local failures. The resulting stresses and cumulative displacements were then printed out.

The modifications in the finite element programme were so extensive that even the smallest wall on beam with

H/L ratio of 0.35 could not be accommodated within the bounds of computer memory that was available. Previously four magnetic tapes were in use and an additional tape was also utilised for temporary storage; even then the programme could not be accommodated. Hence the problem was left at this stage of development for future research. However the final version of the Fortran Programme, including stress redistribution, has been appended to this thesis. A plain beam of smaller dimensions has been checked through, showing satisfactory performance of the programme.

TABLE 4.1 : E AND ν FOR COMPOSITE MATERIAL FROM COMPUTER SIMULATION (STRESS 1.00 Kgm/Sq.Cm.)

S NO	Nature of Specimen	ΔV cms	ΔH cms	$\epsilon_v = \frac{\Delta V}{27.5}$	$\epsilon_H = \frac{\Delta H}{20.0}$	E equiv Kg/Sq.cm	$\nu_{equiv.}$
1	Non-composite, aggregate only	0.00009780	0.00007122	3.556×10^{-6}	0.3556×10^{-6}	281,000	0.1000
2	Composite Random	0.0001489	0.00002077	5.417×10^{-6}	1.038×10^{-6}	184,600	0.1915
3	"	0.0001517	0.00001876	5.517×10^{-6}	0.938×10^{-6}	181,200	0.1700
4	"	0.0001397	0.00001630	5.082×10^{-6}	0.815×10^{-6}	196800	0.1604
5	"	0.0001428	0.00001625	5.193×10^{-6}	0.813×10^{-6}	192,600	0.1565
6	"	0.0001379	0.00001273	5.016×10^{-6}	0.637×10^{-6}	199,400	0.1270
7	"	0.0001406	0.00001752	5.112×10^{-6}	0.876×10^{-6}	195,600	0.1714
8	"	0.0001448	0.00001528	5.266×10^{-6}	0.764×10^{-6}	189,900	0.1451
9	"	0.0001438	0.00001635	5.229×10^{-6}	0.818×10^{-6}	191,300	0.1564
10	"	0.0001392	0.00002014	5.062×10^{-6}	1.007×10^{-6}	197,500	0.1989
11	"	0.0001445	0.00001401	5.253×10^{-6}	0.700×10^{-6}	190,300	0.1331
12	Non-composite, Mortar only	0.0002235	0.00004064	8.128×10^{-6}	2.032×10^{-6}	123,000	0.2500
					Average (2 to 11)	191,920	0.16103

TABLE 4.2 : THEORETICAL PREDICTIONS OF E AND COMPUTER RESULTS

(in Kg/Cm^2 Units)

S.No.	Nature of specimen	Volume Fraction, g Agg/ Total	Parallel Model (1)	Series Model (2)	Combination Models: Harmonic Average (3)	Combination Models: Arithmatic Average (4)	E Computer Simulation
1	Aggregate Only	1.00	281000	281000	281000	281000	281000
2	Composite Random	0.47	196100	167100	180400	181600	184,600
3	"	0.47	196100	167100	180400	181600	181,200
4	"	0.53	206710	175100	189700	190805	196,800
5	"	0.52	205130	173900	188100	189515	192,600
6	"	0.55	209850	178000	187400	193925	199,400
7	"	0.51	203570	172500	186700	188035	195,600
8	"	0.51	203570	172500	186700	188035	189,900
9	"	0.52	205130	173900	188100	189515	191,300
10	"	0.56	211410	179500	194200	195455	197,500
11	"	0.49	200430	169700	183700	185065	190,300
12	Mortar only	0.00	123000	123000	123000	123000	123000

TABLE 4.3: THEORETICAL PREDICTIONS OF ν AND COMPUTER RESULTS

S.No.	Nature of Specimen	Volume fraction, g Agg/Total	Parallel Model	Series Model	Computer Simulation
1	Non-composite Aggregate	1.00	0.1000	0.1000	0.1000
2	Composite Random	0.47	0.1795	0.1480	0.1920*
3	"	0.47	0.1795	0.1480	0.1700
4	"	0.53	0.1705	0.130	0.1604
5	"	0.52	0.1720	0.140	0.1565
6	"	0.55	0.1675	0.137	0.1270*
7	"	0.51	0.1735	0.142	0.1714
8	"	0.51	0.1735	0.142	0.1451
9	"	0.52	0.1720	0.140	0.1564
10	"	0.56	0.1660	0.132	0.1989*
11	"	0.49	0.1765	0.144	0.1331
12	Non-composite Mortar	0.00	0.2500	0.2500	0.2500

* Values fall beyond theoretical bounds.

TABLE 4.4 : ELASTIC CONSTANTS OBTAINED FROM VARIOUS LOADING PATTERNS

S.No.	Nature of Loading	Nature of specimen	Elastic Constants/Computer		
			E kg/cm^2	ν	G kg/cm^2
1	(Simple Vertical Compression)	Aggregate	281000	0.1000	127727.0
		Mortar	123000	0.2500	49200.0
		Random 0.47/0.53	184600	0.1915	77450.0 $E/2(1+\nu)$
2	(Simple Lateral Compression)	Aggregate	281100	0.1000	122730.0
		Mortar	122900	0.2500	49160.0
		Random 0.47/0.53	187600	0.1732	80000.0 $E/2(1+\nu)$
3	(Restrained Vertical Compression)	Aggregate	282300	0.1040	127800.0
		Mortar	125300	0.2581	49790.0
		Random 0.47/0.53	192300	0.2058	79730.0 $E/2(1+\nu)$
4	(Pure Shear) $G = \tau/\gamma$	Aggregate	-	-	127700.0
		Mortar	-	-	49210.0
		Random 0.47/0.53	-	-	81450.0
5	(Flexure) $E = ML^2/2\delta I$	Aggregate	281000	-	-
		Mortar	123600	-	-
		Random 0.47/0.53	176000	-	-

Data: $E_{aggregate}$ = 281000 kg/sq.cm., $G_{Aggregate}$ = 127727 kg/sq.cm.

$\nu_{aggregate}$ = 0.10

E_{mortar} = 123000 kg/sq.cm., G_{mortar} = 49200 kg/sq.cm.

ν_{mortar} = 0.25

TABLE 4.5 : EFFECT OF MORTAR MODULUS ON STRESSES IN CRITICAL ELEMENTS (BRICKWORK)

S.No.	Element/Stress	Effect of Mortar Modulus (Kg/cm^2) on Stresses					Remarks/Data
		10,000	20,000	40,000	100,000	200,000	
1	138/σ_y Brick	-133.11	-127.27	-119.16	-104.99	-91.78	E_{brick} = 345000 kg/cm^2 ν_{brick} = 0.1
2	172/σ_x (kg/cm^2)	+8.63	+ 8.09	+7.42	+6.10	4.40	Applied Compression = 80.0kg/cm ν_{mortar}= 0.2
3	126/σ_x Mortar	+3.51	+4.13	+4.52	+4.03	+2.57	Thickness of mortar joint 1.25 cm
4	139/σ_y	-3.63	-7.20	-13.60	-30.99	-53.00	

TABLE 4.6 : EFFECTS OF PARAMETER $E_b \nu_m / E_m \nu_b$ ON BRICKWORK

S.No.	Elastic Constants/Data: E_{brick}/ ν_{brick}	E_{mortar}/ ν_{mortar}	$\frac{E_b \cdot \nu_m}{E_m \nu_b}$	Element No.	*Element/Stresses: σ_x	σ_y	Remarks
1	300000/ 0.1	150000/ 0.05	1.0	126	3.02	-44.44	mortar
				139	-0.08	-45.29	
				138	0.66	-90.52	brick
				172	-0.07	-80.56	
2	300000/ 0.25	120000/ 0.10	1.0	126	3.74	-36.56	mortar
				139	0.20	-37.52	
				138	0.19	93.81	brick
				172	0.07	-80.67	
3	345000/ 0.1	200000/ 0.2	3.45	126	2.57	-53.05	mortar
				139	1.69	-53.03	
				138	2.88	-91.78	brick
				172	4.40	-80.22	
4	345000/ 0.1	10000/ 0.2	69.0	126	3.51	-4.16	mortar
				139	1.21	-3.63	
				138	5.97	-133.11	brick
				172	8.63	-80.48	

*Applied stress is uniform vertical compression at 80 kg/sq.cm.

TABLE 4.7 : EFFECT OF MORTAR LAYER THICKNESS ON STRESSES IN CRITICAL ELEMENTS (BRICKWORK)

S.No.	Element No./ Stress	Effect of Mortar Joint Thickness 0.5 cm	1.0 cm	1.25 cm	2.0 cm	Remarks/ Data
1	138/σ_y brick	-92.28	-91.93	-91.78	-91.41	E_{brick} 345000 kg/cm^2 ν_{brick} 0.10
2	172/σ_x (kg/cm^2)	+2.04	+3.70	+4.40	+6.08	E_{mortar} 200000 kg/cm^2
3	126/σ_x mortar	+ 1.82	+ 2.34	+2.57	+3.15	ν_{mortar} 0.20
4	139/σ_y	-53.56	-53.18	-53.03	-52.72	Applied compression = 80 kg/sq.cm (uniform)

TABLE 4.8 : ELASTIC CONSTANTS FOR BRICKWORK FROM THEORY OF COMPOSITE MATERIALS AND COMPUTER SIMULATION

S.No.	Volume fraction g	Simulation Data				Series Model		Computer Simulation	
		E_b kg/cm^2	ν_b	E_m kg/cm^2	ν_m	E kg/cm^2	ν	E	ν
1	0.90	345000	0.10	10000	0.20	79300	0.1053	81000	0.0450
2	0.90	345000	0.10	20000	0.20	131200	0.1053	132100	0.059
3	0.90	345000	0.10	40000	0.20	195700	0.1053	196200	0.086
4	0.90	345000	0.10	100000	0.20	277000	0.1053	270000	0.091
5	0.90	345000	0.10	200000	0.20	321500	0.1053	320000	0.105
6	0.90	300000	0.10	150000	0.05	272700	0.0909	270000	0.090
7	0.90	300000	0.25	120000	0.10	260800	0.2170	258000	0.230
8	0.96	345000	0.10	200000	0.20	335400	0.1020	330000	0.109
9	0.92	345000	0.10	200000	0.20	326000	0.1040	325000	0.108
10	0.84	345000	0.10	200000	0.20	309100	0.1090	310000	0.103

TABLE 4.9 : PROGRESSIVE FAILURE IN SUPPORTING BEAMS

(Please refer to Fig. 4.31)

S.No.	Loading kg/metre	Elements Indicating Failure			Mid-span deflection mm
		Cracking	von-Mises Failure	Yielding of Steel	
1	0000	-	-	-	0.0000
2	3000	-	-	-	0.5610
3	3600	25,28,29,32	-	-	0.6730
4	4200	27,30,31,62, 63	-	-	0.8120
5	4800	59,61	-	-	0.9720
6	5400	93	-	-	1.1140
7	6000	-	-	-	1.2620
8	6600	-	-	-	1.3880
9	7200	1,2	-	-	1.5140
10	7800	3,20,21	-	-	1.6510
11	8400	5,19,22,54, 94	4	-	1.8888

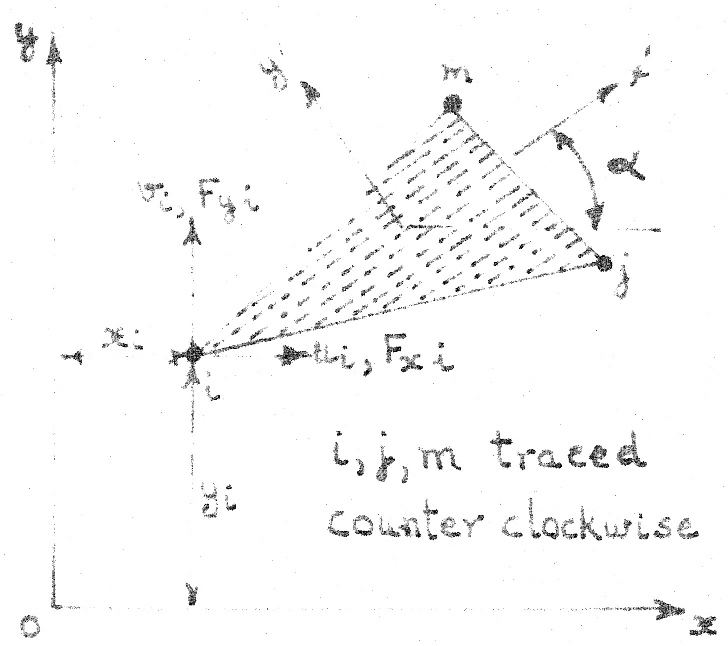


FIG. 4.1. TYPICAL TRIANGULAR ELEMENT

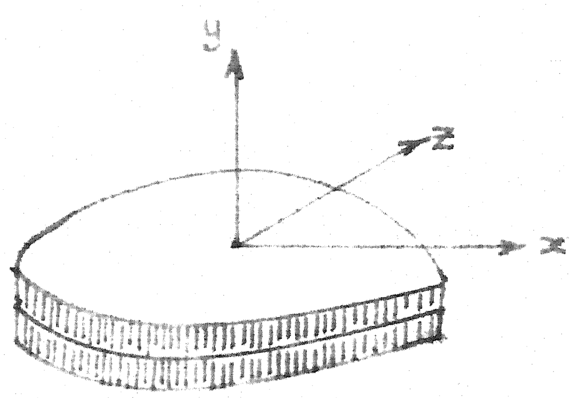


FIG. 4.2. TRANSVERSELY ISOTROPIC MATERIAL

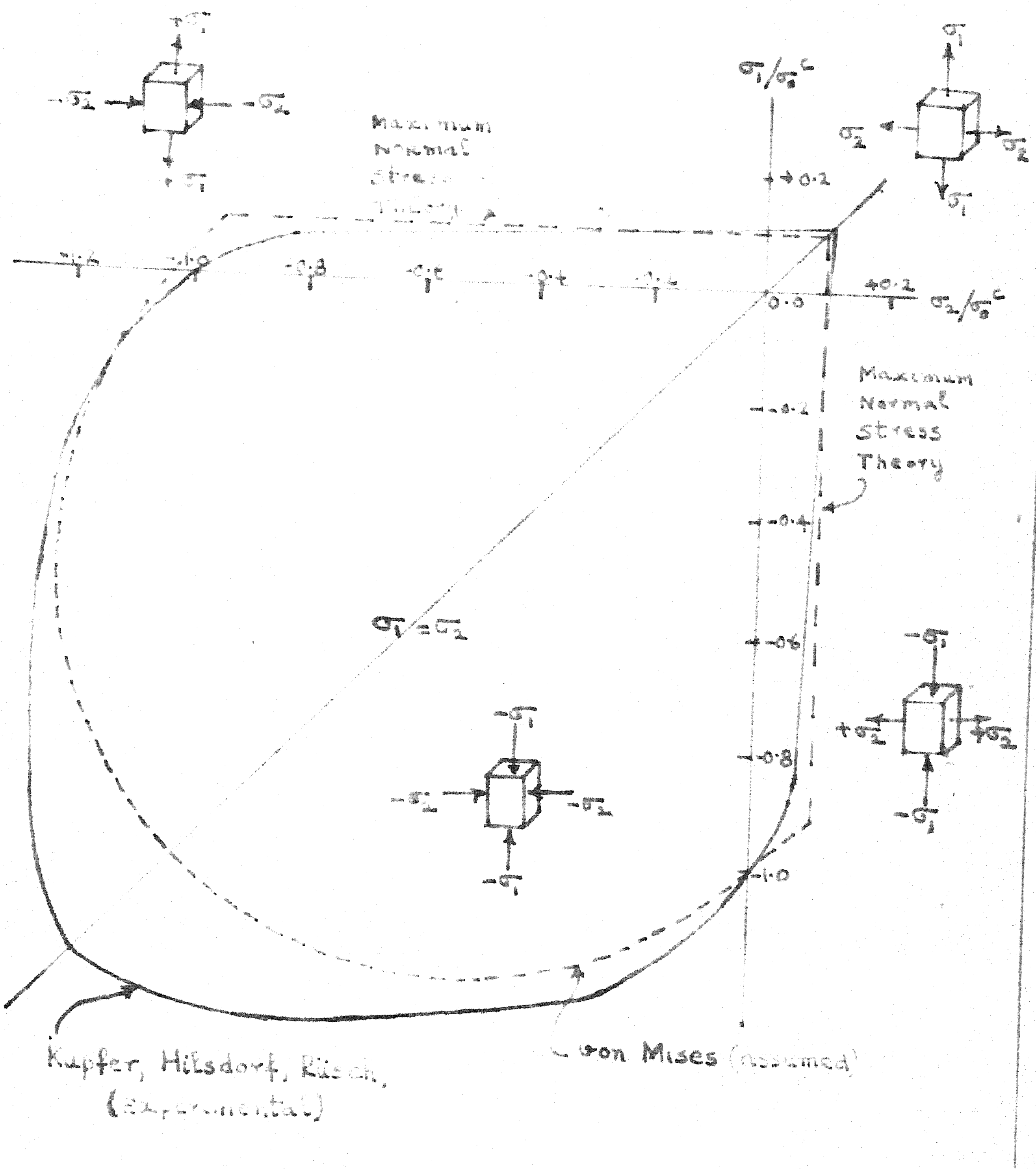


Fig. 4.3. Biaxial Strength of Concrete and Brickwork

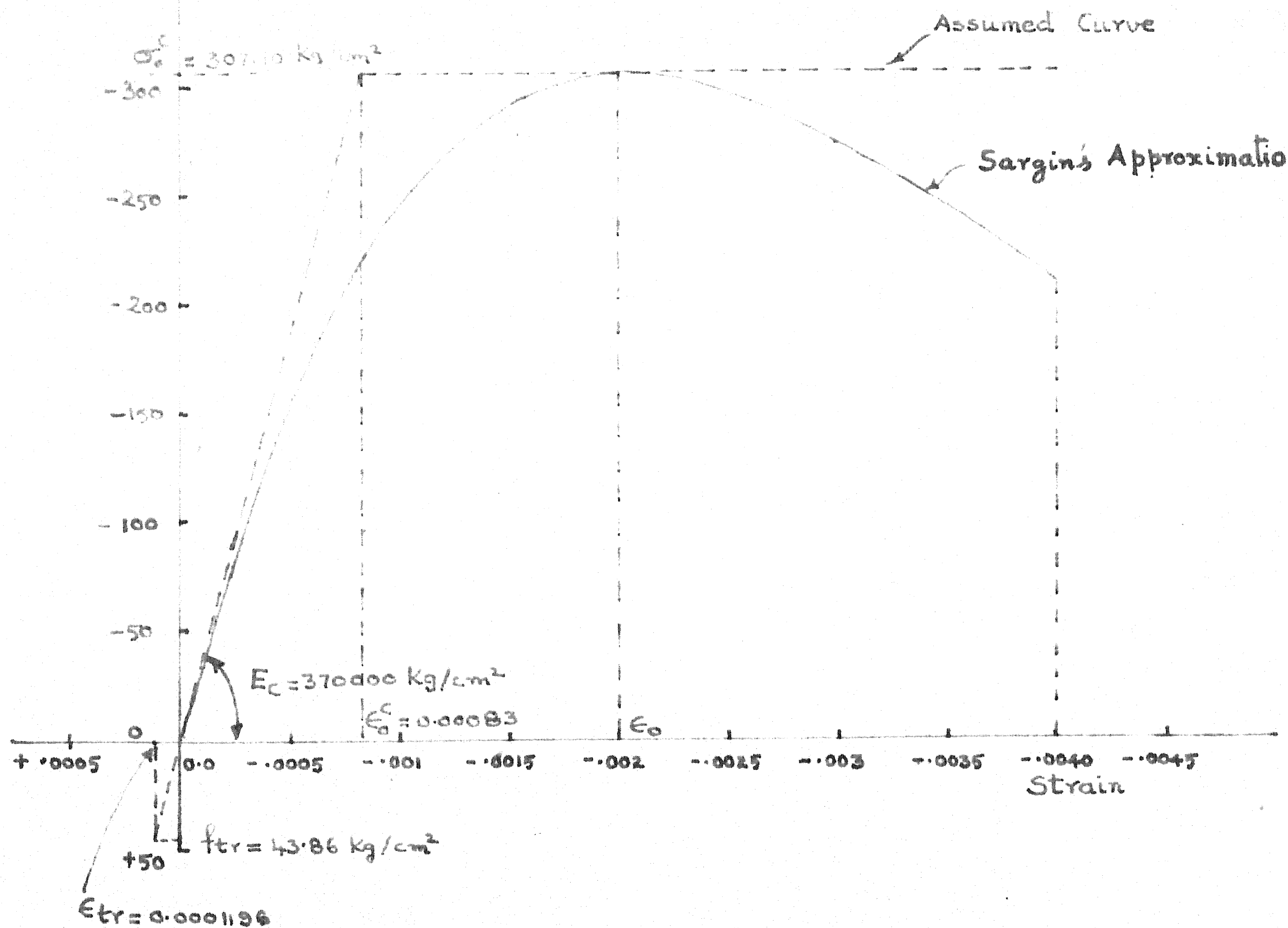


FIG. 4.4. STRESS-STRAIN CURVE FOR CONCRETE AND APPROXIMATIONS

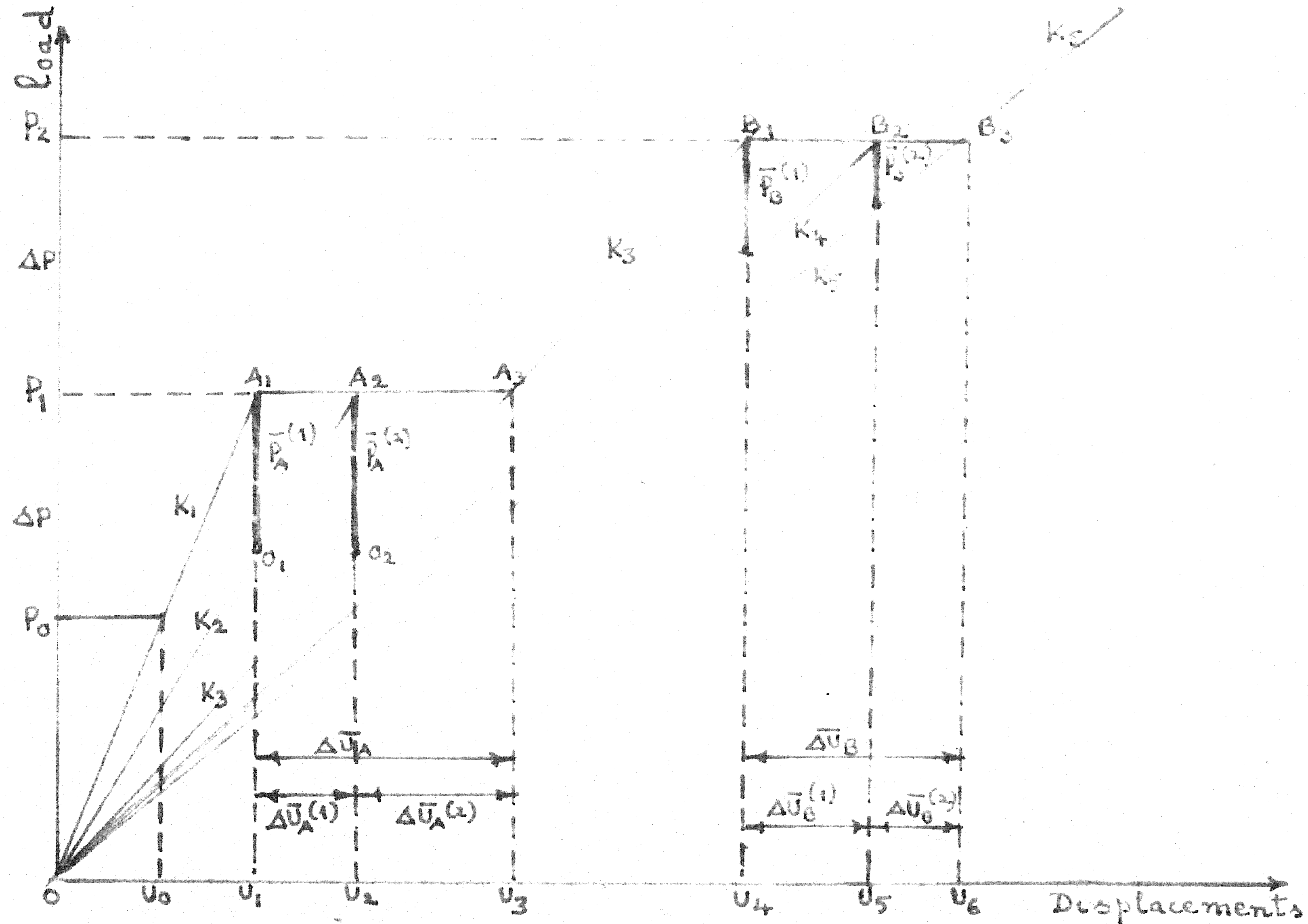


FIG. 4.5. INITIAL STRESS METHOD USING VARIABLE STIFFNESS (SCHEMATIC)

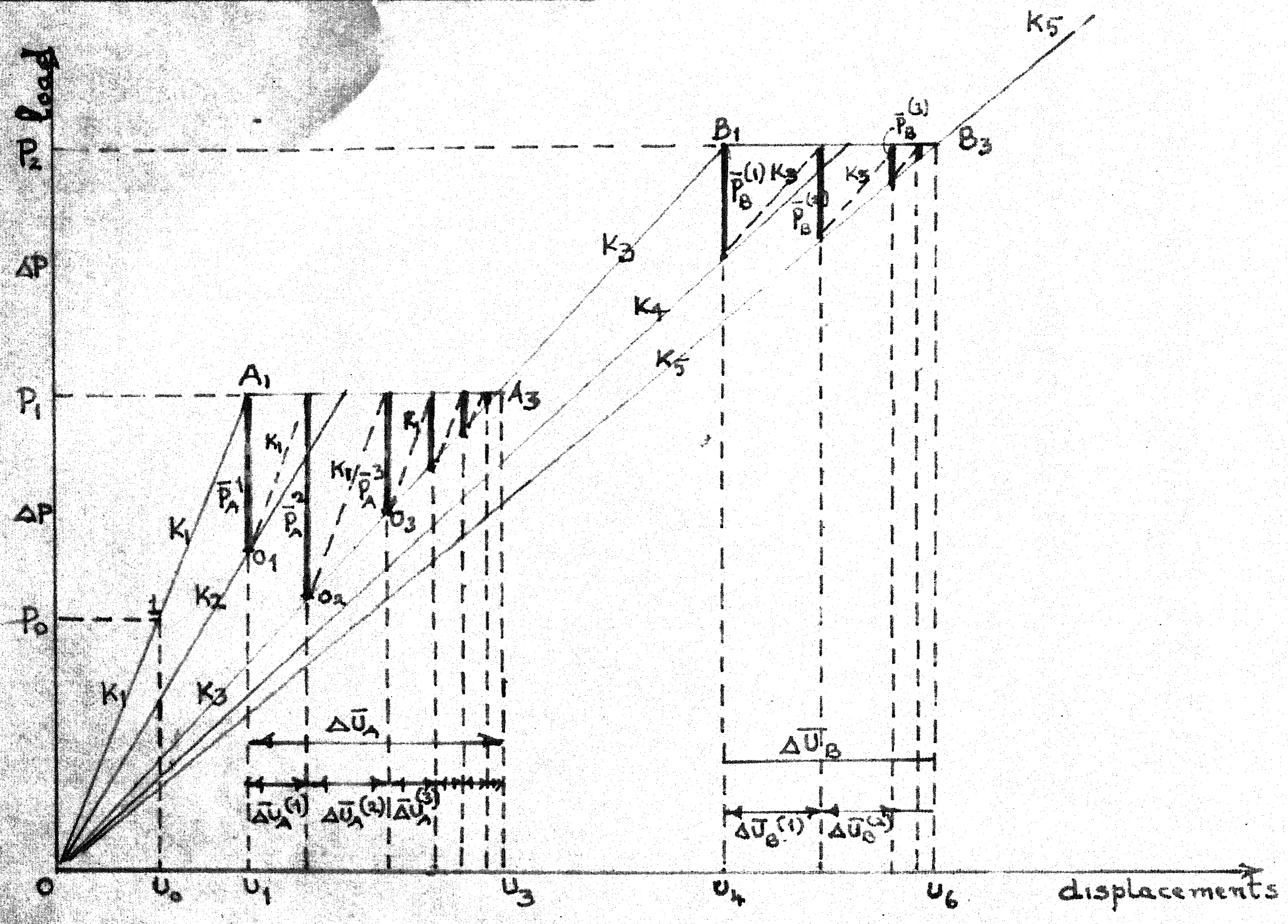


FIG. 4.6. INITIAL STRESS METHOD USING CONSTANT STIFFNESS (SCHEMATIC)

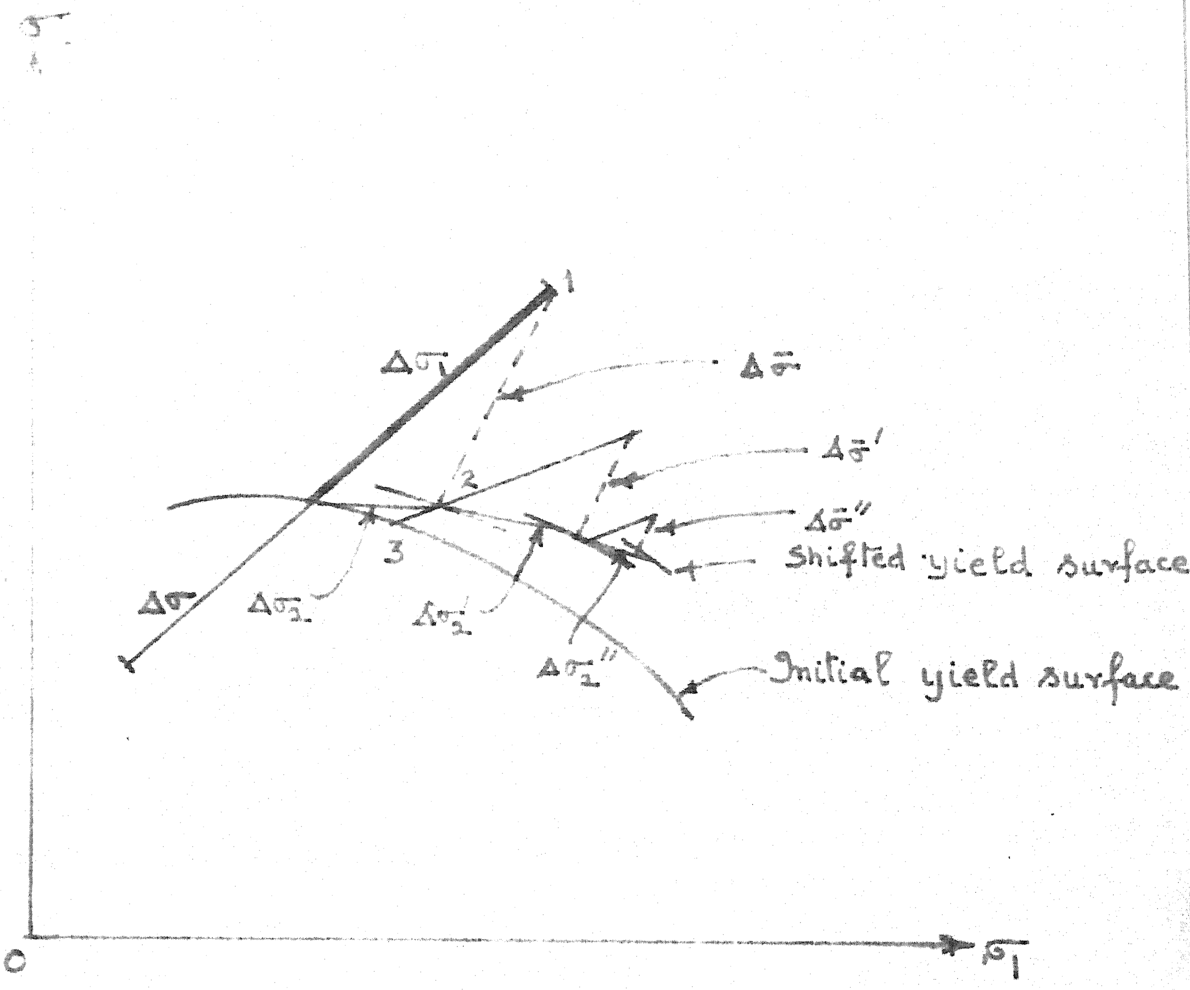


FIG. 4.7. SCHEMATIC ILLUSTRATION OF THE DISTRIBUTION OF PSEUDO STRESSES DUE TO BIAXIAL PLASTICITY OF CONCRETE OR BRICKWORK

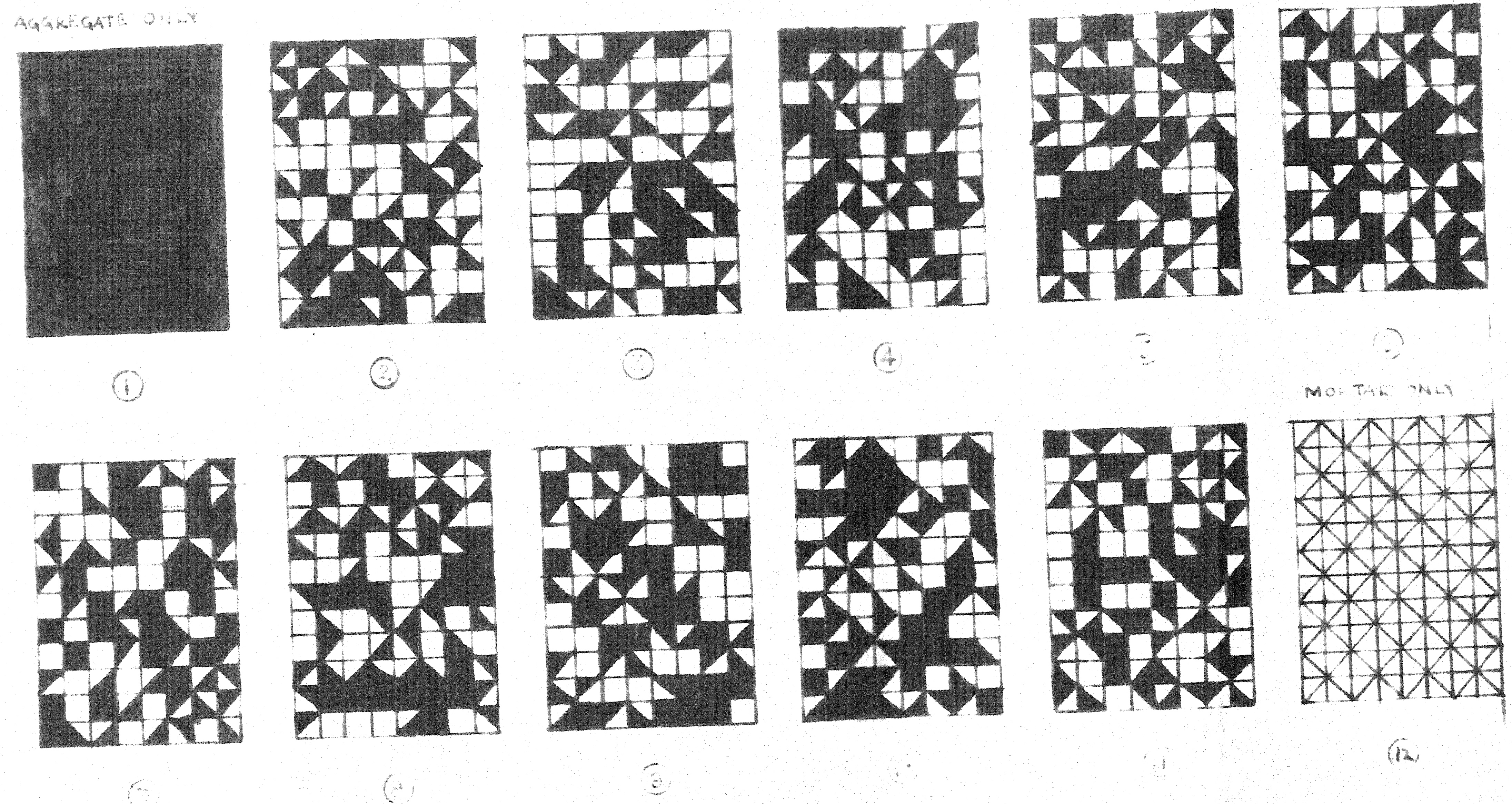


FIG 4.8 RANDOM ARRANGEMENT OF AGGREGATE AND MORTAR IN CONCRETE
(COMPUTER SIMULATION)

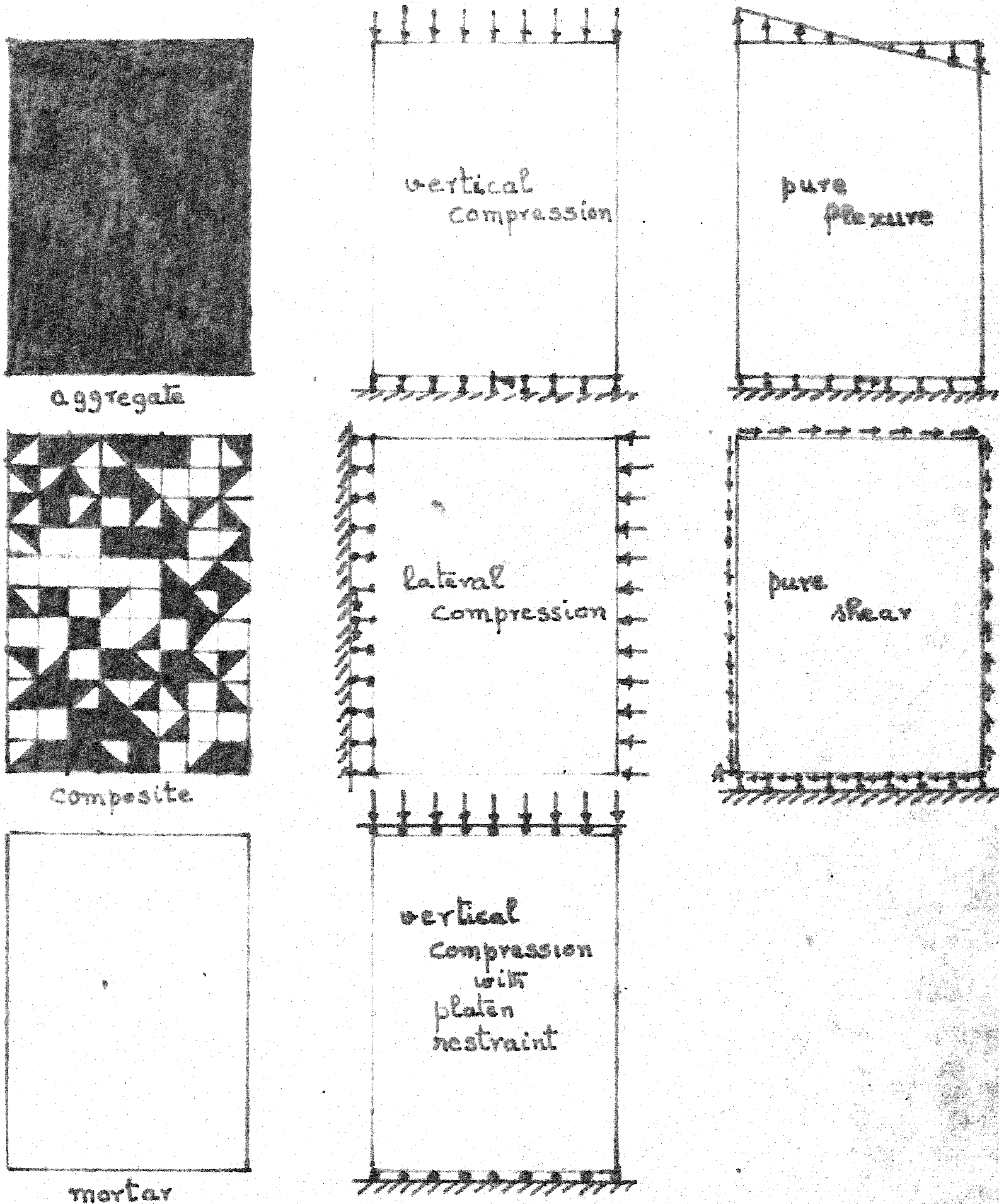


FIG. 4.9. SIMULATED LOADING ON UNIFORM AND RANDOM MATERIAL

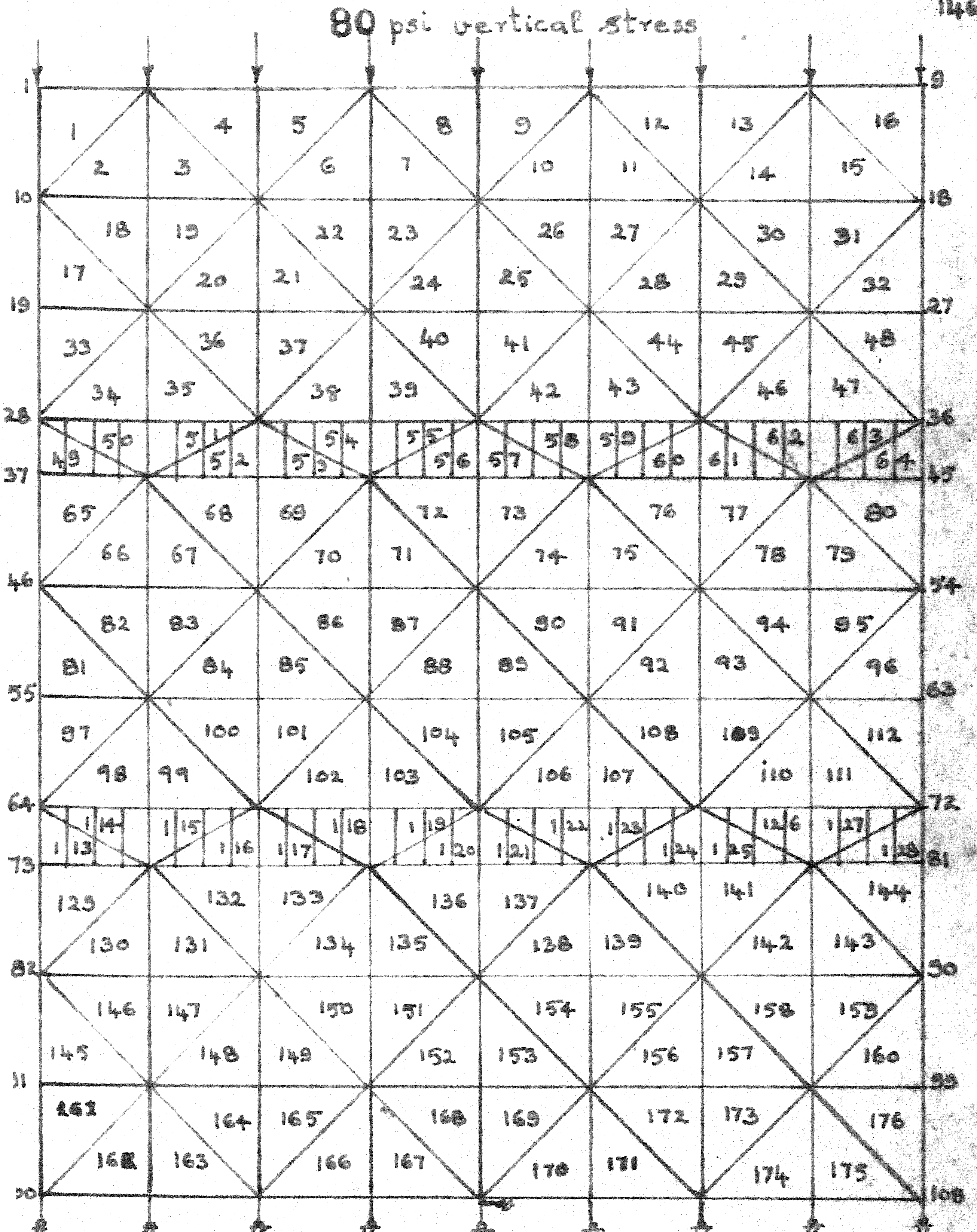


FIG. 4.10. FINITE ELEMENT APPROXIMATION OF BRICKWORK

$E_b = 610000$ psi
$\nu_b = 0.10$

$E_m = 1750,000$ psi
$\nu_m = 0.25$

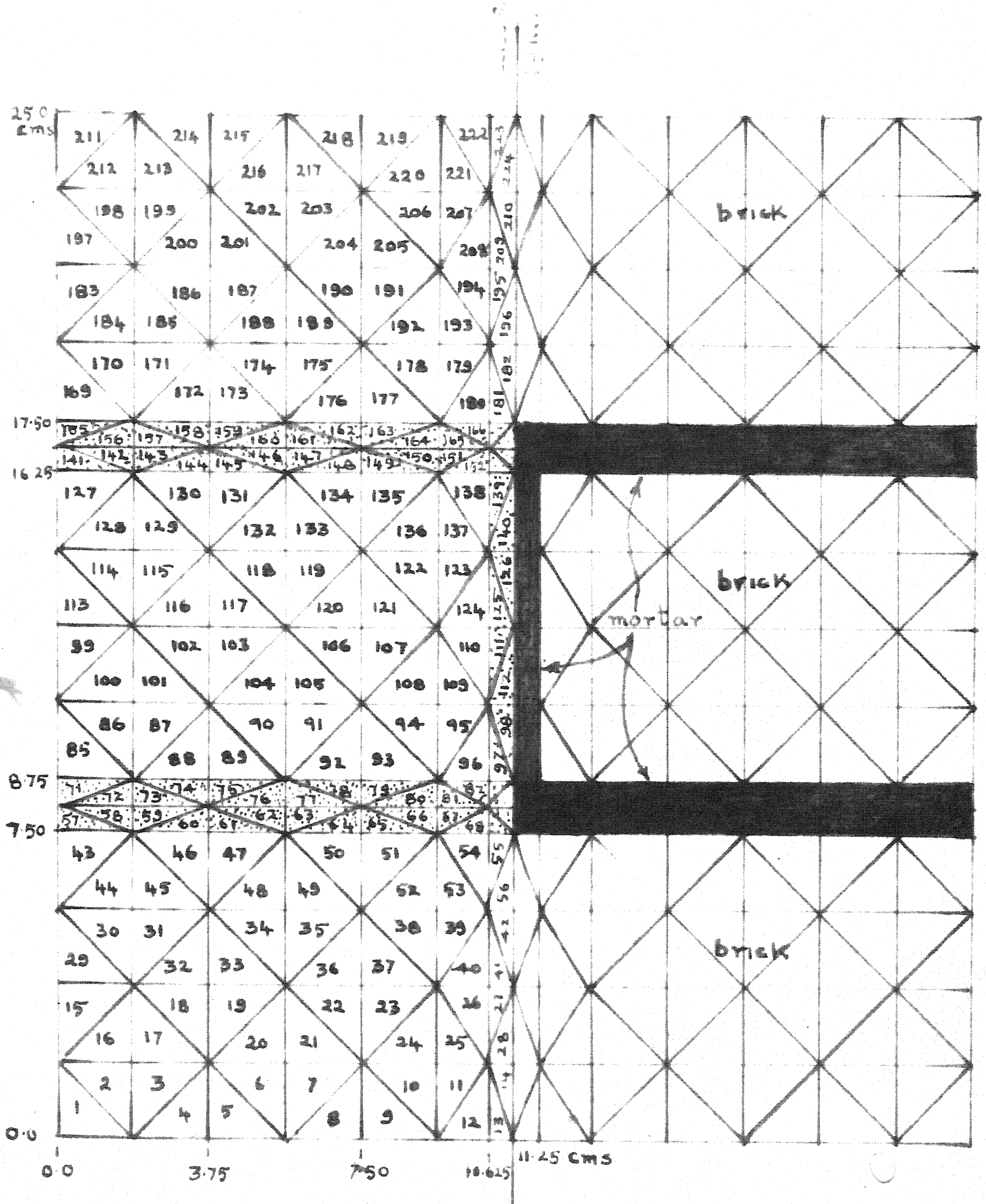


FIG. 4.11. FINITE ELEMENT SUBDIVISION OF BRICKWORK

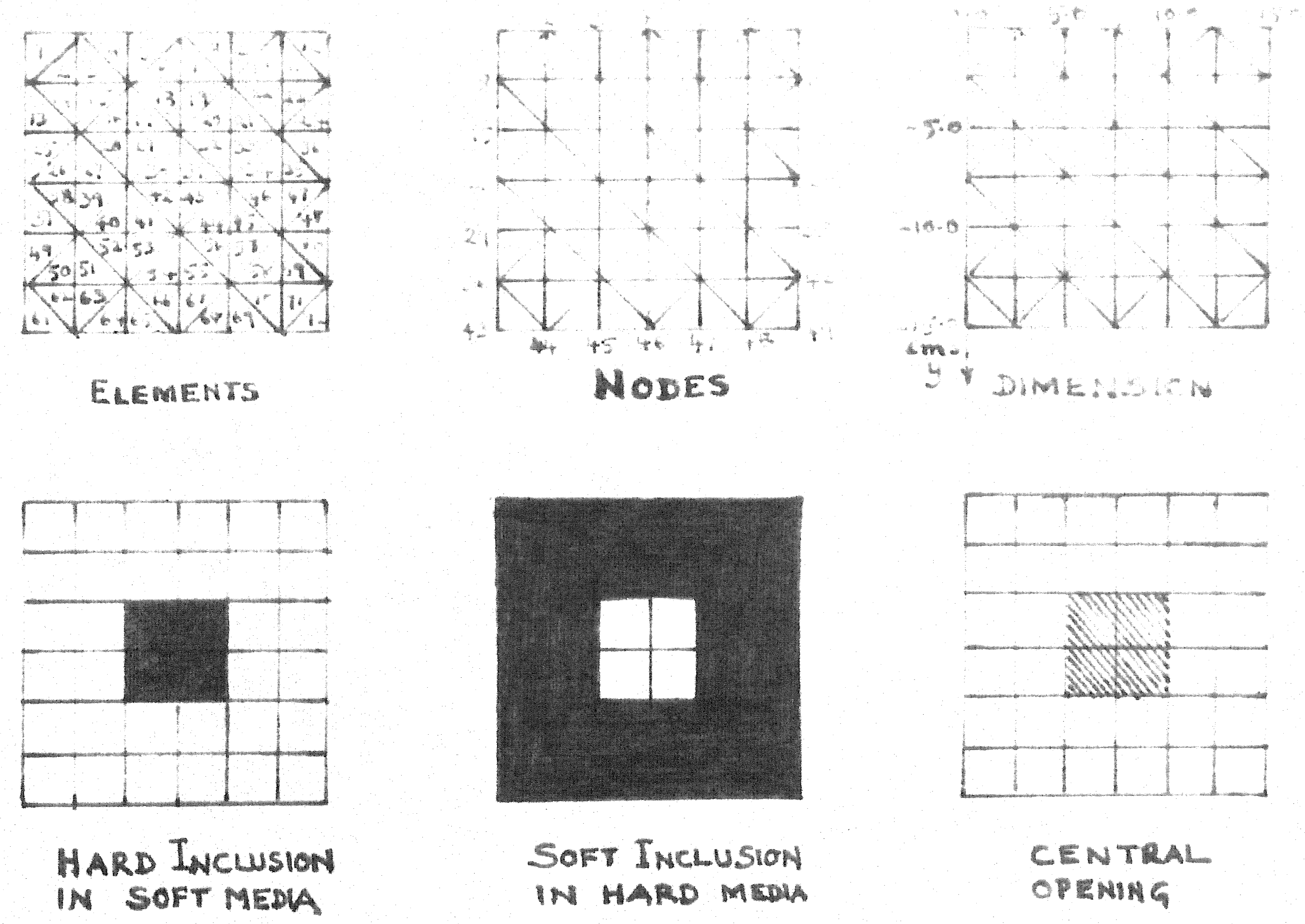


Fig. 4.12. Inclusions and Openings

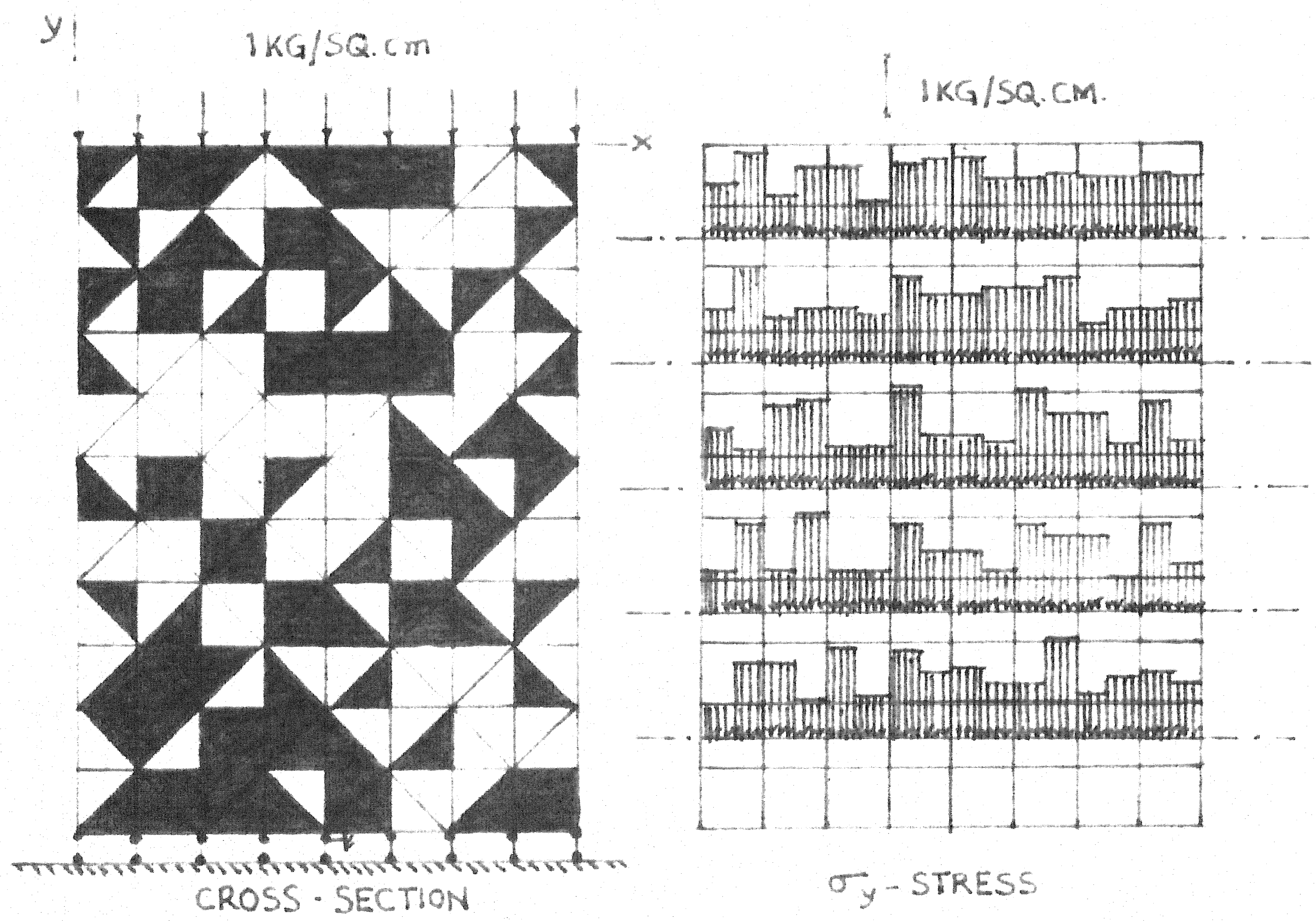
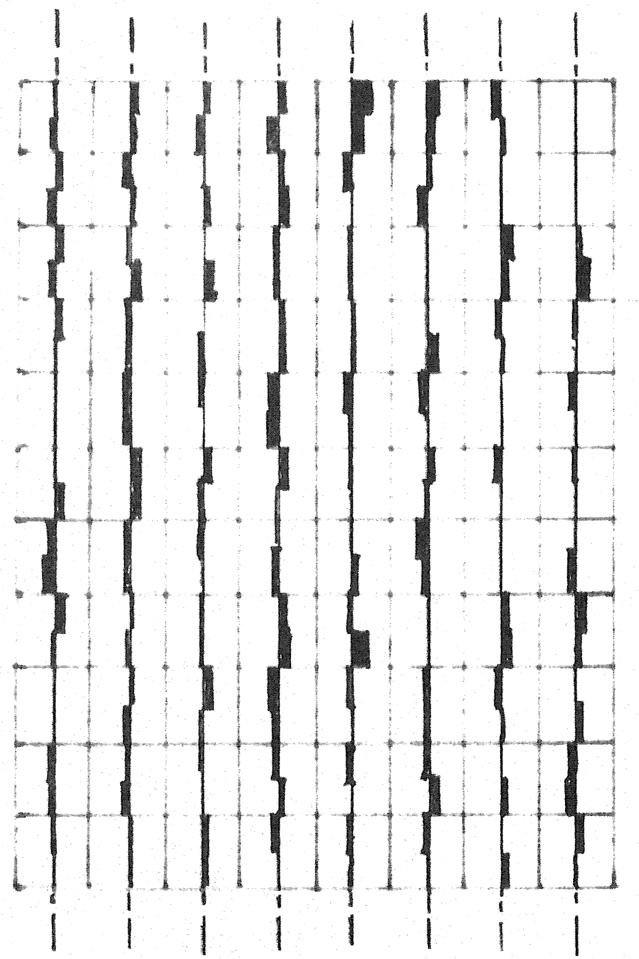
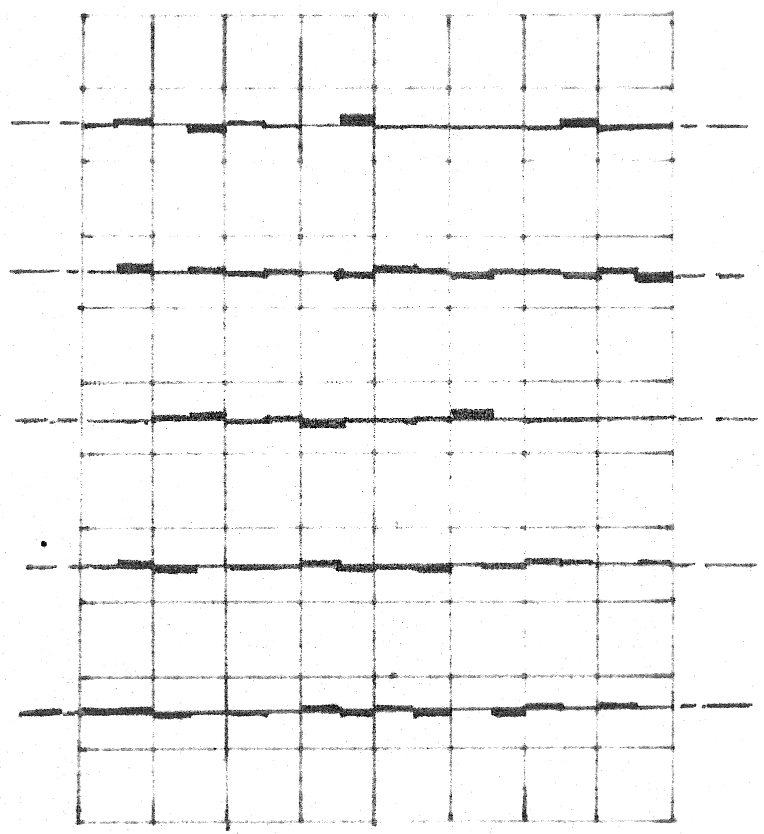


FIG. 4.13(a). STRESS DISTRIBUTION IN NONHOMOGENEOUS MEDIA.



σ_x STRESS



τ_{xy} STRESS

FIG. 4.13(b) STRESS DISTRIBUTION IN NONHOMOGENEOUS MEDIA

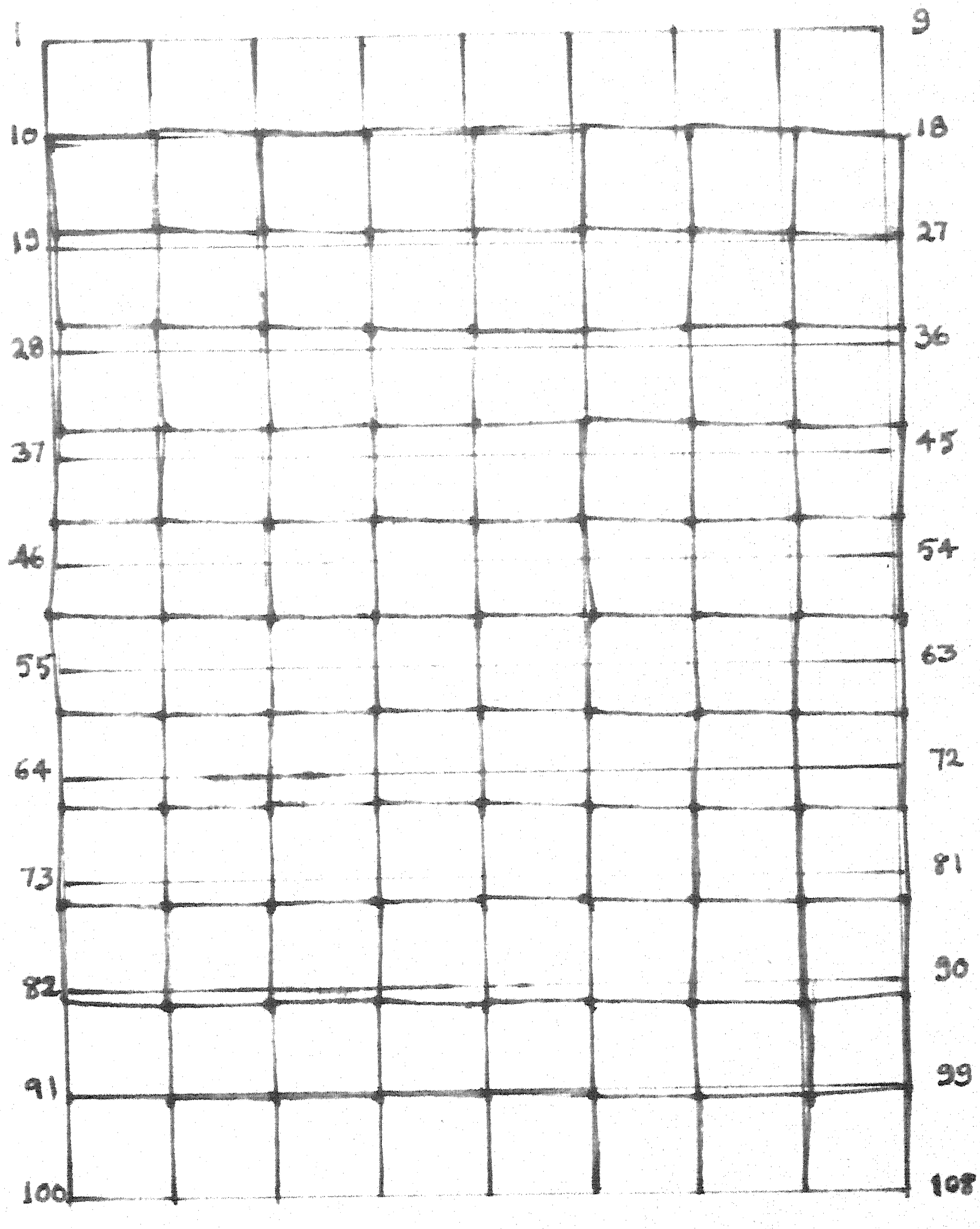


FIG. 4.14. SHAPE OF DEFORMED PRISM

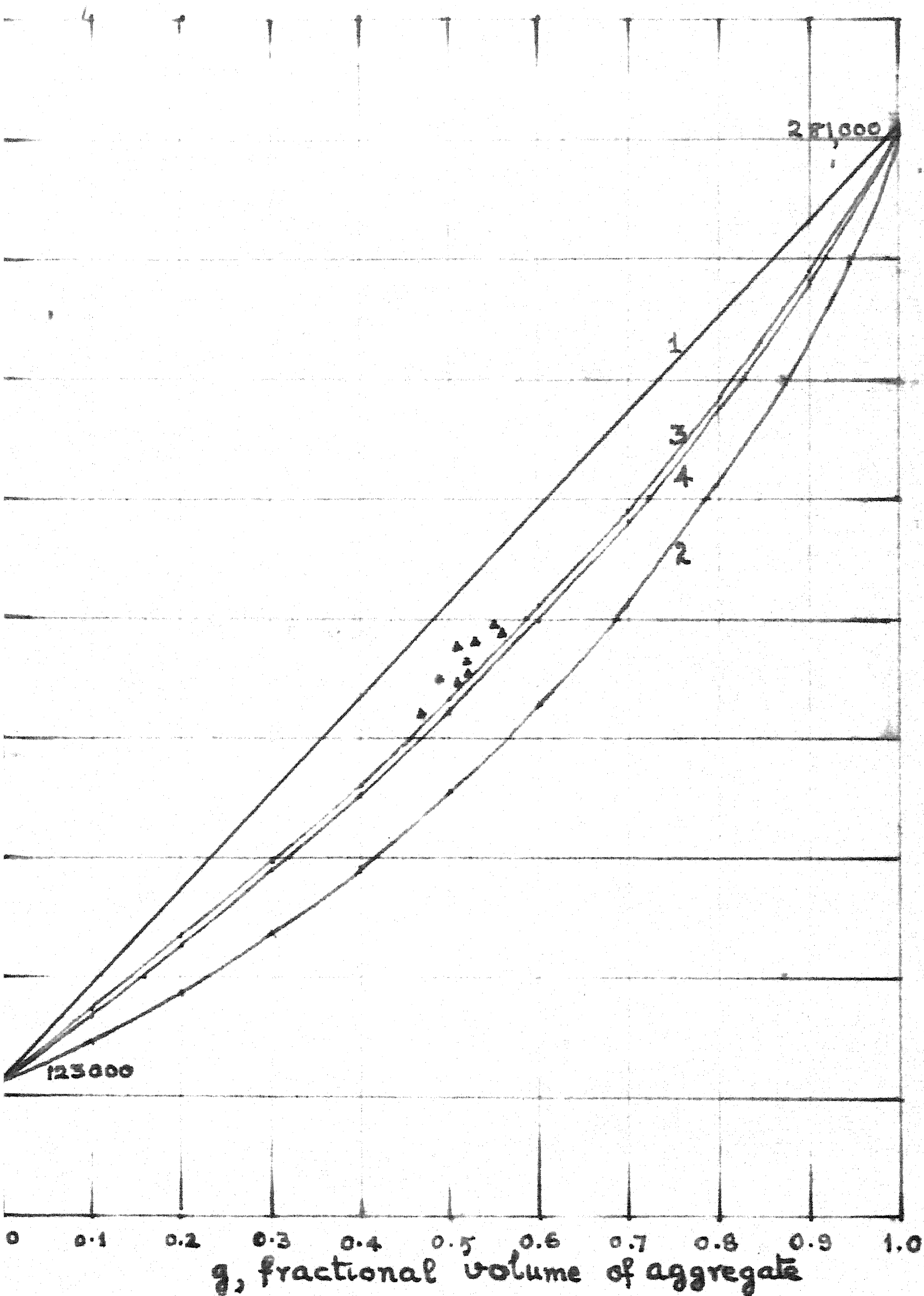


FIG. 4.15. CALCULATED E AS FUNCTION OF VOLUME FRACTION

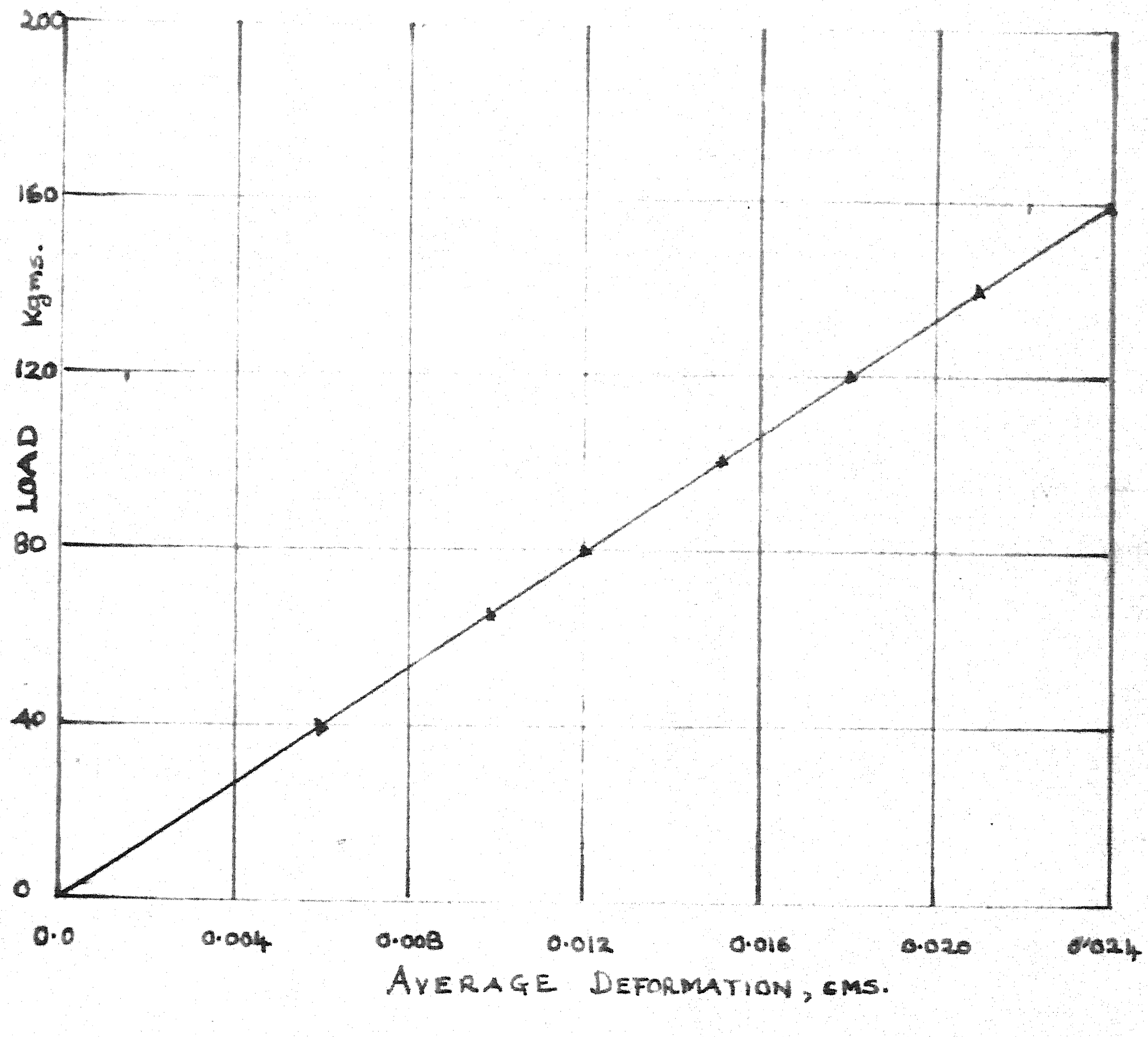


FIG. 4.16. EFFECT OF REDUCTION IN ELEMENT STIFFNESS DUE TO PROGRESSIVE CRACKING.

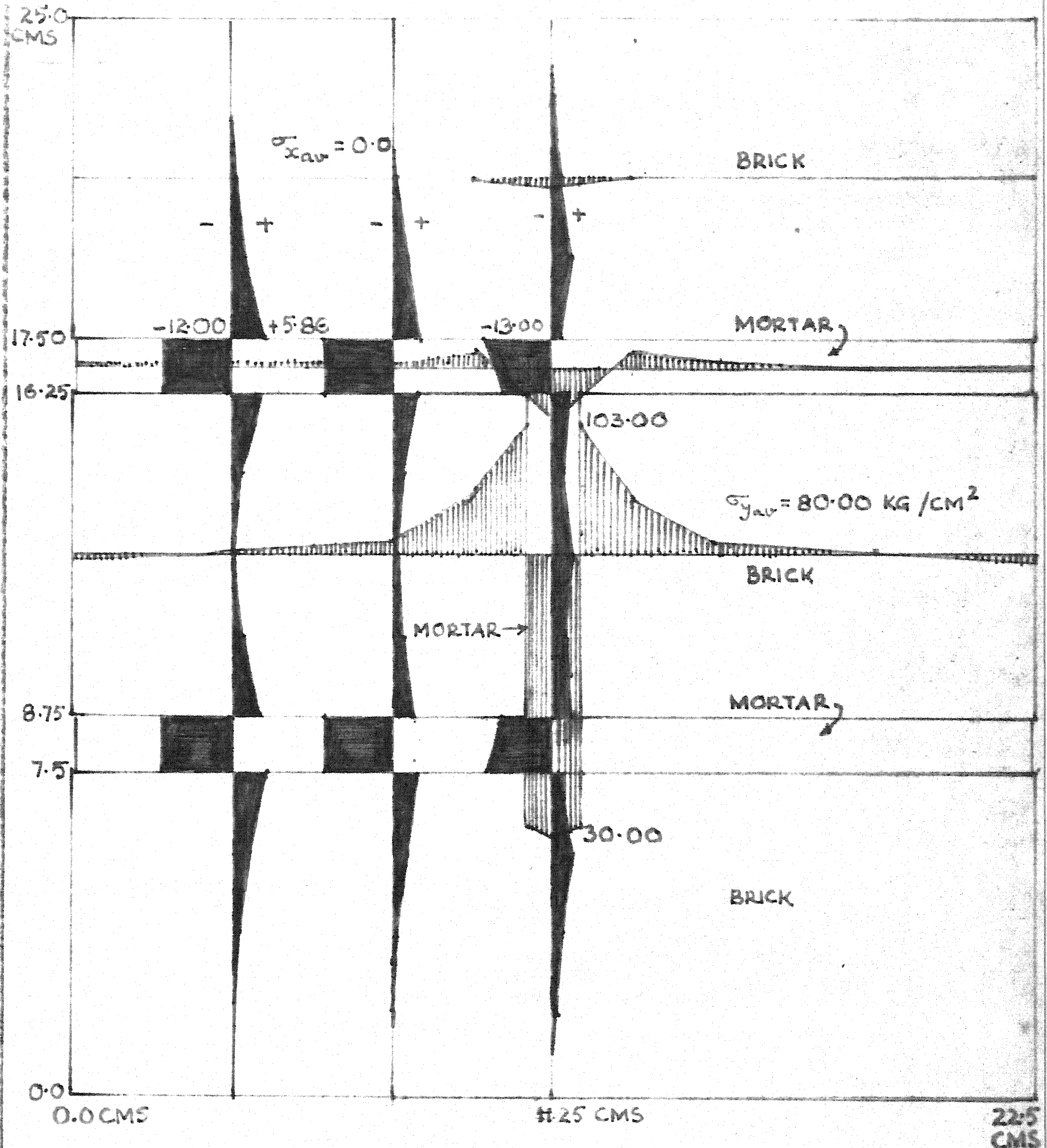


FIG. 4.17. VERTICAL AND LATERAL STRESSES IN BRICKWORK

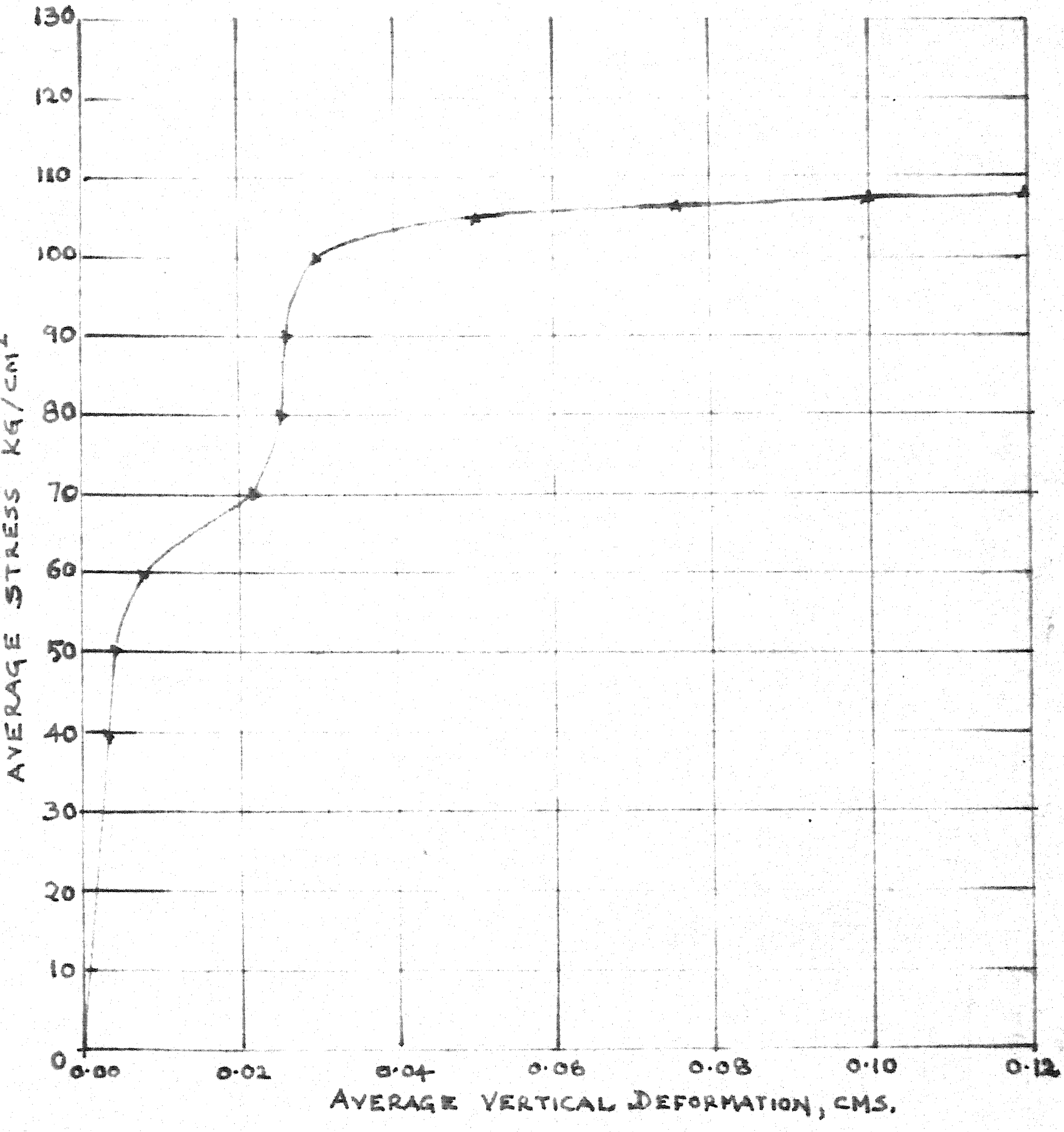


FIG. 4·18· EFFECT OF CRACKING IN MEDIA WITH OPENINGS.

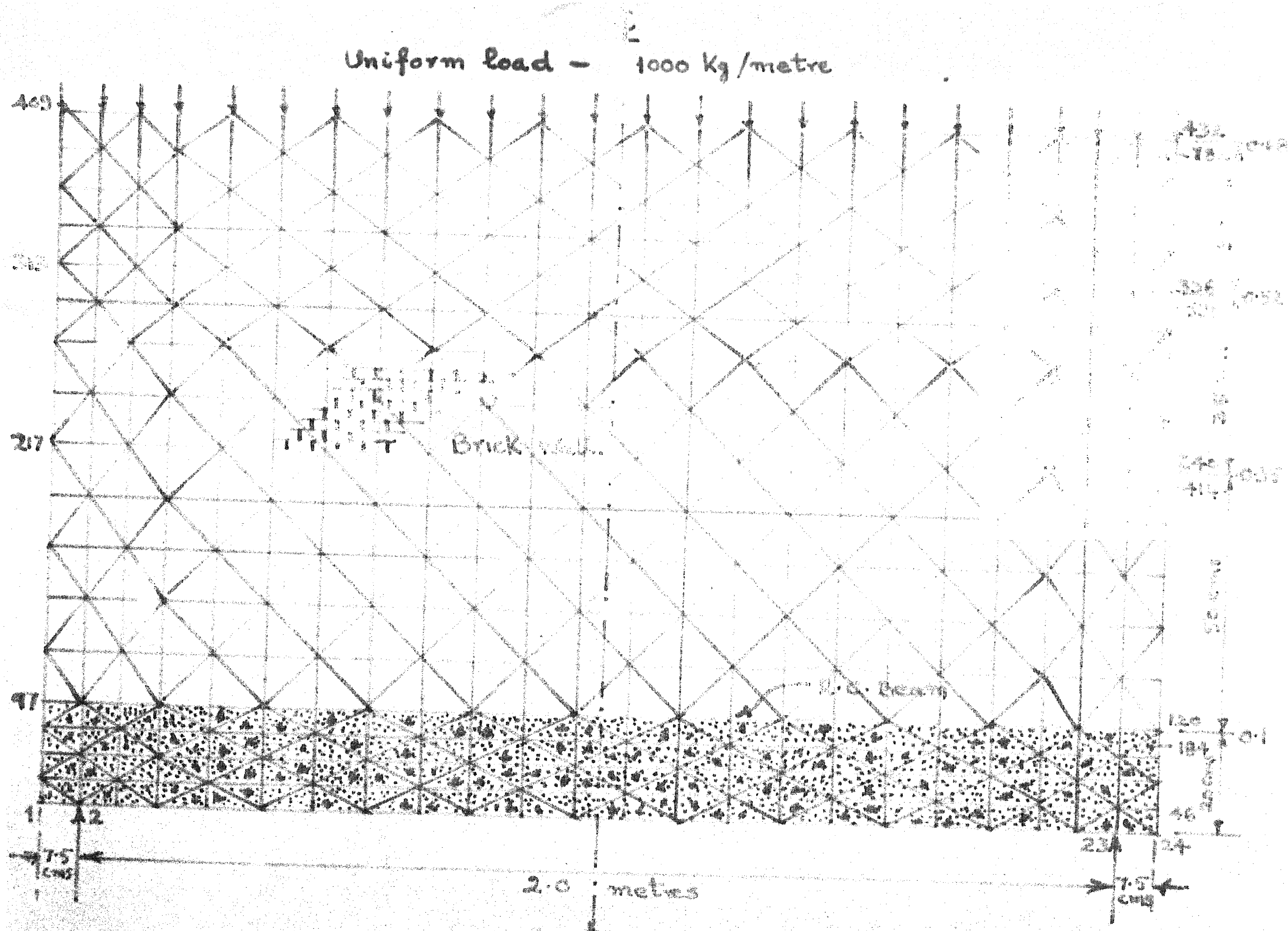


Fig. 4.19. Finite Element Representation of Brick walls on R.C. Beams.

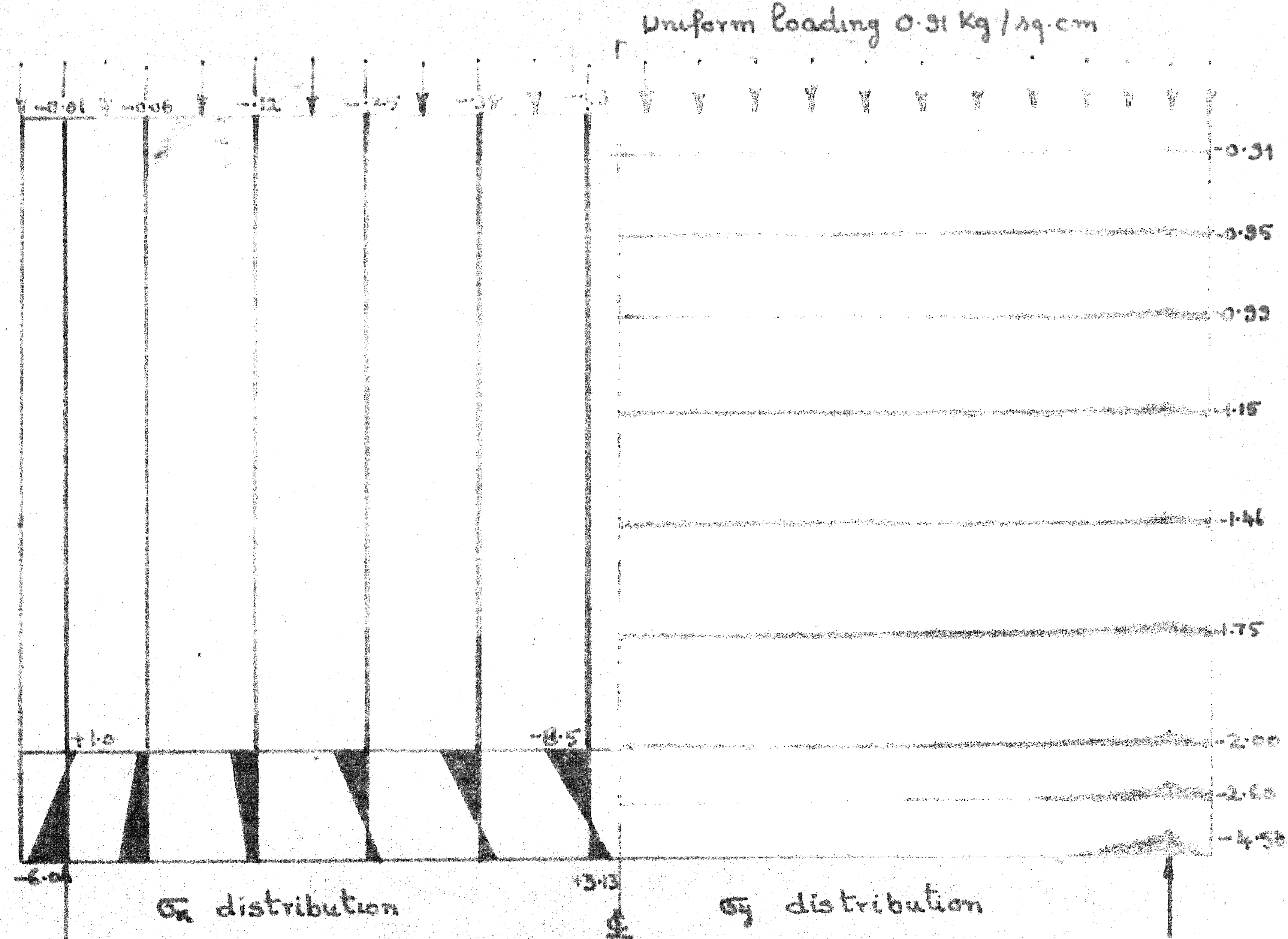


FIG. 4·20. STRESS DISTRIBUTION IN WALLS ON BEAMS (H/L = 0·68)

Fig. 4·21. σ_x - Stress Distribution (H/L Ratio: 0·52)

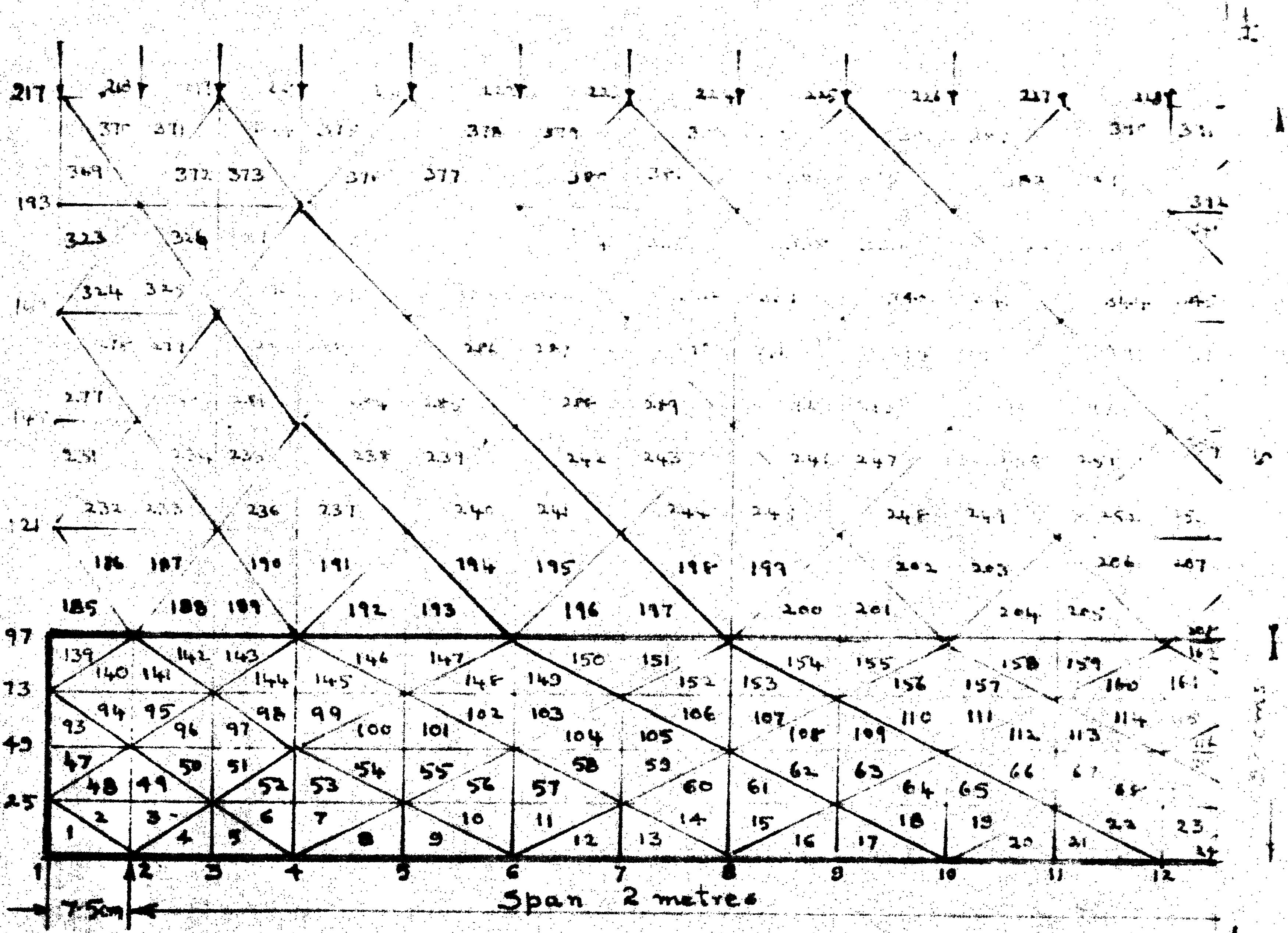


FIG. 4·22. FINITE ELEMENT DIVISION OF WALLS ON BEAMS (H/L = 0·35)

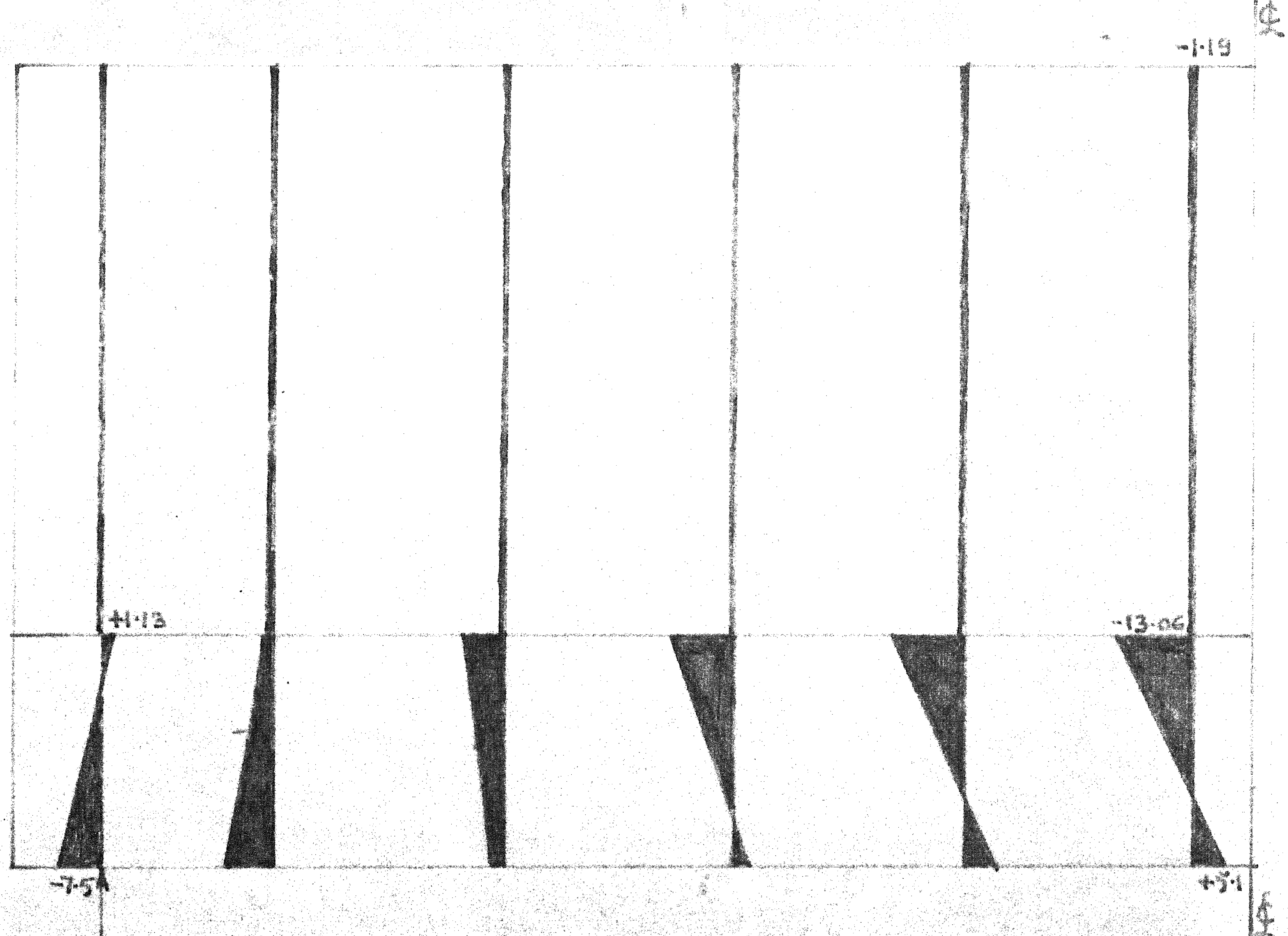


Fig. 4·23· σ_x-Distribution in Walls on Beams (H/L = 0·35)

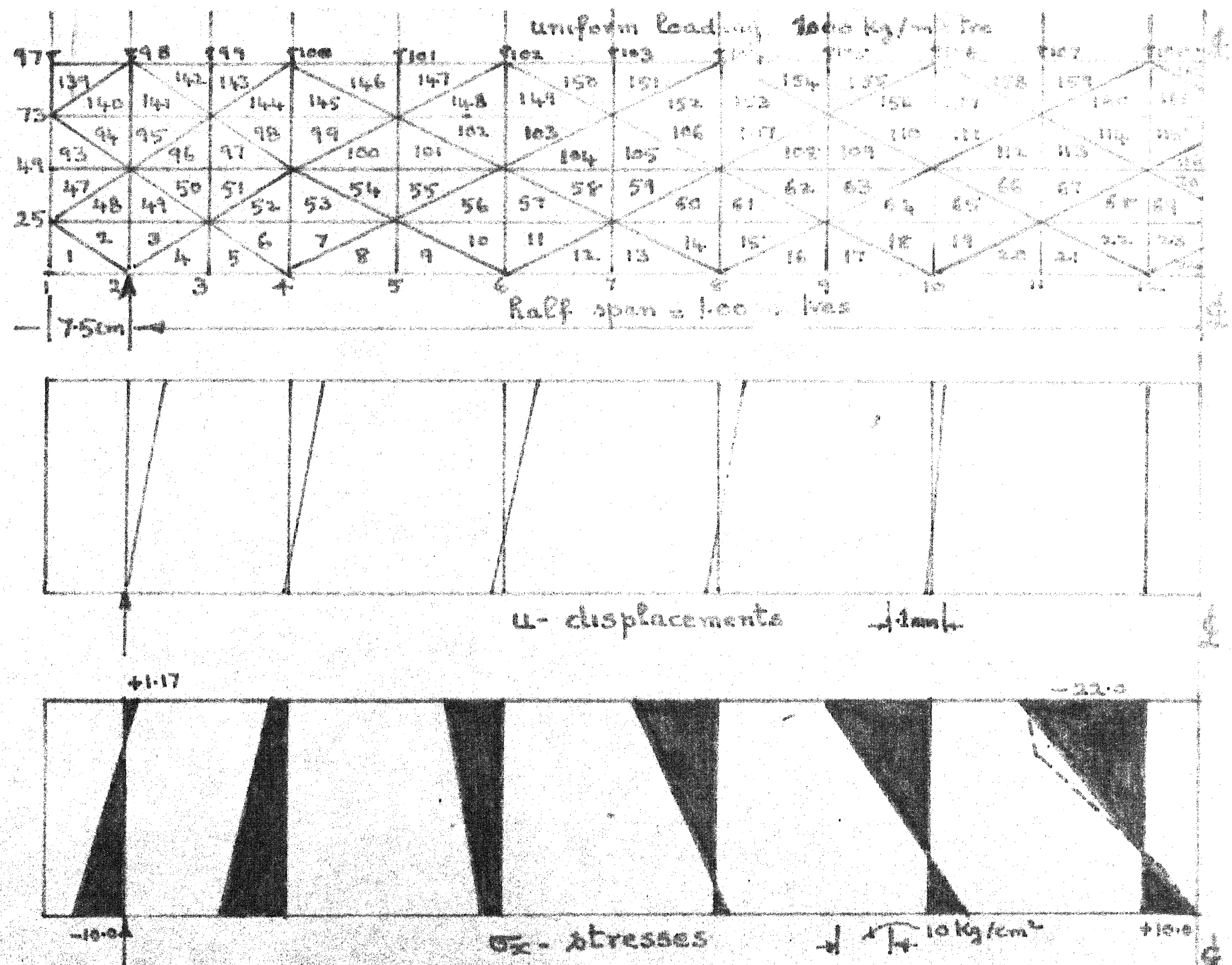


FIG. 4.25. FINITE ELEMENT DIVISION, DISPLACEMENTS AND σ_x-STRESSES IN R.C. BEAM

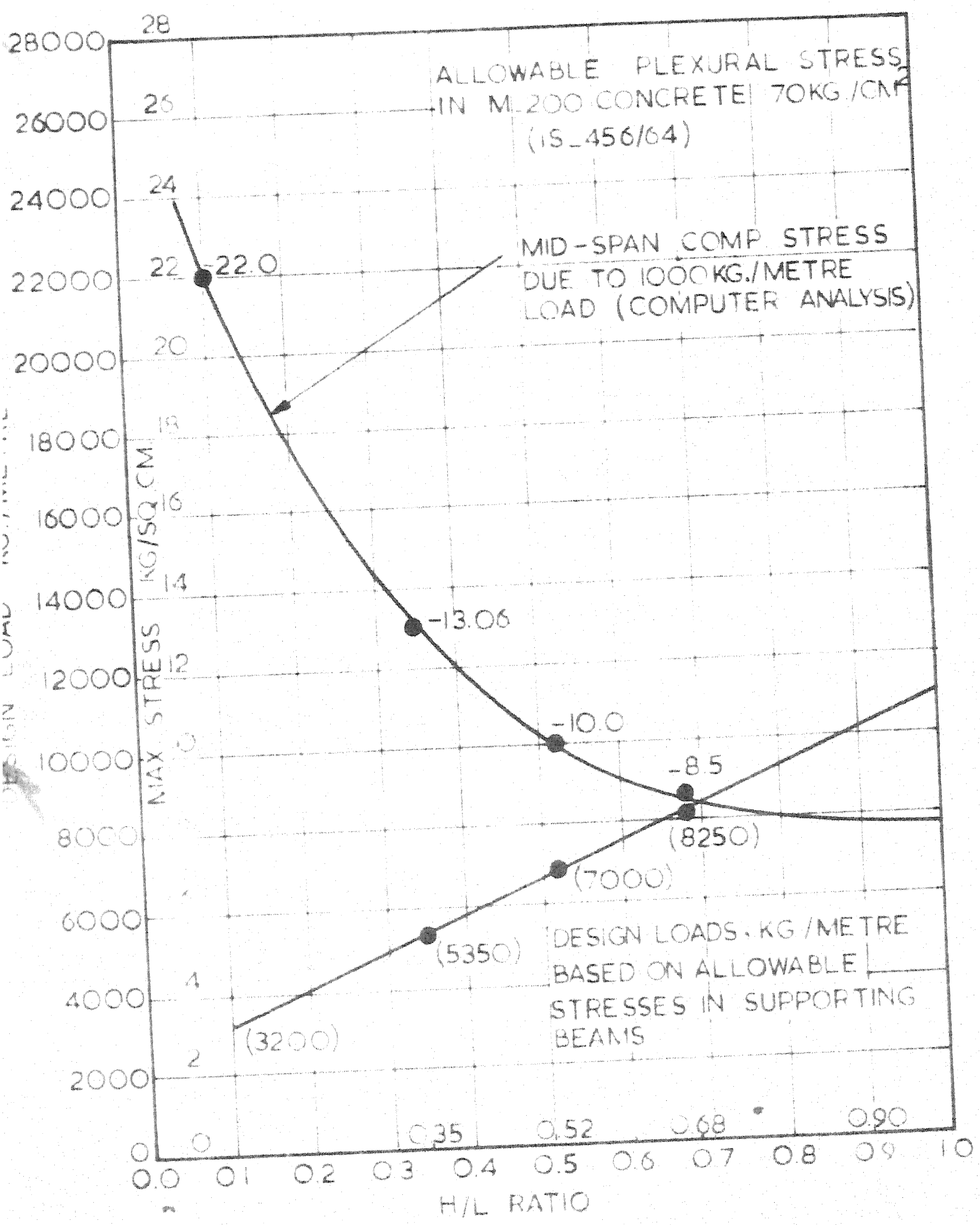


FIG. 4.26 _ PROPOSED DESIGN FOR WALLS ON BEAMS BASED ON ALLOWABLE STRESSES

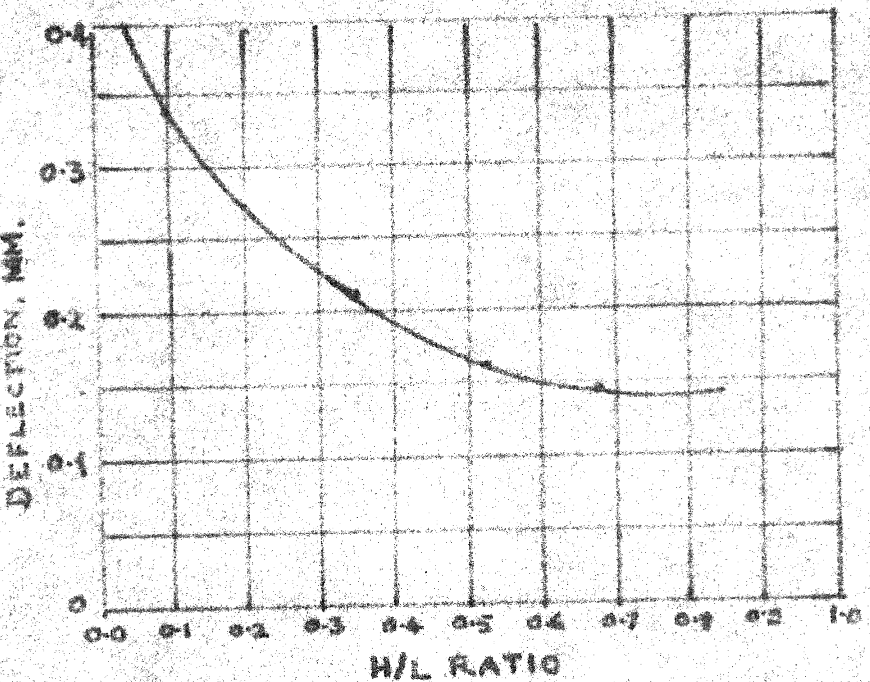


FIG. 4.27(a). H/L RATIO VS. CENTRAL DEFLECTION

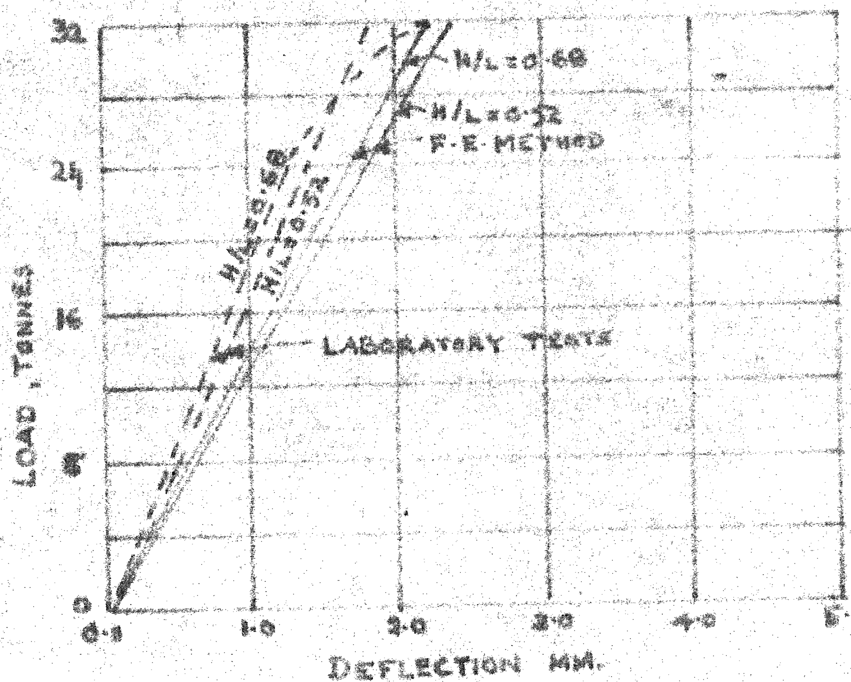


FIG. 4.27(b) LOAD VS. CENTRAL DEFLECTION

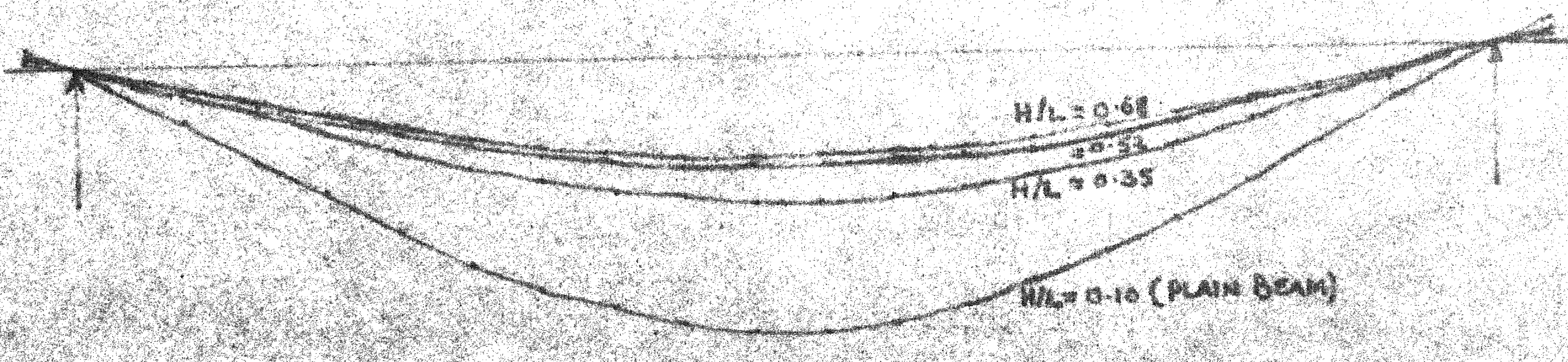


FIG. 4.27(c) DEFLECTION IN BOTTOM FIBRES OF SUPPORTING BEAMS

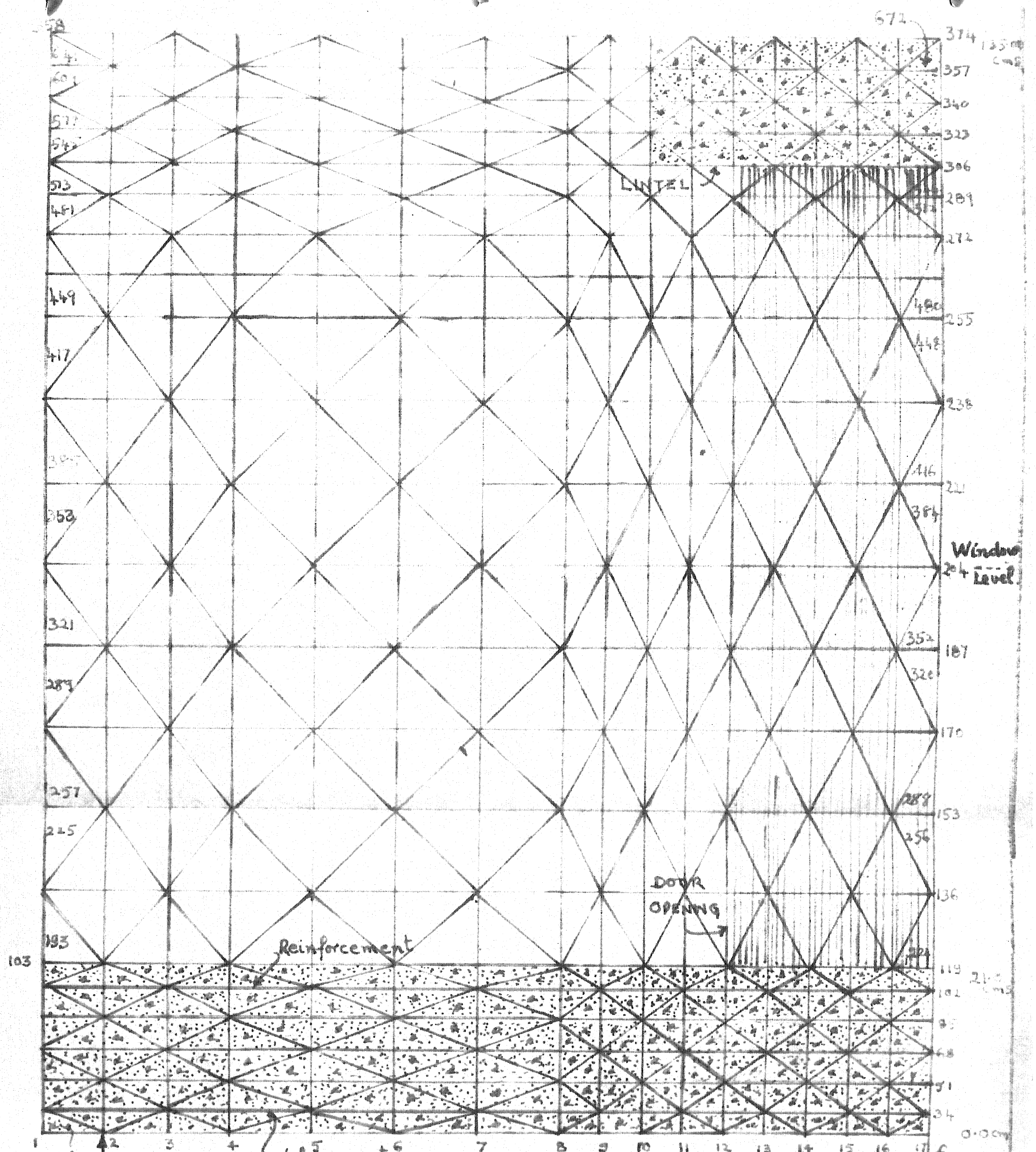


FIG. 4.28. WALLS ON BEAMS WITH OPENINGS (H/L = 0.68) (symmetrical)

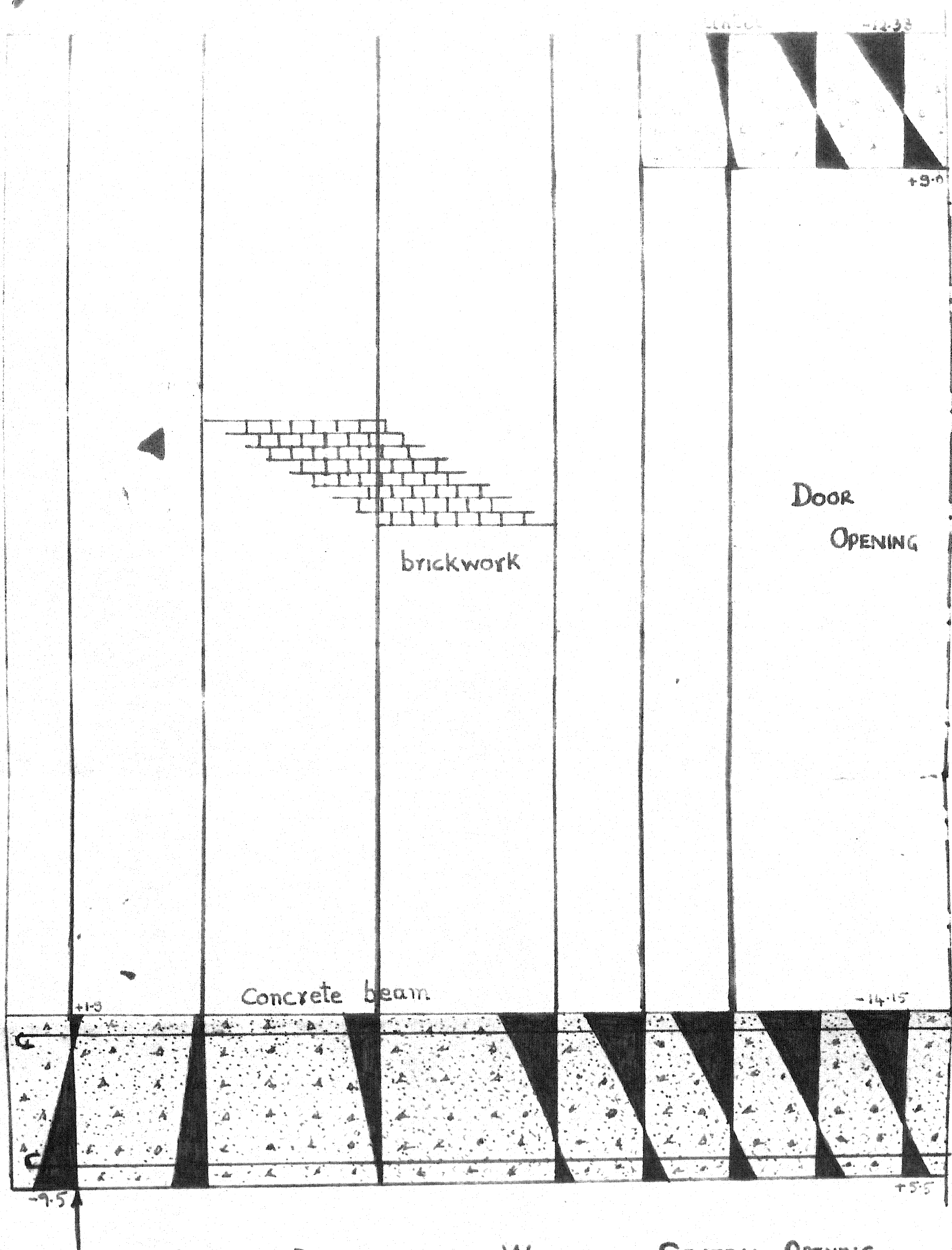


Fig. 4.29. σ_x - Distribution for Walls with Central Opening.

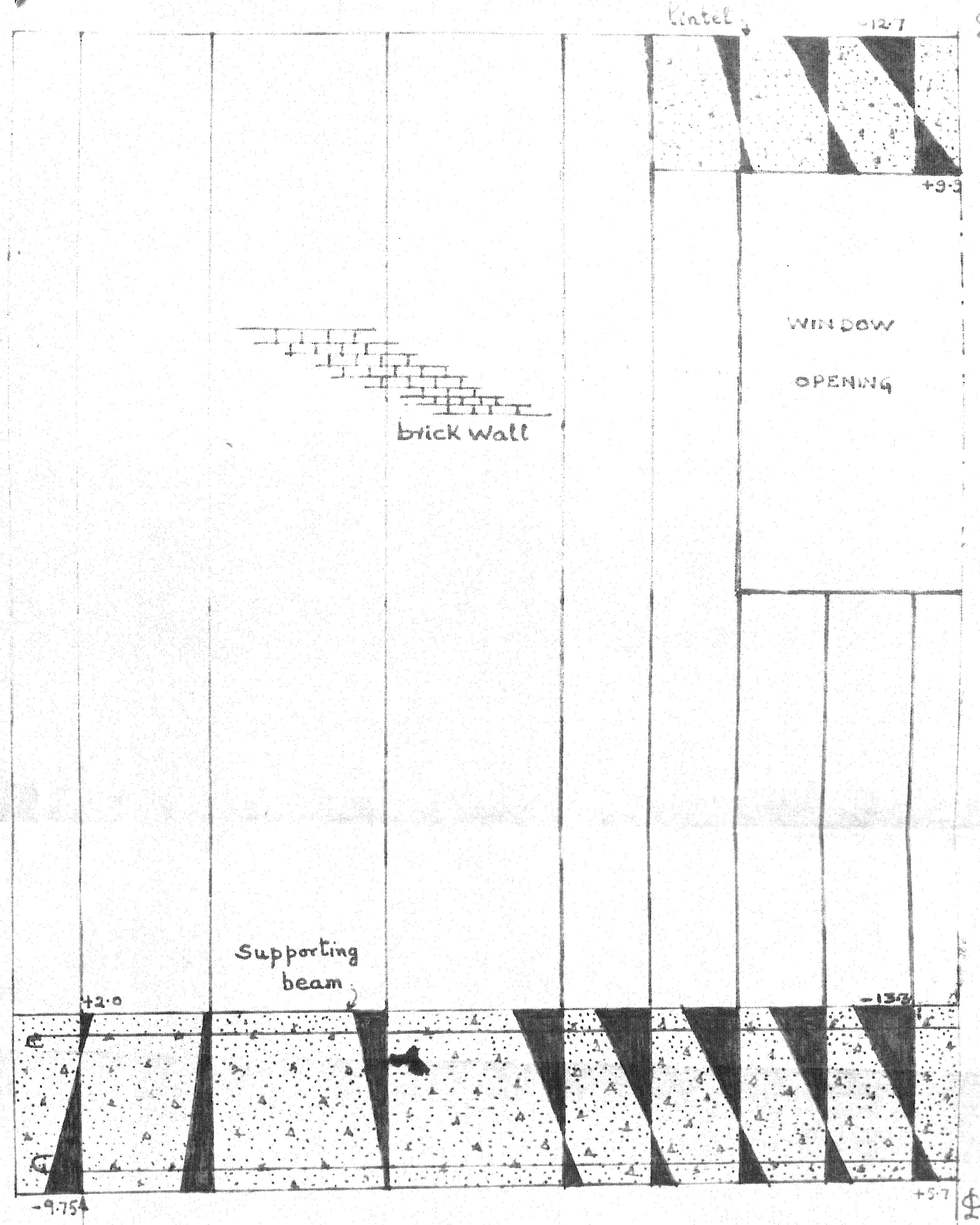


Fig. 4.30. σ_x Stresses in Walls with Window Opening

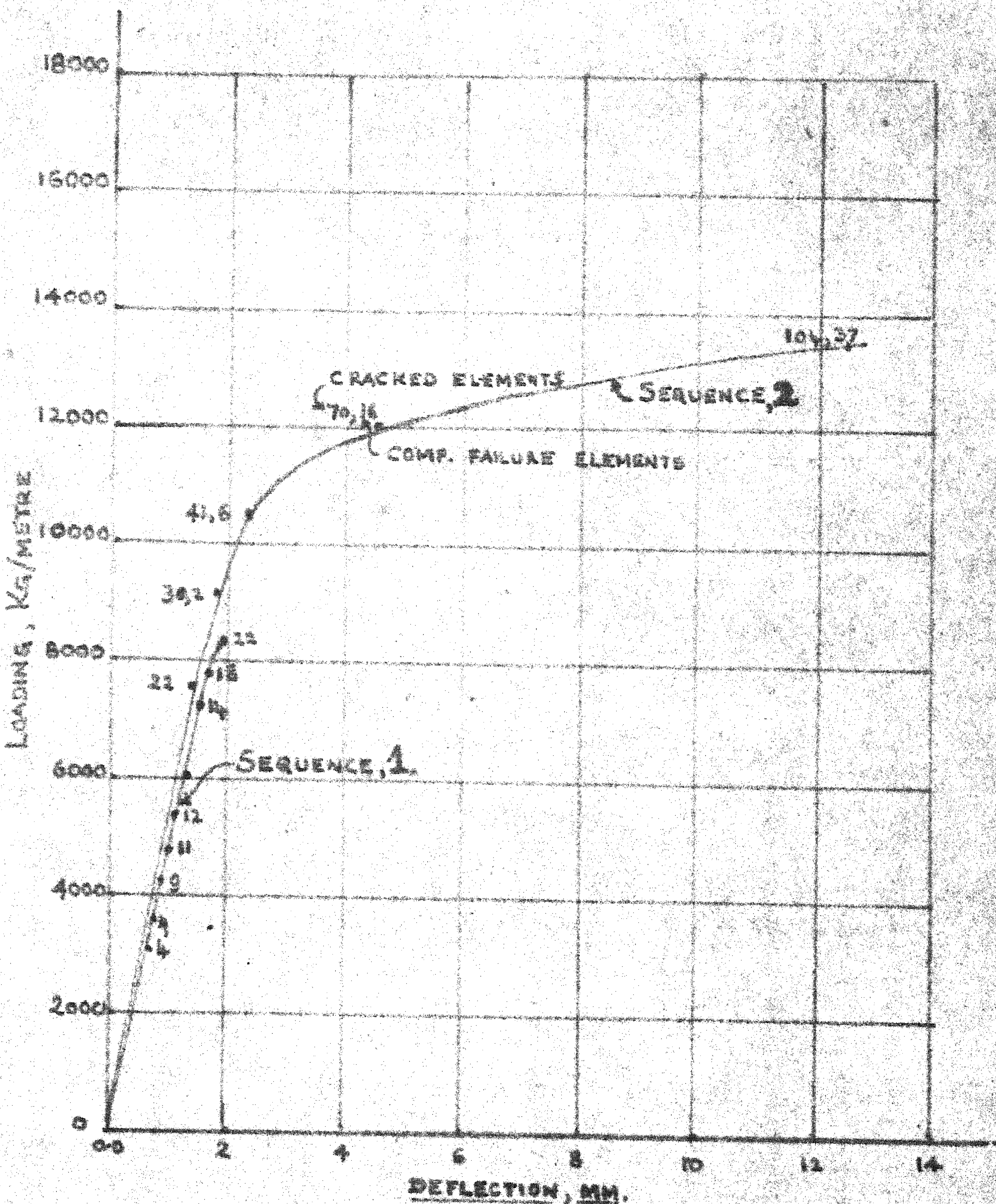


FIG. 4.31. EFFECT OF CRACKING AND LOCAL COMPRESSION FAILURES ON LOAD-DEFLECTION CURVES
(Computer Simulation)

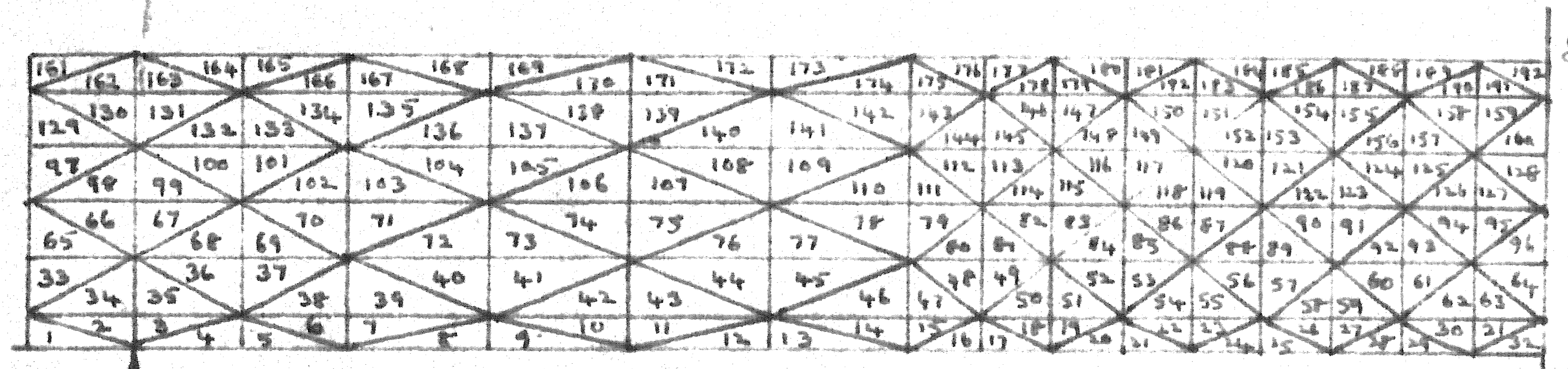


FIG. 4.32(a). ELEMENTS IN SUPPORTING BEAM (PROGRESSIVE LOADING)

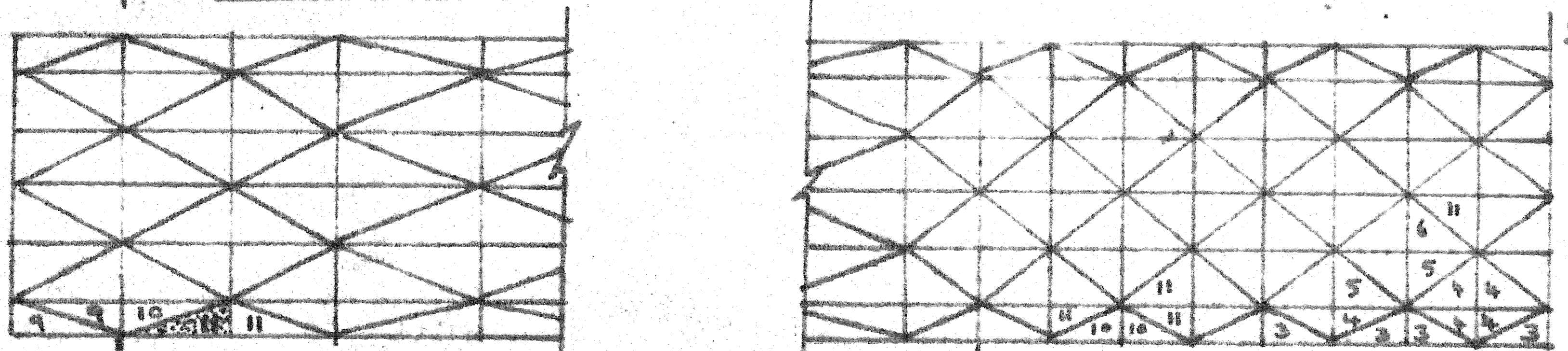


FIG. 4.32(b) and (c) CRACKING AND YIELDING AS PER LOADING SEQUENCE IN TABLE 4.1

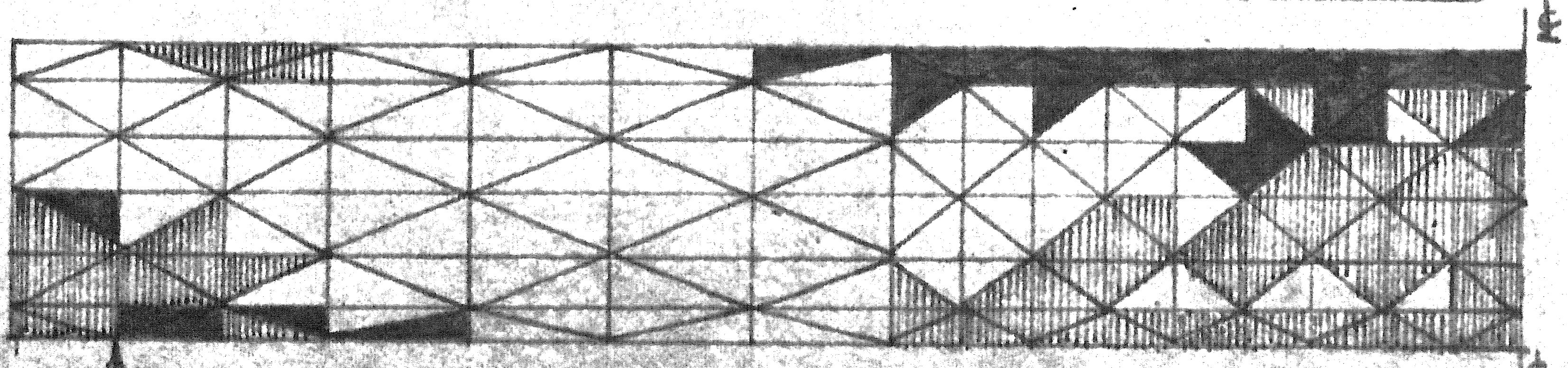


FIG. 4.32(d) LOCAL FAILURES IN VARIOUS ELEMENTS AT COLLAPSE LOAD

CHAPTER V

EXPERIMENTAL INVESTIGATIONS

Previous investigations on 'walls on beams' have been reviewed in Chapter 2 and it has been indicated that the height/span ratio, effect of openings, and the details of the foundation beam have all been studied in some detail. However, the nature of loading, compressive at top level and tensile loading at the foundation beam level, has not received adequate attention. This aspect of the problem has been taken to be the primary objective of the present investigation. Variations in height/span ratio, different mortar proportions, and the effects of openings which have been all studied by earlier investigators were also considered as secondary variables in this investigation. Subject to the restriction imposed by the availability of sophisticated loading and measuring equipment, the tests had to be planned on a rational basis to yield the maximum of results. A brief description of the experimental programme follows wherein sufficient attention has been given to basic properties of materials which have been investigated theoretically, besides the behaviour of walls on beams.

5.1. Basic Details

In this section, these aspects of the experimental programme which are common to all tests are briefly presented.

a) Materials. Bricks available at Kanpur, India were hand moulded, dried in the sun and burnt in country kilns with coal or firewood. One lot of 5000 bricks was obtained commercially and used in the tests. The bricks had average dimensions of 22.85 cms. x 11.06 cms. x 6.75 cms. The average compressive strength of bricks was found to be 345 kg/cm.^{2}, as per ASTM tests. The modulus of rupture had an average value of 42.6 kg/cm.^{2} The Young's modulus E and Poisson's ratio could not be reliably ascertained as the strain gauge readings were erroneous.

Cement used was Portland cement satisfying the relevant Indian Standard specifications. Four different proportions of mortar were used, 1:3, 1:5, 1:8 and 1:10, the proportions of cement and sand being measured by volume. No lime was added to the mortar, contrary to what has been observed in most foreign tests. Different walls were built with the above mortar proportions. The average compressive strengths of these mortars in descending order, as measured by 7 cm. cubes, are 206.0, 43.1, 21.10 and 11.7 kg/cm.^{2} The split tensile strengths are 22.8, 4.10, 0.0, 0.0 kg/cm.^{2} for the mortar proportions as above.

The brickwalls were fabricated by experienced masons. The average mortar thickness in bedding layers is 1.7 cms. The vertical joints varied in thickness from 1.5 cms to 2.0 cms. Control tests on brickwork cubes are reported else-

where. The walls were 11 cms. average thickness, and to maintain uniform thickness of mortar joints, wooden reepers of required thickness were kept as guides while the mortar layers (to receive the bricks) were prepared. Thus the width of mortar was 9.0 cms., as against the brickwall thickness of 11.0 cms. The top layers of the brickwork had a 2.0 cms. thick levelling course of mortar and undulations, if any, were made up by plaster of paris to receive the steel distribution girders.

Concrete. The foundation beams which supported the brickwalls were of reinforced concrete, the average size being 214 cms x 21 cms x 15 cms. While the brickwork was 11.0 cms in width, the supporting beam had a 15 cm width. The concrete mix used was of constant 1:2:3 proportions by weight with water-cement ratio of 0.5. The beams were cast in appropriate wooden moulds and vibrated with needle vibration in several layers. The cylinder compression tests indicated an average value of 370.0 kg/cm^2 and the split tensile strength was found to 32.4 kg/cm^2.

Reinforcement used was smooth, round mild steel bars of 10 mm diameter, two at top and two at bottom. The yield stress of this reinforcement was found to be 3160.0 kg/cm^2. The Young's modulus was found to be 2.045×10^6 kg/cm^2. The stirrups were of 6 mm. diameter, smooth round bars, spaced at a constant 15 cms intervals. The yield strength of stir-

rups was found to be 3000 kg/cm.^{2}

b) <u>Loading</u>. The load simulated in the tests was uniformly distributed load. For compression loading two 30 tons hydraulically operated jacks, fed by a power operated 20 H.P., high pressure (10000 psi) pump were used. The loads could be maintained reasonably constant by suitable control valves. The load from the jacks was distributed through a system of girders on to the wall, to simulate a uniformly distributed load. The reactions from the jacks were transmitted to the heavy test floor, through a system of reaction girders and tension rods. The test floor had reaction holes at 50 cms intervals and this imposed restrictions on the span and loading points. For the tension loading at bottom level of beam, hooks were embedded in the concrete beam with appropriate anchorage and bond devices, and were pulled down by tension bars connected to reaction girders and these girders were once again loaded by the hydraulic jacks mentioned earlier. The twin box type test floor with loading facilities at the bottom of the test floor was ideally suited for the purpose. The loading details for compression and tensile loading are sketched in Figs. 5.1 and 5.2.

c) <u>Instrumentation</u>

<u>Deflections</u>. In deep girders the deformations at various horizontal levels are known to be different. Yet dial deflectometers of 0.02 mm least count were used to measure

rups was found to be 3000 kg/cm^2.

b) Loading. The load simulated in the tests was uniformly distributed load. For compression loading two 30 tons hydraulically operated jacks, fed by a power operated 20 H.P., high pressure (10000 psi) pump were used. The loads could be maintained reasonably constant by suitable control valves. The load from the jacks was distributed through a system of girders on to the wall, to simulate a uniformly distributed load. The reactions from the jacks were transmitted to the heavy test floor, through a system of reaction girders and tension rods. The test floor had reaction holes at 50 cms intervals and this imposed restrictions on the span and loading points. For the tension loading at bottom level of beam, hooks were embedded in the concrete beam with appropriate anchorage and bond devices, and were pulled down by tension bars connected to reaction girders and these girders were once again loaded by the hydraulic jacks mentioned earlier. The twin box type test floor with loading facilities at the bottom of the test floor was ideally suited for the purpose. The loading details for compression and tensile loading are sketched in Figs. 5.1 and 5.2.

c) Instrumentation

Deflections. In deep girders the deformations at

deflections at the reinforced concrete beam bottom level only. Dial gauges were used to monitor lateral tilting, if any, of these slender walls. If there were eccentricities in the fabrication of the walls or the tilting of the foundation beams, (the lateral dial gauges indicating their nature and magnitude at top and bottom of the walls) corresponding adjustments were made in loading arrangement to counteract these effects. None of the specimens failed prematurely by tilting.

Slip. The slip between bottom most brick course and the R.C. beam was measured with the help of two dial gauges but the slips were too small in magnitude to be recorded.

Strain. At first, metal studs were patiently affixed to the bricks and concrete beams, to monitor the lateral and vertical strains; the erratic readings of the Whittmore demountable strain meters forced the discontinuation of this process and 6 in. single wire 120 ohm electrical resistance strain gauges were used to measure longitudinal strains and 1.5 cm. flat-grid 120 ohm strain gauges were used to record vertical strains and lateral splitting strains across the 11 cms thickness of brickwork. Large number of gauges were used and a battery of switching and balancing units of the S.R.4 Baldwin-Lima-Hamilton make were used to monitor the strains at more than one measuring stations.

Measurement of strains in brickwork and concrete using electrical strain gauges requires care and precision

if reliable records are aimed at. The best practice for preparing the surface, affixing the gauges and water-proofing were adopted and reasonable results were obtained subject to local stress concentrations, proximity of cracks and so on. The lead wires had to be not more than 4 meters in length to avoid the capacitance build up and compensating gauges had the same lead wire length as the measuring gauges. The gauge factor of the various gauges was set at 2.05 (constant) even though different lots were used with gauge factors varying from 2.01 to 2.10. The error due to this aspect for large strains was neglected while analysing the results.

d) Loads. The jacks and the associated hydraulic system were calibrated using a proving ring of 50000kg capacity and at small loads, 5 percent error was found to be inevitable. The specimens were loaded and unloaded several times to 25 percent of the first crack load to check the performance of the measuring and loading systems. Sometimes tests had to be postponed for a day or two until all things worked to satisfaction. The loading was continued at prescribed intervals so as to have adequate readings in the elastic and post-cracking stage, but near the collapse load adequate readings could not be recorded in order to remove valuable measuring equipment protecting them from possible damage as the specimen failed. Since the hydraulic loading was under remote control the loads could be recorded at failure accurately, and manual reading in proving rings etc. were avoided.

e) Other Aspects

Marginal Protections. The jacks and distribution girders were chained to rigid points in the loading apparatus to hang on even when the specimens failed. Only in one case there was a sudden premature collapse and the masonry literally flew apart. Otherwise gradual failures were noticed. To prevent lateral tilting marginal lateral restraint with wooden cross beams, lightly prestressed by threaded rods were used. This was found to be quite effective during instrumentation and testing. Otherwise the specimens, particularly of large depth/span ratios could have been tilted by a light push with the hand. In the case of the deepest beam with depth/span ratio of 0.90, the specimen was encased all round by an open crate of reepers of small dimensions which were supported by the brick masonry itself, not interacting in any way with the specimen itself. This precaution was taken to prevent bricks flying around at collapse.

f) Crack Observations. Cracks were marked with black marker as they progressed with loading. No efforts were made to measure the cracks with a hand microscope, even though they were available for use. The chief interest in crack observation was in identifying the nature and type of failure.

5.1.1. Description of Specimens Loaded in Compression

In addition to the details furnished in the previous section, the following additional data regarding the

compression specimens are of interest. The following variables were studied under compressive loading.

	a) Depth/span ratios	:	0.35, 0.52, 0.68 and 0.9
	b) Mortar strengths	:	1:3, 1:5, 1:8 and 1:10
and	c) Size and location of openings	:	Door or window at centre, Door or window at quarter span.

The size of specimens can be obtained from Fig.5.3. The span for 'walls on beams' was kept constant at 2.00 metres and the heights were varied. The foundation beams were of the same dimensions and concrete mix and were doubly reinforced with same number of rods. Adequate control tests on materials were conducted for each specimen and are reported elsewhere. The concrete beams were first cast, rolled into position on level concrete supports. The concrete supports had on top 37 mm steel plates of 25 cm x 25 cm size with rollers of 32 mm diameter followed by a similar top plate. Both supports approximated the hinged condition rather than being hinged at one end and with roller bearings at the other end. This was done to ensure stability of the wall. There could have been lateral thrusts on these bearings and the performance during tests indicated that the concrete supports did not slide on the test floor and hence these thrusts should have existed. Considerable time was taken to bring the concrete beams resting absolutely level with the help of plaster of paris packed-in to compensate for unevenness of beams during casting. It has

been reported that wooden moulds were used in fabricating the R.C. Beams. Steel moulds would have proved better in controlling the dimensions and angles of the concrete beams. Slight deviation in angles from the right angle can create large eccentricities as the brickwalls were built-up. Bricks were soaked in water before laying on mortar beds. The size and shape of bricks were kept under control with judicious rejection of non-uniform bricks. The mortar thickness on bed joints was controlled as described earlier. However the vertical joints had to be adjusted with the eye. The brick-work was brought up to the required height approximately and additional mortar layer was used on top to make up the desired depth/span ratio. The walls were continuously cured with wet sacks until instrumentation had to be arranged for. After allowing the brickwork to dry for 72 hours, strain gauges were pasted. The vertical and horizontal spacing of strain gauges varied for each specimen, so as to avoid local blemishes, minor cracks and so on and detailed measurements of the final position of strain gauges were recorded. Perspex pieces were affixed with Araldite wherever deflection measurements had to be made so that the dial gauges could easily adjust for tilts, rotations and deflection of the beam. It has already been reported that the brickwalls were of 11.0 cms thickness and the concrete beams were of 15.0 cm thickness.

5.1.2. Description of Instrumentation for Compressive Loading

The following is a general description of the nature of the instrumentation adopted for compression tests. Some of these measurements were omitted as time consuming and unnecessary as the test series progressed and additional readings were recorded in some cases to throw light on previously observed behaviour patterns. The general pattern of instrumentation is sketched in Fig. 5.4.

The vertical deflection at beam bottom level of the beam were recorded by dial deflectometers. A maximum of five gauges were used and in most tests only the central deflection was recorded. The lateral deflections were measured to indicate whether the specimen tilted. A total of six gauges were used three at top of the wall and three at the bottom and as loading progressed these gauges were watched. At the slightest indication of lateral movement, the loading was stopped, the cause of eccentricity ascertained, correction made, if any, and the loading was continued. This process was found to be necessary at the first few loading, unloading cycles. Once the loading was found to be satisfactory the test continued with suitable increments of loading.

The slip between the brick layer and top of concrete beam were recorded using two dial gauges attached to the top reaction plate, which adjusted for rotation of the deep

girder at supports as the loading progressed. Thus rotation effects were nullified, but slips alone were recorded. This was done at both ends of the wall to observe unsymmetrical behaviour and in almost all cases the slip was negligible and the use of these gauges was discontinued.

The vertical strain distribution across a horizontal section at the bottom most brick layer was recorded by SA 10 120 ohm resistance gauges of Indian manufacture and about nine gauges were used to monitor the strains. The horizontal strain distribution at several vertical section were recorded by SA 130, single wire 120 ohms strain gauges. A minimum of 6 gauges and a maximum of 30 gauges were used to monitor the horizontal strains. The very large compressive stresses concentrating at the supports, created cracking and vertical splitting of these thin brickwalls and lateral strains across the thickness of the wall and vertical strains above the supports were also measured at several bricks to throw light on this phenomenon.

The cracks were marked by black markers as the cracks progressed. The strain gauge readings were found to be erratic if the cracks were close to a strain gauge. The interpretation of strains after cracking is bound to be erroneous and more so when stresses are computed from these using large elastic moduli for brick and concrete.

Steel strains in the reinforcement were not measured and such measurements could have yielded valuable information regarding redistribution of moments between R.C. beams and brickwalls.

5.1.3. Description of Specimens Loaded in Tension

Five specimens were loaded in tension, the following variables being considered:

a) H/L = 0.35, 0.52 and 0.68 (Mortar 1:3)

b) Mortar 1:3, 1:5 and 1:8, (H/L = 0.52).

The basic difference between compression and tension specimens was in the loading pattern only, the load in the tension specimens being applied through embedded hooks in the concrete beam. The concrete beam of the same dimensions as in compression tests series, had 22 mm diameter U hooks embedded in the concrete at 1/8, 3/8, 5/8, 7/8 points of the span at 50 cms. intervals. The concrete beams were cast upside down, so that the projecting hooks could be accommodated. The concrete mix proportions, the method of casting, curing, details of control specimens and the method of constructing the brick masonry were identical with the compression test specimens. A 15 cm x 22 cm x 10 mm plate was welded to the 'U' hooks to provide adequate bearing resistance for the tension hooks. Slight deviations in the alignment of these hooks created enormous difficulties during loading. Special

tension bars with sliding pins were made to transmit the load coming on the distribution girders to the concrete beam. The twin box culvert type of test floor available in the structural laboratory was ideally suited for loading from underneath. (Fig. 5.2). The dimension of specimens can be obtained from Fig. 5.5.

5.1.4. Description of Instrumentation for Tensile Loading

The instrumentation for tensile loading was of the same set-up as described for compression loading. It was evident from the very first load increment that tensile loading creates separation in the mortar beds between the brickwork and the R.C. beam. The separation was predominant at 25 percent of the ultimate load and thereafter the reinforced concrete was taking the load with the wall simply riding along acting as a self supporting arch on the supports. Even though visible slips at ends of the deep brick wall were not noticed, slip measurements would have given valuable information. The central deflection and strain measurements in vertical and horizontal strain gauges were of interest. Lateral deflection gauges were employed to indicate tilting, if any. However, tilting could not be compensated for in tensile loading as in the case of compression loading, by shifting the jack position. Fortunately lateral tilt was negligible and none of the specimens failed by tilting. Cracks were of

primary interest in tensile loading and were traced with black marker as the loading progressed.

5.1.5. Description of Plain R.C. Beam Tests

It was necessary to compare the flexural load carrying capacity of 'walls in beams' with those of the plain R.C. beams supporting the walls. In most cases the R.C. beams were cracked under full 'walls on beams' test, but their ultimate strength was by no means exhausted. The mode of failure in compression tests was primarily due to the brick failure. Such supporting beams were salvaged and subjected to two point loading and their behaviour has been recorded. The tensile loading invariably led to the foundation beam failing in flexure after separating from the brickwork in the midspan. A separate specimen was hence cast to examine its behaviour without a wall on top. Tests on this beam indicated a higher ultimate load than those in compression tests (previously described). This could be due to extra bond and anchorage steel in the tension specimens and the specimen not being cracked due to prior loading. The instrumentation for these tests essentially consisted of the central deflection measurement with dial gauges.

5.2. Results of Control Tests on Bricks

5.2.1. General

The variation in the properties of bricks has been claimed to be one of the reasons for large safety factors associated with brickwork. The dimensions have variations, the strength of bricks in compression and tension is not uniform, the density is also varying in nature. These variation in turn generate a randomness in the basic properties of brickwork and hence the assumption of homogeneity customarily made in solutions based on the theory of elasticity is violated. The nature and magnitude of the interaction introduced by this non-homogeneity can be studied only if the nature of fundamental variations are reasonably established. Tests were conducted on bricks and simple statistical analyses were made.

5.2.2. Dimensional Tolerances

The bricks commercially available at Kanpur, India were hand moulded, dried in the sun and burnt in country kilns with firewood or coal. Because of this labour-intensive method of manufacture variations in dimensions and density should have been present and these variations were studied on a sample of 30 bricks from a bulk supply of 5000 bricks and the results are given in Table 5.1. Simple statistical analysis showed that the length, breadth and depth of bricks had

coefficients of variation of 0.78%, 0.99% and 0.51% . The area and volume of bricks had coefficients of variation of 4.12% and 5.5% . The coefficient of variation for density was found to be 3.7% . The average dimensions of bricks were found to be 22.85 cms x 11.06 cms x 6.75 cms. The average area was found to be 252.8 sq.cm., flat-wise and the average density was 1.710 gms/cc. The average of volumes turned out to be 1705 cubic centimetres. The average weight for self-weight calculations works out to 2.91 kgms/brick. The results are surprisingly consistent for a much maligned material such as brick.

5.2.3. Strength in Compression

Because of the unevenness in the brick surface and also the 'frog' or recess in the top surface, compression tests conducted even after filling the recess with 1:3 mortar and levelling the surface with a thin layer of mortar at top and bottom, did not yield consistent results. As such bricks were cut to size in a power saw and compression tests were conducted as per ASTM C67-62. The tests were conducted in Olsen's 200 tons U.T.M. The results indicated a strength of 345 kg/cm^2. Tests conducted on full length bricks, without cutting, but with mortar filling and facing showed lower strength. The results indicated a strength of 242 kg/cm^2. The results obtained from full brick tests were only 70 percent

of the strength obtained on cut bricks. If engineered Brick Masonry is to be achieved then standardization of the size of specimens for strength of brickwork deserves special attention.

5.2.4. Modulus of Rupture Tests

The modulus of rupture tests were conducted on 7 inches span with the bricks bending flatwise and the load being applied centrally. Standard fixtures available with OLSEN's U.T.M. were used for the purpose. The test results indicated the modulus of rupture value as 42.6 kg/cm^2.

5.3. Results of Control Tests on Mortar

5.3.1. General

The effect of mortar strength on the strength of brickwork and walls on beams was of primary consideration and special attention was devoted to this aspect during testing. The nonuniformity in brickwork, particularly due to workmanship is mainly due to the mason adding the required amount of water to achieve workability as and when he builds up the several layers. Even though measured quantity of water was given for all batches, there were less or more water used depending upon the humidity in the surrounding atmosphere and the degree of wetness of soaked bricks used in the construction. Mortar cubes and discs were produced as per

standard specifications (sometimes vibrated in the concrete casting table) and hand tamped until water appeared on the surface. Steel moulds were used for the mortar cubes but wooden moulds, specially fabricated, were used for the discs. After 24 hours of curing under wet cloth, they were soaked in water, air-dried on the testing date and tested in the Baldwin compressing testing machine under standard rate of loading. Split tests were conducted with plywood strips placed diametrically opposite to each other.

5.3.2. Compression Tests on Mortar Cubes (7 cm)

Mortar proportions 1:3, 1:5, 1:8 and 1:10 were used in the masonry work. Since a large number of walls were built of 1:3 mortar, the test results for 1:3 mortar are more numerous. Compressive strengths of mortars as obtained from cube tests (7 cm) are given in Tables 5.2, 5.3, 5.4, and 5.5. It is seen that the coefficient of variation (35.67) as computed in Table 5.2 is quite large showing the variability in mortar strength. Mortar is an important source of non-homogeneity.

5.3.3. Split Tension Tests on Mortar Discs

Due to non-availability of disc moulds in large numbers, fewer tests were conducted on discs and discs made

of 1:8 and 1:10 mortars showed negligible tensile strength. The 1:10 mortar specimens failed even during handling and tests could not be conducted on such specimens. All proportions were by volume and w/c ratio was attempted at 1.0, but as described earlier, there were variations depending upon ambient temperature and humidity. The split tension results are given in Table 5.6. The discs were of 7.5 cms diameter and 5 cms in thickness.

5.4. Results of Control Tests on Concrete

5.4.1. General

Concrete was used in the beams supporting the brick-walls. The beams were of 214 cms x 21 cms size. The mix proportion was 1:2:3 by weight and w/c ratio was maintained at 0.5 by weight. The concrete was mixed in a 5/7 cft, vertical drum concrete mixer for three minutes and the batches were cast in layers in wooden form work of appropriate size and vibrated with one inch diameter needle vibrators. The top was screeded and wet gunny bags were used to cover the top of the beam. The beams were demoulded after 48 hours, covered with gunny bags and kept moist for 21 days. The re-inforcement details in the beams are discussed elsewhere.

5.4.2. Compression Tests on Concrete Cylinders

Concrete cylinders were cast in 6" dia. x 12" steel moulds with a steel base for each batch of concrete mix.

The cylinders were vibrated in the same manner as in the test beam to simulate realistic casting conditions, trowelled level, covered with wet cloth, demoulded after 24 hours and cured in water tanks upto 28 days. The cylinders were removed from water, capped with plaster of paris on both sides, using glass plates, level platform and accurate spirit level and kept ready for tests on the same day as the tests on corresponding concrete beams. Generally two months elapsed between the date of casting the cylinders and testing them.

The compression tests on concrete cylinders were conducted in Olsen's 200 tons U.T.M. and a standard rate of loading of 2000 psi/minute was applied until first crack, and gradual failure was imposed on the specimens by adjusting the rate of loading. The results of compression tests indicated an average cylinder strength of 370 kgms/sq.cm. with a standard deviation of 67.5 kgms/sq.cm. The coefficient of variation is 18 percent and this indicates poor control since the specimens were cast on 10 different days.

5.4.3. Split Tests on Concrete Cylinders

The tensile strength of concrete was obtained from cylinder split tests. Two plywood strips of 1.25 cms width and 5 mm depth were interposed between the concrete cylinders and the machine platforms and a rate of loading of 2000 kgms per minute was maintained until sudden splitting occured.

The average tensile strength was found to be 32.4 kg/sq.cm and the coefficient of variation was found to be 21.75 percent.

5.5. Results of Control Tests on Reinforcement

5.5.1. General

Plain mild steel bars were used for reinforcement in the foundation beam. Two numbers of 10 mm diameter at top and two numbers at bottom were used for all tests. The shear stirrups were of 6 mm dia. bars, uniformly spaced at 15 cms spacing throughout the beam. In none of the tests the steel in the doubly reinforced beams were in distress. It is the brick wall which showed evidence of failure.

5.5.2. Tension Test on Mild Steel Bars

Tension tests on mild bars were conducted in Olsen's 200 tons U.T.M. and the strains were measured by Olsen's LVDT transducer. The stress-strain curve was automatically plotted in a X-Y plotter for the 10 mm diameter bars. The stress-strain curve for mild steel is replotted to convenient scale and is shown in Fig. 5.6. The Young's modulus for 10 mm bars was found to be 2.045×10^6 kg/sq.cm. and the proportional limit was found to be 2700 kgm/sq.cm. The yield stress varied for different specimens and the 0.2 percent yield value was found to be 3160 kg/sq.cm. The percentage elongation was

found to be 38 percent. The yield stress for 6 mm bars was found to be 3000 kgm/sq.cm.

5.6. Results of Control Tests on Brickwork

5.6.1. General Variations in Compression Strength

The most interesting aspect of brickwork is that when bricks and mortar are combined to yield brickwork the resultant strength in vertical compression (uniaxial) is found to be often less than the minimum strength of either of the two materials. For example, the mean strength of bricks only was found to be 345 kg/sq.cm. and that of 1:3 mortar was found to be 206 kg/sq.cm. But the mean compressive strength of bricks with 1.25 cm. mortar facing on one side was found to be 133 kgm/sq.cm. The mean compressive strength of bricks with 1.25 cm. mortar facing on both sides was found to be 110 kgm/sq.cm. Strength of brick cubes in three layers with two mortar joints was found to be 101.3 kgm/sq.cm. The strength of three layer brickwork with two additional mortar facings was found to be evenless at 96.5 kgm/sq.cm. The more the number of layers of mortar the lesser the strength, but the strength approaches an asymptotic minimum value of about 90.0 kgm/sq.cm. The results of various tests on brickwork are given in Tables 5.7 to 5.11.

5.6.2. Modulus of Elasticity of Brickwork

For the analysis of walls on beams the elastic properties of brickwork were required. Use of electrical strain gauges in vertical and horizontal direction gave erroneous results. Hence deflectometers were used to average the movement of machine platforms between which the brickwork was compressed. The results of tests are given in Tables 5.12 and 5.13. However, the Young's modulus obtained from these dial gauge readings was found to be quite low and considering that strong bricks and mortars have been used these moduli were quite low. Inadvertently the deformation of the machine platform itself might have been measured, giving large strains and hence the low Young's Modulus. The measurement of lateral and vertical strains to determine the Poisson's ratio also led to erroneous results and the use of electrical strain gauges for this purpose was not found to be satisfactory. The stress-strain curves for brickwork are plotted in Figs. 5.7(a) and 5.7(b). The stress strain curves show a very low initial modulus and eventually become stiffer in behaviour. This aspect is important in that the masons build brickwork by spreading mortar in layers and just levelling the bricks on top of this mortar and eventually fills the vertical joints with trowel. There is no such thing as consolidation as per standard specifications. Hence the measured mortar strength based on standard tests has no realistic

basis for predicting the mortar strength in brickwork. This weak bedding of mortar in turn creates local flexure and this may be one of the reasons for vertical splitting of brickwork when uniaxially compressed. The low Young's Modulus measured in the tests may be justified on the basis of unconsolidated mortar joints.

5.7. Presentation of Test Data on Walls on Beams

5.7.1. General

Various investigators have already identified the primary variables effecting the behaviour and strength of walls on beams. The height/span ratio, effect of brick and mortar properties on the behaviour of walls, and the effect of openings have all been investigated. The recommendations made for practical design of such beams may however end up in unsafe designs, particularly when the designer is not completely aware of the nature of tests conducted on which these recommendations are based. Most of the design recommendations are based on compression tests. This kind of loading is quite unrealistic in the sense that in multi-storeyed structures or in foundation beams it is only the self-weight of the wall that comes on the beams. The floor load is often transmitted to the reinforced concrete beams directly from the slab at the bottom of 'walls on beams' and not at top of the walls. In practice there is mortar filling or gap between the top of the wall and beam above it. Even though self

weight of walls is quite heavy when compared with the floor loads directly coming on the reinforced concrete beams, the direct loading of the foundation beam is more critical. One is compressive from the top down and the other is tensile directly at the bottom. In tensile loading, arch action, deep girder action, truss action etc., are all missing and the reinforced concrete beams separate from the wall at the bottom most brick layer. The beneficial interaction is no more present and for such tensile loads the recommendations made by various investigators may be safely overlooked. Hence in the present test series compression and tension loading have been taken to be the primary variables. Thus the test results are presented separately. However, in the compressive loading investigation additional variables such as the height/span ratio, effect of mortar strength and effect of openings were studied. In the tensile loading case height/span ratio and effect of mortar strength were the additional variables investigated. Since the study is one of interaction between brickwalls and reinforced concrete beams the basic strength of such concrete beams were also investigated and test results are presented separately.

The nature of tests conducted and the instrumentation have been detailed earlier in the main part of the report. However, a brief summary is presented so that the accompanying test results can be studied and interpreted without reference to the main text. The reinforced concrete beam for all tests

were of the same dimension and concrete strength. The reinforcement details were identical. The beams were of size 214 cms x 21 cms x 15 cms. The width of the beam was 15 cms on which slender brickwalls of 11 cms width were built up to the required height. The reinforcement was two numbers of 10 mm dia. bars at top and two numbers at bottom of the beam. The beams were thus doubly reinforced. The stirrups were of 6 mm diameter at 15 cms. spacings throughout. This reinforced concrete beam was supported on steel pins and appropriate steel plates at 200 cms. span interval. The concrete mix used was 1:2:3 by weight with a water cement ratio of 0.5.

The brickwalls were built near the testing equipment inside the laboratory by experienced masons. The bricks were 22.85 cms x 11.06 cms x 6.75 cms size. The average mortar thickness was 1.70 cms. The width of mortar joints was not the same as the width of bricks. They were 9 cms in width as against the width of bricks of 11.0 cms. Nominal restraining system was used to prevent such walls tilting during loading and before the commencement of loading. The vertical joints were not of uniform thickness. The top layer of the brickwalls were having a uniform mortar layer of 2 cms. thickness, to receive the steel distribution girders on additional plaster of paris levelling layers.

The walls were compressed by two 30 tons jacks fed by a 20 H.P. power operated high pressure (10000 psi) pump

and load was maintained constant by control valves. The loads from the jacks were distributed through a system of girders to simulate uniform loading.

Tensile loading was effected by a system of four hooks embedded in the reinforced concrete beam and same two jacks were used to distribute loads to these tension bars through a complicated system of girders and pins.

The twin-box type test floor available in the institute was eminently suited for loading these specimens. Additional tension rods, reaction girders, bracing systems, roller bearings and hinges were built up as required.

The instrumentation consisted of deflection gauges, slip gauges and lateral deflection meters all using accurate dial gauges mounted on suitable rigid stands. The strains were measured with a large number of electrical strain gauges of the 120 ohms type. The vertical strain measurements in brick work was made on the bottom most brick layer only and the strain gauges had 1.25 cms grids. The horizontal strains were measured with 15 cms long single wire strain gauges. Attempts to measure strains with demountable Whittmore type mechanical strain meters led to erratic results and these were discontinued after the first test.

The electrical strain gauges were also used in some tests to measure lateral-splitting-tension in brickwork.

The slip gauges were discontinued since there was no measureable slip betweer brickwall and the reinforced concrete beams.

The lateral deflection dial gauges were kept on to check whether there is any eccentric loading on the wall. In fact the loading jacks were slightly offset from the theoretical centre line if any such tilting was noticed.

The electrical strain-gauges were hooked on to SR4 portable strain indicators through a battery of 10 channel switch boxes. A number of indicators were simultaneously employed to reduce the time interval required for measurements at each stage of loading.

Most of the specimens were loaded and unloaded two or three times upto 25 percent of the first crack load to create initial compression in mortar and to adjust for eccentricities if any. None of the specimens failed by tilting. Only one specimen failed by collapse of brick work due to sudden release of load at failure and crack pattern could not be recorded.

Cracks were marked as the cracks progressed but only the final crack pattern at failure is presented.

For safety of instruments and personnel, measurements often stopped before failure actually occured. This was unfortunate and lack of remote recording facilities forced

this step to be taken.

5.7.2. Results from Specimens Loaded in Compression

The nomenclature used to designate the compression test specimens, the primary variables and dates of casting, wall fabrication and testing are given in Table 5.14. The first crack load and ultimate loads of such specimens are given in Table 5.15. The corresponding mortar strengths, brick strengths, and concrete strengths from control tests are given in Table 5.16. The load deflection, load-horizontal strain, load-vertical strain, horizontal strain distribution across a vertical section and vertical strain distribution along the bottom-most brick layer are presented in one composite figure for each test, so that a perspective of the behaviour of these walls on beams can be easily obtained. These graphs are plotted in Figs. 5.8 to 5.18. The crack patterns at failure loads are given in Figs. 5.19 to 5.29. Tabulated readings are filed in the department of Civil Engineering as a separate report and are not furnished herein. Critical discussion on the behaviour of these specimens is to be found in the ensuing sections.

5.7.3. Tests on Specimens Loaded in Tension

The nomenclature used to designate the tension test specimens, the primary variables, and the dates of casting,

wall fabrication and testing are given in Table 5.17. The first crack load and ultimate loads of such specimens are given in Table 5.18. The corresponding brick strengths, mortar strengths and concrete strengths from control tests are summarised in Table 5.19. The load deflection, load-horizontal strain, load-vertical strain, horizontal strain distribution across a vertical section and vertical strain distribution along the bottom most brick layer are presented in one composite figure for each specimen so as to present a comprehensive behaviour pattern and these graphs are plotted in Figs. 5.30 to 5.34. The crack patterns at failure loads are given in Figs. 5.35 to 5.39.

5.7.4. Control Tests on Reinforced Concrete Beams

In most of the tests even though the concrete beam was cracked its ultimate capacity was by no means exhausted even when the wall built above it collapsed. These beams were reloaded to failure by two point loading and the ultimate capacities of these beams are recorded in Table 5.20. Deflection measurements were also taken in a few cases and they are tabulated in Table 5.21. These are plotted in Fig. 5.40. The control beam for tensile loading had to be manufactured afresh since tensile loaded walls on beams failed by separation of brick work and eventual failure of the concrete beams by plastification. These results are analysed in subsequent sections.

5.8. Analysis of Test Data

The detailed description of experimental procedures presented earlier identifies the controlling parameters, sources of error and difficulties in the study of walls on beams. In conjunction with the details furnished by Fiorato, Sozen and Gamble (C 6) and Rosenhaupt (C 13), future investigations can be planned to yield more accurate experimental data. The large amount of data accumulated from the tests have not been presented in full. Only those data which are of helpful in the formulation of design recommendations will be analysed in detail.

5.8.1. Basic Behaviour of Brickwork

The brick and mortar themselves will be examined first before discussing the behaviour of brickwork. The physical dimensions of bricks have coefficients of variation C_v, less than one percent. Areas and volumes have coefficients of variation in the order of 5 percent (Table 5.1). The weight of bricks used has a C_v of 0.43 percent. Considering that these bricks are hand made and fired in country kilns, their physical details are quite satisfactory. Considering the strength of bricks, the strength in compression was found to be 345 kg/sq.cm. with C_V = 22 percent, indicating large scatter. The mortar used (Table 5.2) had a mean strength of 206 kg/sq.cm. with C_V = 35.6 percent. The tensile strength of mortar (1:3 by volume) was

found to be 22.8 kg/sq.cm. The components of brickwork namely brick and mortar suffer obviously from poor quality control. The strength of brickwork is bound to be affected by these variations. Observations on the behaviour of brickwork are summarised below.

1) The number of mortar layers in brickwork seems to have a detrimental effect on the compressive strength of brickwork as tabulated below:

a)	One layer of mortar above the brick	133.0 kg/sq.cm.	Table 5.7
b)	Two layers of mortar, above and below	110.0 kg/sq.cm.	Table 5.8
c)	Three courses, five layers of mortar	101.3 kg/sq.cm.	Table 5.9
d)	Three courses, five layers of mortar with vertical joints and loaded perpendicular to bed joints.	96.5 kg/sq.cm.	Table 5.11
e)	Same as above, but loaded parallel to bed joints.	67.5 kg./sq.cm.	Table 5.11

In all these cases, the mode of failure has been predominantly the vertical splitting mode and computer simulation studies have indicated that lateral tension in bricks exist when the $E_b \nu_m / E_m \nu_b$ ratio is rot kept in the order of unity.

2) While the brick has a strength of 345 kg/sq.cm. and the mortar used has a strength of 206 kg/sq.cm., it is unfortunate that the resulting brickwork has a strength of 133.0 to 67.3 kg/sq.cm. The efficiency is less than 50 percent in terms of strength.

3) Earlier investigators have reported strengths of brickwork higher than that of mortar strength (A 4) and this indicates that quality control on the brick laying process itself is important. Brickwork is so familiar that proper attention has not been given in India to the efficient utilisation of bricks and mortar available.

4) Computer simulation studies of the author have indicated that vertical joints induce local flexure and hence tension in brickwork is inherent.

5) The simplest guide line that can be given for the design of crack free work is to keep the ratio $E_b \nu_m / E_m \nu_b$ close to unity. This really means the condition of homogeneity. When E_m approaches E_b, the effect of vertical joints is minimized. In addition, if ν values approach each other, the influence of bed joints in generating lateral stresses is minimized. This concept can be extended to multi-phase composite media.

Regarding the elastic moduli for brickwork (E and ν) the Structural Clay Products Institute (C 20) advocates a

modulus of 1000 f_m' , where f_m' is the 28 day compressive, strength of brick. Nothing is mentioned about the Poisson's ratio, ν. The following observations on elastic constants are valid.

1) According to SCPI, the Young's modulus for brickwork under study should be in the order of 100,000 kg/sq.cm.

2) The computer simulation results are in the order of 310000 kg/sq.cm. (Table 4.8).

3) The theory of composite materials indicates 309100 kg/sq.cm. (Table 4.8).

4) The values measured in the laboratory are in the order of 10000 kg/sq.cm., that is 3 to 10 percent of various theoretical predictions.

5) The above comparison indicates that there is much work to be done in rationalising the elastic stiffness of brickwork.

6) The explanation of the author for the above discrepancies is as below. In assigning the elastic moduli of brick and mortar for purposes of computer simulation and for use in the theory of composite materials the equation 1000 f_m' was used. However, the strength of mortar as measured from vibrated mortar in standard specimens cannot relfect the strength of uncompacted mortar and this is the main reason for reduced modulus. This is confirmed by the appreciating stress-strain

curve obtained by the author (Fig. 5.7) and reports of C.N.S. Rao, in the International Conference on Masonry Structural Systems at the University of Texas (Nov. 1967). It is suggested that the elastic moduli be measured from tests on standard specimens of brickwork until such time when a reasonable theoretical approximation can be proposed.

7) Furthermore the measured elastic moduli parallel to bed joints are seen to be significantly different from the above tests (Table 5.13).

The author has used measured values in the computer simulation of walls on beams. There is very little information on the values of ν. Author's experiments have not yielded reliable results.

5.8.2. Basic Behaviour of Walls on Beams

1. The specimens tested in compression are described in Table 5.14. The first crack and ultimate loads are summarised in Table 5.15. It is clearly seen that the ultimate strengths are in the order of 20.44 to 51.39 tonnes as against the capacity of plain R.C. beams which are in the range of 7.80 to 9.24 tonnes (Table 5.20). Hence it can be concluded that interaction between walls and their supporting beams increases the load carrying capacity by 2.5 to 6 times the capacity of non-interacting systems. Tensile loading of similar walls on beams has

shown a constant capacity of 12.5 tonnes, and the increase in load carrying capacity over and above the plain R.C. beams can thus be neglected in such cases.

The basic behaviour patterns of walls on beams under compressive loading have been plotted in Figs. 5.8 - 5.18. There is a wealth of information in these test data; but the load deflection curves and vertical and horizontal strain distributions are of interest. The following observations are valid:

1) The concentration of vertical strains in the supporting region is reflected by the vertical stress distribution in computer simulation (Fig. 4.20).

2) The concentration of flexural strains in the supporting beams is reflected in the computer simulation. (Figs. 4.20 to 4.24). The brickwall is relatively free from large compressive strains, as indicated by the vertical stress distribution in simulation studies.

3) However, the large vertical strains on either side of openings observed in tests is not reflected in the computer simulation studies (Fig. 4.29 and 4.30). This may be ascribed to the poor attention given to stress concentration effects in the finite element modelling of walls on beams. Finer meshes around openings could have helped.

4) The deflection curves show non-linear trends mainly due to cracking. These will be studied in greater detail in subsequent section.

The crack patterns of ultimate loads are furnished in Figs. 5.19 to 5.29 and these may be viewed in conjunction with the first crack loads given in Table 5.15. The following observations are valid:

1) The first crack loads in the cases of large openings and poor mortar in brickwork are in the order of 40 percent of ultimate loads.

2) In plain walls with rich mortars (1:3 and 1:5 by volume) and window openings, the first crack loads are in the range of 48.5 percent to 73.5 percent.

3) Cracks are not acceptable in brick masonry construction and in general 33.1/3 percent of the observed ultimate loads may be taken as working loads. First crack loads are compared later on with the working loads obtained from computer simulation.

4) The crack patterns show extensive separation between mortar and bricks and shear cracks in brickwalls at ultimate loads. Furthermore the supporting beams are found to be cracked extensively. The tensile cracks over the supports has been predicted in the computer simulation studies.

5) It is clear that crack limited states are important in the design of walls on beams. While there is reserve strength of significant magnitude in the brickwalls interacting with their supporting beams, this reserve cannot be exploited in design until the local cracking behaviour of brickwork is controlled.

The case of tensile loading at supporting beam level has been given particular attention in this study. The load-response characteristics are plotted in figures 5.30 to 5.34. The first crack and ultimate loads have been given in Table 5.18. The crack patterns at ultimate loads have been plotted in Figs. 5.35 to 5.39. The following observations are valid.

1) The ultimate loads in tensile loading are insensitive to height-span ratios and mortar strengths indicating little interaction, if any.

2) The first crack loads are in the order 18 to 29 percent of ultimate loads.

3) Cracking and separation between mortar and bricks are extensive.

4) The supporting beam could be pulled down at mid-span by 3 cms without the brickwall riding along. At the very early stages of loading the brickwall separated from the reinforced concrete beams.

5) It is thus obvious that interaction between brick-walls and their supporting beams cannot be relied upon in the case of floor loads being directly transmitted to the beams of the framed building.

6) Only in the case of direct compressive loadings can the interaction be given attention.

Additional tests conducted on salvaged beams are indicated in Table 5.20 and deflections are plotted in Fig. 5.40. In comparison with the strength of walls on beams under tensile loading the observed ultimate strength of these salvaged beams are in the order of 75 percent. This is mainly due to the fact that two point loading was used at 1/3rd span intervals in the tests on salvaged beams, whereas uniformly distributed loads were simulated in the tension tests on walls on beams. Thus the strength of plain R.C. Beams can be taken to be that obtained in the tensile tests on walls on beams.

In the following sections the effects of individual parameters will be given due attention.

5.8.3. H/L Ratio Effect

The overall performance of a structural member is best studied through load-deflection curves. The effect of H/L ratio has been separated from the others in the load deformation graphs plotted in Fig. 5.41. It is seen that H/L ratios

of 0.52, 0.68 and 0.90 have deflections which are quite close. The tensile loading cases have been plotted to indicate the behaviour of plain R.C. beams without brickwalls. The elastic deflections in walls on beams of H/L ratio greater than 0.5 (built with 1:3 mortar) are 25 percent of the plain beams. The computer simulation values have also been plotted in the same figure showing reasonable correlation. Besides the ultimate loads, the deflection curves also indicate the presence of beneficial interaction.

5.8.4. Effects of Mortar Strengths

The effect of mortar strength has been separated and the corresponding load deflection curves are as indicated in Fig. 5.42. Mortar proportions of 1:3 and 1:5 have more or less similar effects and 1:8 and 1:10 mortar proportions are better avoided. The 1:5 mortar is affected by cracking as seen from significant nonlinearity in the load-deflection curves. Thus 1:3 mortar seems to be ideal for brickwalls interacting with supporting beams. It has already been suggested on the basis of computer simulation that it is better to bring up the mortar strength to the level of the strength of bricks in use. Thus the experimental observations strengthen this recommendation. It has already been reported that the strength of mortar had little influence on the strength of walls on beams loaded in tension.

5.8.5. Effects of Openings

The effects of openings have been isolated in the load-deflection curves given in Fig. 5.43. It is seen that plain walls and walls with central door and window openings have more or less identical elastic response. It has been pointed out that this will be the case as per computer simulation studies. Applied load seems to be transferred to the supports thus reducing bending effects on the beam-wall system. This is not however true in the case of eccentric openings. Eccentric door openings reduce interaction effects significantly. It is thus concluded that interaction benefits should be neglected in the case of eccentric openings, particularly when the eccentricity is in the order of 1/4 of span length.

TABLE 5.1 : VARIATION IN DIMENSIONS AND DENSITY OF USED IN TESTS ON WALLS ON BEAMS

S.No.	Length cms	Breadth cms	Depth cms	Area sq.cms	Volume cc	Weight kgms	Density gms/cc
1	23.0	11.0	6.6	253.0	1670	2.80	1.678
2	23.6	11.5	6.8	271.5	1818	3.10	1.707
3	22.8	11.0	6.7	250.8	1680	3.00	1.786
4	23.1	11.4	7.0	263.5	1843	3.00	1.628
5	22.8	10.9	6.6	249.5	1640	2.80	1.708
6	22.6	11.0	6.9	248.6	1714	2.95	1.721
7	22.6	11.1	6.8	251.8	1705	2.80	1.643
8	22.9	11.0	7.1	251.9	1789	3.10	1.733
9	22.8	11.5	6.9	262.2	1811	2.90	1.602
10	22.7	10.9	6.8	247.6	1684	2.80	1.662
11	23.4	10.8	6.8	252.5	1720	2.95	1.716
12	22.0	11.0	6.8	242.0	1646	2.80	1.702
13	23.0	11.0	6.8	256.3	1745	2.95	1.691
14	22.8	10.9	6.6	248.5	1642	2.90	1.767
15	23.4	10.8	6.6	252.5	1668	2.90	1.750
16	22.7	11.0	6.8	249.7	1698	2.90	1.708
17	22.4	11.4	6.7	260.0	1742	2.95	1.693
18	21.5	10.0	6.8	215.0	1462	2.80	1.915
19	22.5	10.6	6.6	238.5	1577	2.80	1.778
20	22.8	11.0	6.8	250.8	1706	3.00	1.759
21	23.2	11.4	6.6	264.4	1745	2.9	1.662
22	22.7	10.6	6.2	240.5	1491	2.7	1.810

TABLE 5.1 : (CONTD.) VARIATION IN DIMENSIONS AND DENSITY OF USED IN TESTS ON WALLS ON BEAMS

S.No.	Length cms	Breadth cms	Depth cms	Area sq.cms	Volume cc	Weight kgms	Density gms/cc
23	23.2	11.5	7.0	266.8	1868	3.1	1.660
24	22.9	11.1	6.7	254.2	1704	2.9	1.703
25	23.2	11.4	6.9	264.4	1841	3.1	1.683
26	22.9	11.0	6.4	251.9	1613	2.8	1.735
27	23.0	11.1	6.9	255.3	1729	2.9	1.678
28	23.4	11.2	7.0	262.0	1835	3.1	1.689
29	22.3	11.7	6.7	260.8	1750	2.8	1.600
30	22.70	10.9	6.5	247.8	1609	2.8	1.741
Mean	22.85	11.06	6.75	252.8	1705	2.91	1.710
	0.1785	0.1098	0.0345	10.40	94.C	0.0126	0.063
C_V	0.78%	0.99%	0.51%	4.12%	5.5%	0.43%	3.7%

TABLE 5.2 : COMPRESSION TESTS ON 7cm MORTAR CUBES, PROPORTION 1:3 BY VOLUME

S.No.	Date of Casting	Date of Testing	Failure Load kg.	Comp. Stress f_m kg/sq.cm	$f_m-\bar{f}_m$	$(f_m-\bar{f}_m)^2$
1	27.7.71	21.9.71	8700	177.5	28.5	812
2	27.7.71	21.9.71	15100	310.0	104.0	10816
3	27.7.71	21.9.71	15400	314.0	108.0	11664
4	8.8.71	21.9.71	9000	184.0	22.0	484
5	8.8.71	21.9.71	12900	263.0	57.0	3249
6	8.8.71	21.9.71	11450	233.0	27.0	729
7	22.8.71	21.9.71	10250	200.8	5.2	27
8	22.8.71	21.9.71	10700	218.0	12.0	144
9	22.8.71	21.9.71	9100	186.0	20.0	400
10	22.8.71	21.9.71	7600	155.0	51.0	2601
11	22.8.71	21.9.71	10800	220.0	14.0	196
12	22.8.71	21.9.71	10200	208.0	2.0	4
13	22.8.71	21.9.71	11100	226.0	20.0	400
14	22.8.71	21.9.71	9150	187.0	19.0	361
15	22.8.71	21.9.71	10100	206.0	0.0	0
16	22.8.71	21.9.71	9150	186.0	20.0	400
17	22.8.71	21.9.71	11300	230.0	24.0	576
18	22.8.71	21.9.71	13500	275.0	69.0	4761
19	28.8.71	1.11.71	15650	320.0	114.0	12996

TABLE 5.2 : (CONTD.) COMPRESSION TESTS ON 7cm MORTAR CUBES, PROPORTION 1:3 BY VOLUME

S.No.	Date of Casting	Date of Testing	Failure Load kg.	Comp. Stress f_m kg/sq.cm	$f_m-\bar{f}_m$	$(f_m-\bar{f}_m)^2$	σ^2
20	28.8.71	1.11.71	16050	326.0	120.0	14400	$\sigma^2 = \frac{177528}{33}$
21	28.8.71	1.11.71	16000	325.0	119.0	14161	
22	28.8.71	1.11.71	8750	178.0	28.0	784	$\sigma^2 = 5380$
23	28.8.71	1.11.71	13100	266.0	40.0	1600	
24	28.8.71	1.11.71	16050	326.0	120.0	14400	
25	12.9.71	1.11.71	4500	92.0	114.0	12996	$\sigma = \sqrt{5380}$
26	12.9.71	1.11.71	5950	122.0	84.0	7056	$\sigma = 73.5$
27	12.9.71	1.11.71	9900	204.0	2.0	4	
28	12.9.71	1.11.71	7900	172.0	44.0	1936	
29	12.9.71	1.11.71	4950	101.0	105.0	11025	
30	12.9.71	1.11.71	4400	90.0	116.0	13456	$C_v = \frac{73.5x}{20}$
31	12.9.71	1.11.71	5900	120.0	86.0	7396	= 35.6%
32	25.9.71	1.11.71	2600	53.0	153.0	23409	
33	25.9.71	1.11.71	6900	141.0	65.0	4225	
			$\bar{f}_m$	206.0	Σ	177528	

TABLE 5.3 : COMPRESSION TESTS ON 7cm MORTAR CUBES (PROPORTION 1:5 BY VOLUME)

S.No.	Date of Casting	Date of Testing	Failure Load Kg.	Comp. Stress fm Kg/sq.cm.	Remarks
1	12.9.71	1.11.71	2250	46.0	
2	12.9.71	1.11.71	1750	33.5	
3	12.9.71	1.11.71	2050	42.0	
4	12.9.71	1.11.71	2500	51.0	
			Average	43.1 Kg/sq.cm.	

TABLE 5.4 : COMPRESSION TESTS ON 7cm MORTAR CUBES (PROPORTION 1:8 BY VOLUME)

S.No.	Date of Casting	Date of Casting	Failure Load Kg.	Comp.Stress f_m, Kg/sq.cm	Remarks
1	19.9.71	1.11.71	750	15.2	
2	19.9.71	1.11.71	1500	30.4	
3	19.9.71	1.11.71	400	8.2	
4	19.9.71	1.11.71	1000	20.4	
5	19.9.71	1.11.71	1250	25.5	
6	19.9.71	1.11.71	1000	20.4	
7	19.9.71	1.11,71	1350	27.5	
			Average	21.10 Kg/sq.cm.	

TABLE 5.5 : COMPRESSION TESTS ON 7cm MORTAR CUBES

(PROPORTION 1:10 BY VOLUME)

S.No.	Date of casting	Date of testing	Failure Load, Kg.	Comp. Stress f_m, Kg/sq.cm.	Remarks
1	26.9.71	1.11.71	480	9.8	
2	26.9.71	1.11.71	570	11.6	
3	26.9.71	1.11.71	585	12.0	
4	26.9.71	1.11.71	560	11.2	
5	26.9.71	1.11.71	600	12.2	
6	26.9.71	1.11.71	655	13.4	
			Average:	11.7 Kg/sq.cm.	

TABLE 5.6 : SPLIT TESTS ON MORTAR DISCS

(SIZE: 7.5 cms. DIAMETER x 5cms. THICKNESS)

S.No.	Mortar Proportion	Date of casting	Date of testing	Splitting Load Kg.	Tensile Strength Kg/sq.cm.	Remarks
1	1:3	8.8.71	21.9.71	1070	18.0	
2	1:3	8.8.71	21.9.71	945	16.0	
3	1:3	22.8.71	21.9.71	1375	23.0	
4	1:3	22.8.71	21.9.71	750	12.5	
5	1:3	22.8.71	1.11.71	2000	33.5	
6	1:3	22.8.71	1.11.71	2000	33.5	
				Average	22.0 Kg/sq.cm.	
1	1:5	12.9.71	1.11.71	215	3.60	
2	1:5	12.9.71	1.11.71	250	4.20	
3	1:5	26.9.71	1.11.71	270	4.50	
				Average	4.10 Kg/sq.cm	
1	1:8	19.9.71	1.11.71	0	0.0	Specimens failed under negligible load.
2	1:8	19.9.71	1.11.71	0	0.0	
3	1:8	19.9.71	1.11.71	0	0.0	
				Average	0.0 Kg/sq.cm	

* 1:10 mortar discs failed even during handling.

TABLE 5.7: ULTIMATE STRENGTH OF BRICKS IN UNIAXIAL COMPRESSION, WITH ONE LAYER OF 1:3 MORTAR, 1.25 cms THICK

S.No.	Size cms	Area sq.cm.	Ultimate Load Kgms.	Ultimate Stress Kgm/sq.cm.	Remarks
1	22.85x11.06	253.0	28250	112.0	
2	22.85x11.06	253.0	48750	192.0	
3	22.85x11.06	253.0	32000	126.0	Failed by vertical
4	22.85x11.06	253.0	25000	99.0	splitting of
5	22.85x11.06	253.0	38250	151.0	specimens.
6	22.85x11.06	253.0	26500	105.0	Brick by itself
7	22.85x11.06	253.0	43500	172.0	has an average
8	22.85x11.06	253.0	35000	138.0	strength of 345 Kg/sq.
9	22.85x11.06	253.0	32000	126.0	cm.
10	22.85x11.06	253.0	27250	108.0	

Average: 133.0 Kg/sq.cm.

TABLE 5.8 : ULTIMATE LOAD OF BRICKS IN COMPRESSION WITH TWO 1.25 CMS LAYERS OF 1:3 MORTAR AT TOP AND BOTTOM

S.No.	Area sq.cm.	Ultimate Load Kgms	Ultimate Stress, f_b Kgms/sq.cm.	$(f_b-\bar{f}_b)$	$(f_b-\bar{f}_b)^2$	$\sigma^2 = \frac{\Sigma(f_b-\bar{f}_b)^2}{n}$	Remarks
1	253.0	25500	103.0	7.0	0049	$\sigma^2 = 406$	
2	253.0	29750	118.0	8.0	0064	$\sigma \doteq 20.0$	
3	253.0	25750	110.0	0.0	0000	$\sigma = 20.0$	Bricks
4	253.0	29750	118.0	8.0	0064		generally
5	253.0	19750	78.0	32.0	1024	$C_v = \frac{20x100}{110}$	failed
6	253.0	20750	82.0	28.0	0784	$= 18.2\%$	by
7	253.0	23750	94.0	16.0	0256		splitting
8	253.0	25500	103.0	7.0	0049		
9	253.0	29000	115.0	5.0	0025		
10	253.0	25500	102.0	8.0	0064		
11	253.0	25000	99.0	11.0	0121		
12	253.0	29250	116.0	6.0	0036		
13	253.0	22000	87.0	23.0	0529		
14	253.0	23750	94.0	16.0	0256		
15	253.0	32000	126.0	16.0	0256		
16	253.0	24750	98.0	12.0	0144		
17	253.0	29000	115.0	5.0	0025		
18	253.0	28000	111.0	1.0	0001		
19	253.0	23000	91.0	19.0	0361		

TABLE 5.8 (CONTD.) : ULTIMATE LOAD OF BRICKS IN COMPRESSION WITH TWO 1.25 CMS LAYERS OF 1:3 MORTAR AT TOP AND BOTTOM

S.No.	Area Sq.cm.	Ultimate Load Kgms	Ultimate stress, f_b kgm/sq.cm	$(f_b-\bar{f}_b)$	$(f_b-\bar{f}_b)^2$	$\sigma \frac{\sum(f_b-\bar{f}_b)^2}{n}$	Remarks
20	253.0	28750	114.0	4.0	0016		
21	253.0	20550	81.0	29.0	0841		
22	253.0	29250	116.0	6.0	0036		
23	253.0	26000	103.0	7.0	0049		
24	253.0	29750	118.0	8.0	0064		
25	253.0	23000	91.0	19.0	0361		
26	253.0	33250	132.0	22.0	0484		
27	253.0	26750	106.0	4.0	0016		
28	253.0	30750	121.0	11.0	0121		
29	253.0	26000	103.0	7.0	0049		
30	253.0	26500	105.0	5.0	0025		
31	253.0	30000	119.0	9.0	0081		
32	253.0	27000	107.0	3.0	0009		
33	253.0	24750	98.0	12.0	0144		
34	253.0	43000	170.0	60.0	3600		
35	253.0	39000	152.0	42.0	1764		
36	253.0	25250	100.0	10.0	0100		
37	253.0	30250	120.0	10.0	0100		

TABLE 5.8 (CONTD: ULTIMATE LOAD OF BRICKS IN COMPRESSION WITH TWO 1.25 CMS LAYERS OF 1:3 MORTAR AT TOP AND BOTTOM

S.No.	Area Sq.cm.	Ultimate Load/Kgm.	Ultimate Stress Kg/sq.cm.	$(f_b-\bar{f}_b)$	$(f_b-\bar{f}_b)^2$	$\sigma^2=\Sigma(f_b-\bar{f}_b)^2/n$	Remarks
38	253.0	39250	153.0	43.0	1849		
39	253.0	26750	106.0	40.0	0016		
40	253.0	26250	104.0	6.0	0036		
41	253.0	29000	115.0	5.0	0025		
42	253.0	25000	99.0	11.0	0121		
43	253.0	16750	66.0	44.0	1936		
44	253.0	35250	140.0	30.0	0900		
45	253.0	41500	164.0	54.0	2916		
46	253.0	35500	141.0	31.0	961		
47	253.0	27250	108.0	2.0	0004		
48	253.0	28750	114.0	4.0	0016		
49	253.0	28000	111.0	1.0	0001		
50	253.0	29750	118.0	8.0	0064		

Average 110.0kg/sq.cm.

TABLE 5.9 : COMPRESSION STRENGTH OF BRICK CUBES WITH THREE COURSES OF BRICKS (1:3)

S.No.	Date of Casting	Date of testing	Mortar Proportion	Size, cms Lxbxh	Ultimate Load Kgms	Ultimate Stress Kg/sq.cms
1	19.7.71	13.9.71	1:3	23.5x22.5x22.5	51250	98.9
2	19.7.71	13.9.71	1:3	23.0x22.0x22.5	61000	120.0
3	19.7.71	13.9.71	1:3	22.0x23.0x23.0	53500	105.4
					Average	101.3 Kg/sq.cms.

TABLE 5.10 : COMPRESSIVE STRENGTH OF BRICK CUBES WITH THREE COURSES OF BRICKS (1:8)

S.No.	Date of Casting	Date of Testing	Mortar Proportion	Size,cms Lxbxh	Ultimate Load Kgms	Ultimate Stress Kg/Sq.cm.
1.	13.8.71	13.9.71	1:8	22.5x22.0x22.0	37250	75.3
2.	13.8.71	13.9.71	1:8	22.5x23.0x21.5	41000	79.0
3.	13.8.71	13.9.71	1:8	22.0x22.5x22.0	33250	67.2
					Average	73.4 Kg/sq.cms.

TABLE 5.11(a): COMPRESSION TESTS ON BRICK PRISMS, WITH TOP AND BOTTOM MORTAR FACING OF 12 mm THICKNESS LOAD PERPENDICULAR TO BEDDING*

S.No.	Date of Casting	Date of Testing	Mortar Proportion	Size of Prism, cms Lxbxh	Ultimate Load, Kgm	Ultimate Stress Kg/sq.cm	Remarks
1	25.4.72	15.8.72	1:3	22.2x10.5x25.0	24000	102.0	
2	25.4.72	15.8.72	1:3	22.2x10.6x25.0	25000	106.0	1)Vertical
3	25.4.72	15.8.72	1:3	22.5x10.5x25.5	17500	74.5	splitting failure
4	25.4.72	15.8.72	1:3	22.8x10.5x24.8	24500	98.0	
5	25.4.72	15.8.72	1:3	22.5x10.5x25.0	24000	102.0	2)Density 1.98gm/cc
						Average: 96.5 Kg/sq.cm	

* Middle layer had a vertical mortar joint also.

TABLE 5.11(b) : COMPRESSION TESTS ON BRICK PRISMS, WITH 12 mm MORTAR FACING ALL ROUND**
(LOAD PARALLEL TO JOINTS)

S.No.	Date of Casting	Date of Testing	Mortar Proportion	Size of Prism, Cms Lxbxh	Ultimate Load, Kgms	Ultimate Stress Kg/sq. cm.	Remarks
1	27.4.72	15.8.72	1:3	24.0x11.7x26.0	20500	73.5	1) Vertical Splitting Failure
2	27.4.72	15.8.72	1:3	24.5x11.3x26.0	18350	66.8	
3	27.4.72	15.8.72	1:3	24.5x11.5x25.0	20550	73.2	
4	27.4.72	15.8.72	1:3	24.6x11.3x26.0	12550	45.0	2) Density= 1.88 gm/cc
5	27.4.72	15.8.72	1:3	24.5x11.5x25.0	16550	59.0	
					Average	67.5	

** Middle Brick had a horizontal mortar joint also.

TABLE 5.12 : STRESS-STRAIN READINGS FOR UNIAXIALLY COMPRESSED BRICK PRISMS WITH MORTAR LAYERS HORIZONTAL

S.No.	Load Kgms	Stress Kg/sq.cm.	Deformation (1) mmx10^2	Deformation (2) mmx10^2	Average Deformation (mm)	Strain x10^{+3}	Youngs Modulus E, from graph
			SPECIMEN - 1				
1	0	0	0		0.00	0.000	E =
2	2500	10.75	233	200	2.17	8.68	10600 kg/ sq.cm.
3	5000	21.50	303	267	2.85	11.40	
4	7500	32.25	344	306	3.25	13.00	
5	10000	43.00	382	347	3.65	14.50	
6	12500	53.75	413	377	3.95	15.80	
7	15000	64.50	440	404	4.22	16.88	
8	17500	75.25	464	430	4.47	17.88	
9	20000	86.00	488	453	4.70	18.80	
10	22500	96.75	512	477	4.95	19.80	
11	24000	FIRST CRACK AND FAILURE					
			SPECIMEN - 2				
1	0	0.00	0	0	0.00	0.00	E = 10,000 Kg/sq.cm
2	5000	21.50	320	280	3.00	12.00	
3	10000	43.00	400	360	3.80	15.20	
4	15000	64.50	457	415	4.36	17.44	
5	20000	86.00	510	473	4.91	19.64	
6	22500	96.75	536	500	5.18	20.72	
7	25000		(FIRST CRACK) FAILURE LOAD				

TABLE 5.12 (CONTD.) : STRESS-STRAIN READINGS FOR UNIAXIALLY COMPRESSED BRICK PRISMS WITH MORTAR LAYERS HORIZONTAL

S.No.	Load, Kg	Stress Kg/sq.cm	(1) Deforma- tion mmx10^2	(2) Deforma- tion mmx10^2	Average deforma- tion (mm)	Strain x10^{+3}	Youngs Modulu E, from graph
			SPECIMEN-3				
1	0	0.00	0	0	0.00	0.00	10000 kg/ sq.cm
2	5000	21.50	244	200	2.22	8.88	
3	10000	43.00	310	267	2.88	11.52	
4	15000	64.50	364	320	3.42	13.68	
5	20000	86.00	414 (FIRST CRACK)	375	3.95	15.80	
6	22500	96.75	449 (FAILURE LOAD)	407	4.28	16.92	
			SPECIMEN-4				
1	0	0.00	0	0	0.00	0.00	8020 Kg/sq. cm.
2	5000	21.50	352	308	3.30	13.20	
3	10000	43.00	432	392	4.12	16.48	
4	15000	64.50	492	452	4.72	18.88	
5	20000	86.00	552 (FIRST CRACK)	512	5.32	21.28	
6	22500	96.75	583	542	5.62	24.48	
7	24000		(FAILURE LOAD)				

TABLE 5.13 : STRESS-STRAIN READINGS FOR UNIAXIALLY COMPRESSED BRICK PRISMS WITH MORTAR LAYERS VERTICAL

S.No.	Load, Kg	Stress Kg/sq.cm	(1) Deformation mm x 10^2	(2) Deformation mm x 10^2	Average deformation (mm)	Strain x 10^{+3}	Youngs Modulus E, from graph
			SPECIMEN-1				
1	0	0.00	0	0	0.00	0.00	E = 4800 Kg/sq. cm.
2	2500	8.90	252	825	2.68	10.72	
3	5000	17.80	330	362	3.46	13.84	
4	7500	26.70	460	415	4.38	17.52	
5	10000	35.60	522	457	4.90	19.60	
6	12500	44.50	558	492	5.25	21.00	
			(FIRST CRACK)				
7	15000	53.40	600	532	5.66	22.64	
8	20500		(FAILURE LOAD)				
			SPECIMEN-2				
1	0	0	0	0	0.00	0.00	E = 4000 Kg/sq. cm.
2	2500	8.90	165	203	1.84	7.36	
3	5000	17.80	240	278	2.59	10.36	
4	7500	26.70	302	335	3.18	12.72	
5	10000	35.60	360	397	3.78	15.12	
6	12500	44.50	406	445	4.25	17.00	
7	15000	53.40	465	500	4.82	19.28	
8	17500	62.30	525	565	5.45	21.80	
9	18350		(FAILURE LOAD)				

TABLE 5.13 (CONTD.) : STRESS-STRAIN READINGS FOR UNIAXIALLY COMPRESSED BRICK PRISMS WITH MORTAR LAYERS VERTICAL

S.No.	Load, Kg	Stress Kg/sq.cm	Deformation (1) mmx10^2	Deformation (2) mmx10^2	Average deformation (mm)	Strain x10^{+3}	Young Modulus E, from graph
			SPECIMEN-3				
1	0	0.00	0	0	0.00	0.00	E = 5000 Kg/sq. cm.
2	2500	8.90	340	376	3.58	14.32	
3	5000	17.80	410	446	4.28	16.92	
4	7500	26.70	460	503	4.81	19.24	
5	10000	35.60	500	540	5.20	20.80	
6	12500	44.50	533	572	5.52	22.08	
7	15000	53.40	562 (FIRST CRACK)	614	5.88	23.52	
8	17500	62.30	625	666	6.45	25.80	
9	20000	71.20	672	711	6.92	27.68	
10	20555		(FAILURE LOAD)				
			SPECIMEN-4				
1	0	0.00	0	0	0.00	0.00	E = 4600 Kg/ sg.cm.
2	2500	8.90	117	155	1.36	5.44	
3	5000	17.80	177	220	1.98	7.92	
4	7500	26.70	224	267	2.45	9.80	
5	10000	35.60	260	305	2.82	11.28	
6	12500	44.50	293	337	3.15	12.60	
7	15000	53.40	(FIRST CRACK)				
7	15000	53.40	380	423	4.01	16.04	
8	16550		(FAILURE LOAD)				

TABLE 5.14 : TESTS ON SPECIMENS LOADED IN COMPRESSION-IDENTIFICATION AND CHRONOLOGICAL DETAILS

S.No.	Specimen	H/L Ratio	Mortar Strength	Openings	Date of casting R.C. Beam	Date of Building Wall	Date of Testing
1	A_1	0.35	1:3	Nil	15.6.71	19.7.71	27.8.71
2	B_1	0.52	1:3	Nil	11.12.70	15.4.71	9.6.71
3	C_1	0.68	1:3	Nil	15.12.70	15.4.71	12.7.71
4	D_1	0.90	1:3	Nil	27.7.71	8.8.71	20.9.71
5	B_2	0.52	1:5	Nil	18.12.70	7.7.71	5.8.71
6	B_3	0.52	1:8	Nil	17.12.70	18.7.71	19.8.71
7	B_4	0.52	1:10	Nil	25.6.71	18.7.71	22.8.71
8	C_2	0.68	1:3	Door at Centre	5.7.71	22.8.71	23.10.71
9	C_3	0.68	1:3	Door at 1/4 Span	6.8.71	22.8.71	12.10.71
10	C_4	0.68	1:3	Window at Centre	25.8.71	28.8.71	28.9.71
11	C_5	0.68	1:3	Window at 1/4 Span	21.8.71	28.8.71	16.10.71

TABLE 5.15 : FIRST CRACK AND ULTIMATE LOADS OF COMPRESSION TEST SPECIMENS

S.No.	Specimen No.	H/L Ratio	Mortar Strength	Openings	First Crack Load Tonnes	Ultimate Load Tonnes	First Crack/ Ultimate
1	A_1	0.35	1:3	Nil	14.60	20.44	0.71
2	B_1	0.52	1:3	Nil	29.20	45.55	0.64
3	C_1	0.68	1:3	Nil	29.00	46.72	0.62 Sudden Collapse
4	D_1	0.90	1:3	Nil	23.36	47.88	0.485
5	B_2	0.52	1:5	Nil	17.52	33.29	0.735
6	B_3	0.52	1:8	Nil	8.76	23.36	0.375
7	B_4	0.52	1:10	Nil	17.52	30.95	0.565
8	C_2	0.68	1:3	Door opening at centre	17.52	43.80	0.400
9	C_3	0.68	1:3	Door opening at 1/4 span	11.68	28.616	0.415
10	C_4	0.68	1:3	Window at centre	36.79	51.392	0.720
11	C_5	0.68	1:3	Window at 1/4 span	17.52	32.12	0.540

TABLE 5.16: TESTS ON SPECIMENS LOADED IN COMPRESSION – CONTROL TEST DATA FOR MATERIALS, AVERAGE OF LARGE NUMBER OF TESTS

S. No.	Specimen No.	H/L Ratio	Mortar Strength	Openings	Brick Strength		Mortar Strength		Concrete Strength	
					Comp Kg/cm^2	Tension Kg/cm^2	Comp Kg/cm^2	Tension Kg/cm^2	Comp Kg/cm^2	Tension Kg/cm^2
1	A_1	0.35	1:3	Nil	242	42.6	206.0	22.8	370.0	32.4
2	B_1	0.52	1:3	Nil	242	42.6	206.0	22.8	370.0	32.4
3	C_1	0.68	1:3	Nil	242	42.6	206.0	22.8	370.0	32.4
4	D_1	0.90	1:3	Nil	242	42.6	206.0	22.8	370.0	32.4
5	B_2	0.52	1:5	Nil	242	42.6	43.10	4.10	370.0	32.4
6	B_3	0.52	1:8	Nil	242	42.6	21.10	0.0	370.0	32.4
7	B_4	0.52	1:10	Nil	242	42.6	11.70	0.0	370.0	32.4
8	C_2	0.68	1:3	Door at centre	242	42.6	206.0	22.8	370.0	32.4
9	C_3	0.68	1:3	Door at 1/4 span	242	42.6	206.0	22.8	370.0	32.4
10	C_4	0.68	1:3	Window at centre	242	42.6	206.0	22.8	370.0	32.4
11	C_5	0.68	1:3	Window 1/4 span	242	42.6	206.0	22.8	370.0	32.4

TABLE 5.17 : TESTS ON SPECIMENS LOADED IN TENSION

IDENTIFICATION AND CHRONOLOGICAL DETAILS

S.No.	Specimen No.	H/L Ratio	Mortar Strength	Openings	Date of Casting R.C. Beam	Date of Building Wall	Date of Testing
12	A_2	0.35	1:3	Nil	11.9.71	19.9.71	4.11.71
13	B_5	0.52	1:3	Nil	4.9.71	12.9.71	6.11.71
14	B_6	0.52	1:5	Nil	7.9.71	12.9.71	6.11.71
15	B_7	0.52	1:8	Nil	10.9.71	19.9.71	5.11.71
16	C_6	0.68	1:3	Nil	13.9.71	26.9.71	9.11.71

TABLE 5.18 : FIRST CRACK AND ULTIMATE LOADS OF TENSION TEST SPECIMENS (WALLS ON BEAMS)

S.No.	Specimen No.	H/L Ratio	Mortar Strength	Openings	First Crack Load	Ultimate Load (Tonnes)	First Crack Ultimate
12	A_2	0.35	1:3	Nil	2.336^T	12.702	0.186
13	B_5	0.52	1:3	Nil	3.504^T	11.972	0.290
14	B_6	0.52	1:5	Nil	3.504^T	12.284	0.285
15	B_7	0.52	1:8	Nil	2.236^T	12.202	0.176
16	C_6	0.68	1:3	Nil	2.336^T	12.848	0.192

TABLE 5.20 : SUMMARY OF FLEXURE TESTS CONDUCTED ON R.C. BEAMS SALVAGED FROM FULL WALL TESTS

S.No.	Beam Source from Previous Tests	Casting Date	Testing Date	Ultimate Load	Remarks
17	Beam B_4, H/L = 0.52 1:10 Mortar	25.6.71	4.11.71	7.800^T	
18	Beam B_3, H/L = 0.52 1:8 Mortar	17.12.70	4.11.71	7.818^T	
19	Beam D_1, H/L = 0.90 1:3 Mortar	27.7.71	29.10.71	8.18^T	Load-deflection measured
20	Beam C_4, H/L = 0.68 Window Opening at Centre	28.8.71	29.10.71	9.24^T	"
21	Fresh specimen with tension hooks and bond plates incorporated	15.9.71	8.11.71	13.14^T	"

TABLE 5.21 : LOAD-DEFLECTION TESTS ON SALVAGED BEAM SPECIMENS FROM EARLIER TESTS (TWO POINT LOADING)

S.No.	Load Tons	Deflection (mm)			Remarks
		Specimen 19	Specimen 20	Specimen 21	
1	0.0	0.0	0.00	0.00	
2	1.168	0.69	0.87	0.22	
3	2.336	1.66	1.92	0.57	
4	3.504	2.70	3.07	1.35	
5	4.672	3.66	4.07	2.26	
6	5.84	4.66	5.10	3.25	
7	7.008	6.21	6.37	4.20	
8	8.176	-	7.30	5.10	
9	9.244	-	9.00	6.17	
10	10.512	-	-	7.25	

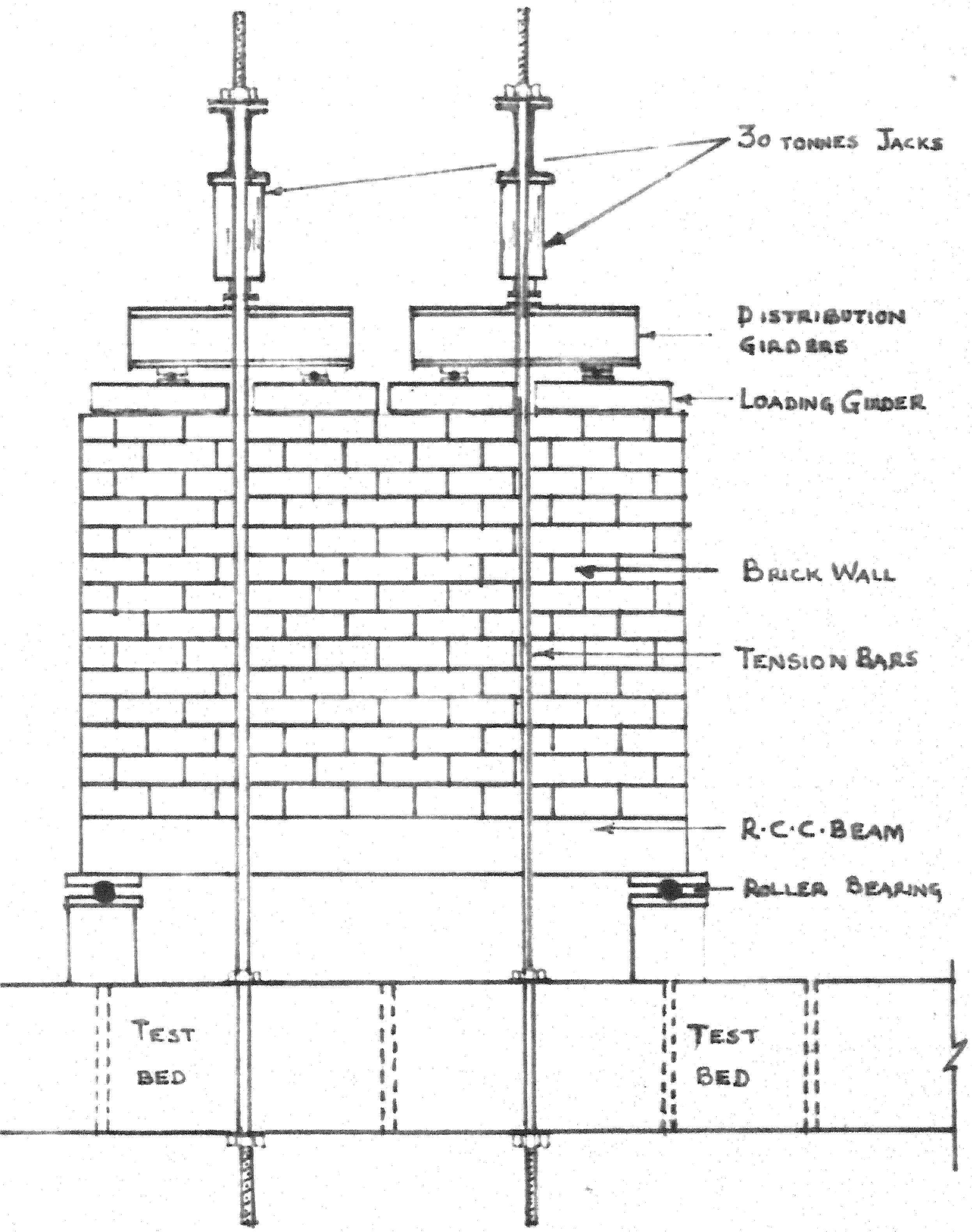


FIG. 5.1. COMPRESSION LOADING ARRANGEMENT.

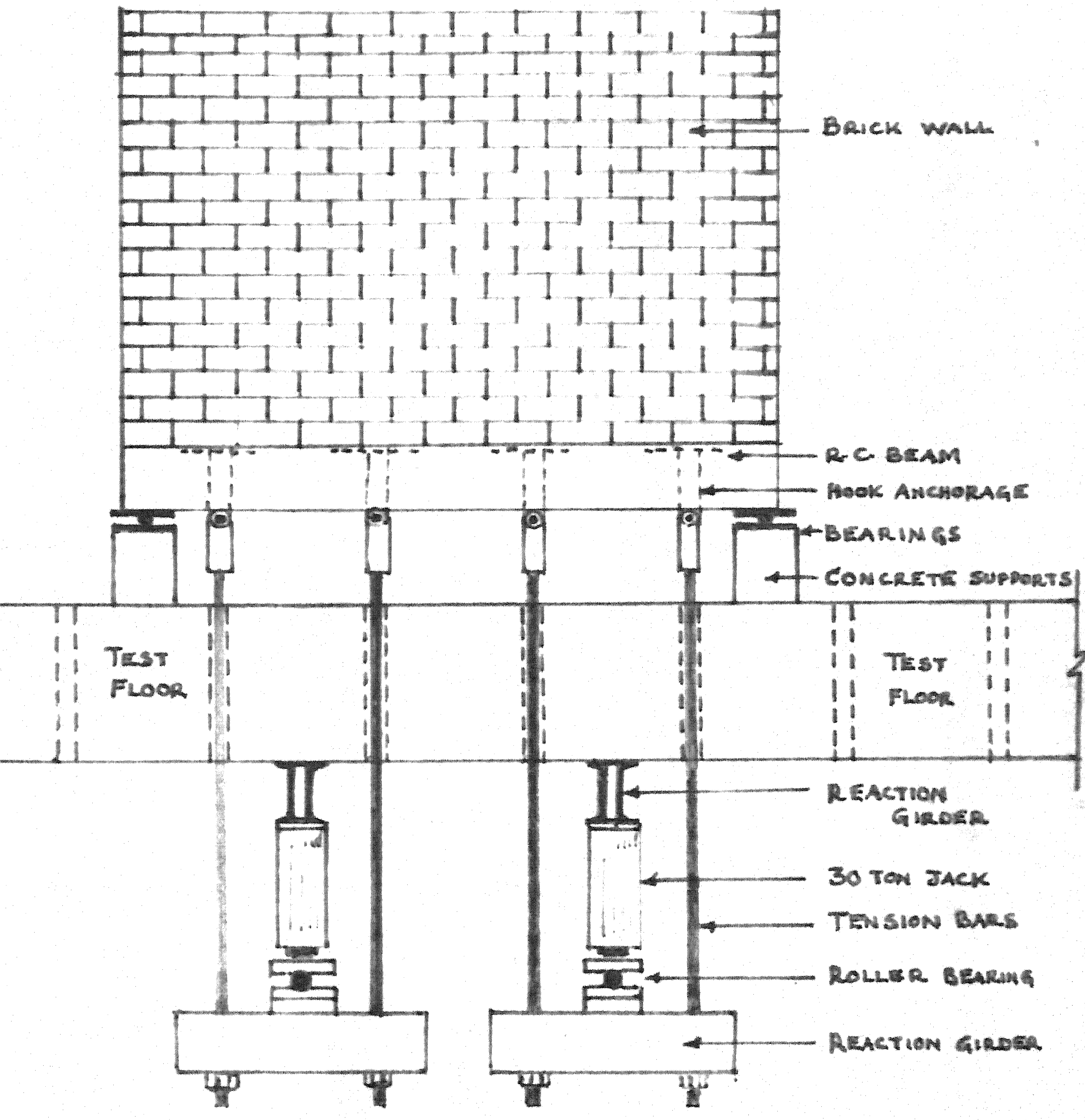


FIG. 5.2 TENSILE LOADING ARRANGEMENT

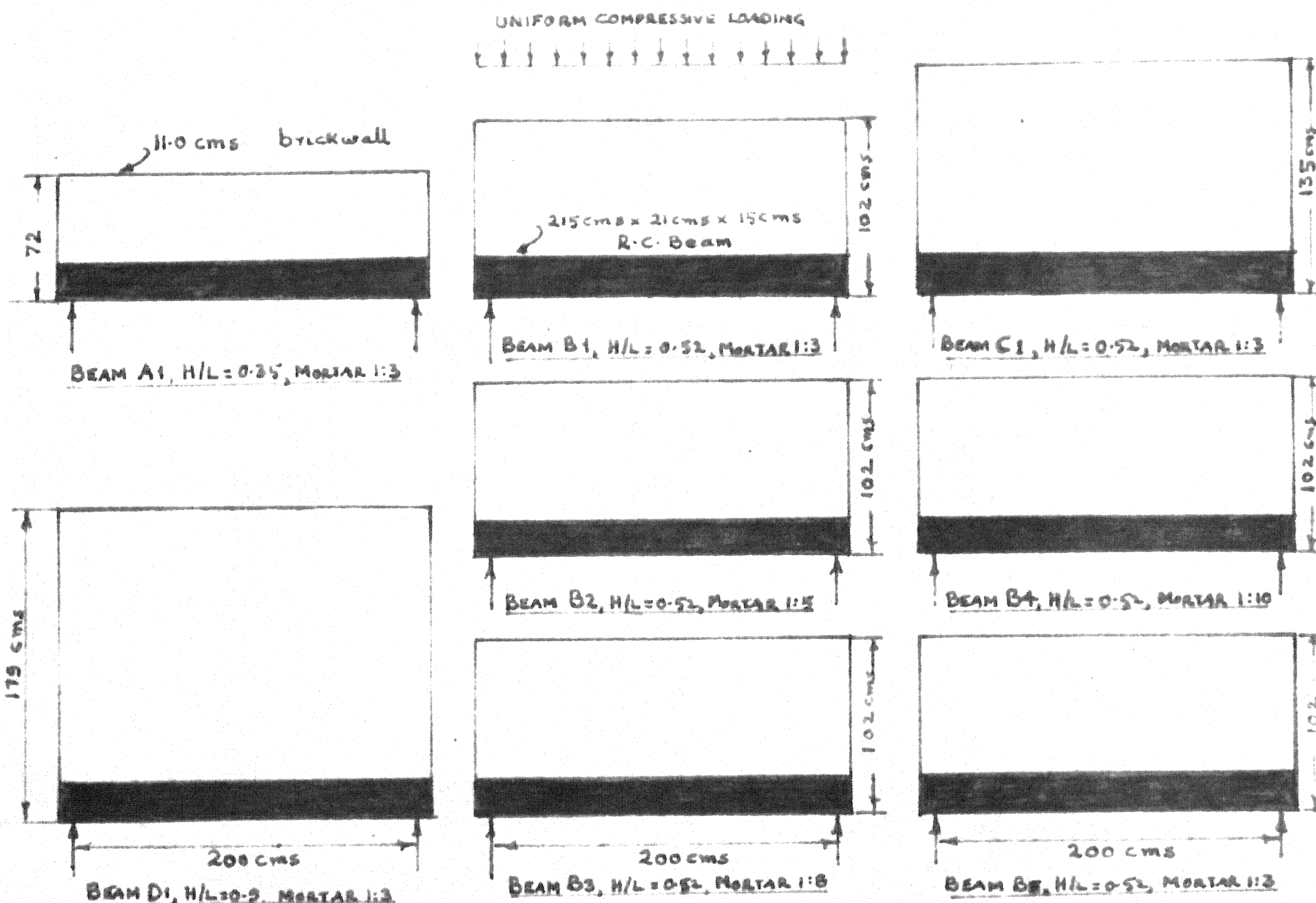


FIG. 5·3(a), DETAILS OF SPECIMENS UNDER COMPRESSIVE LOADING

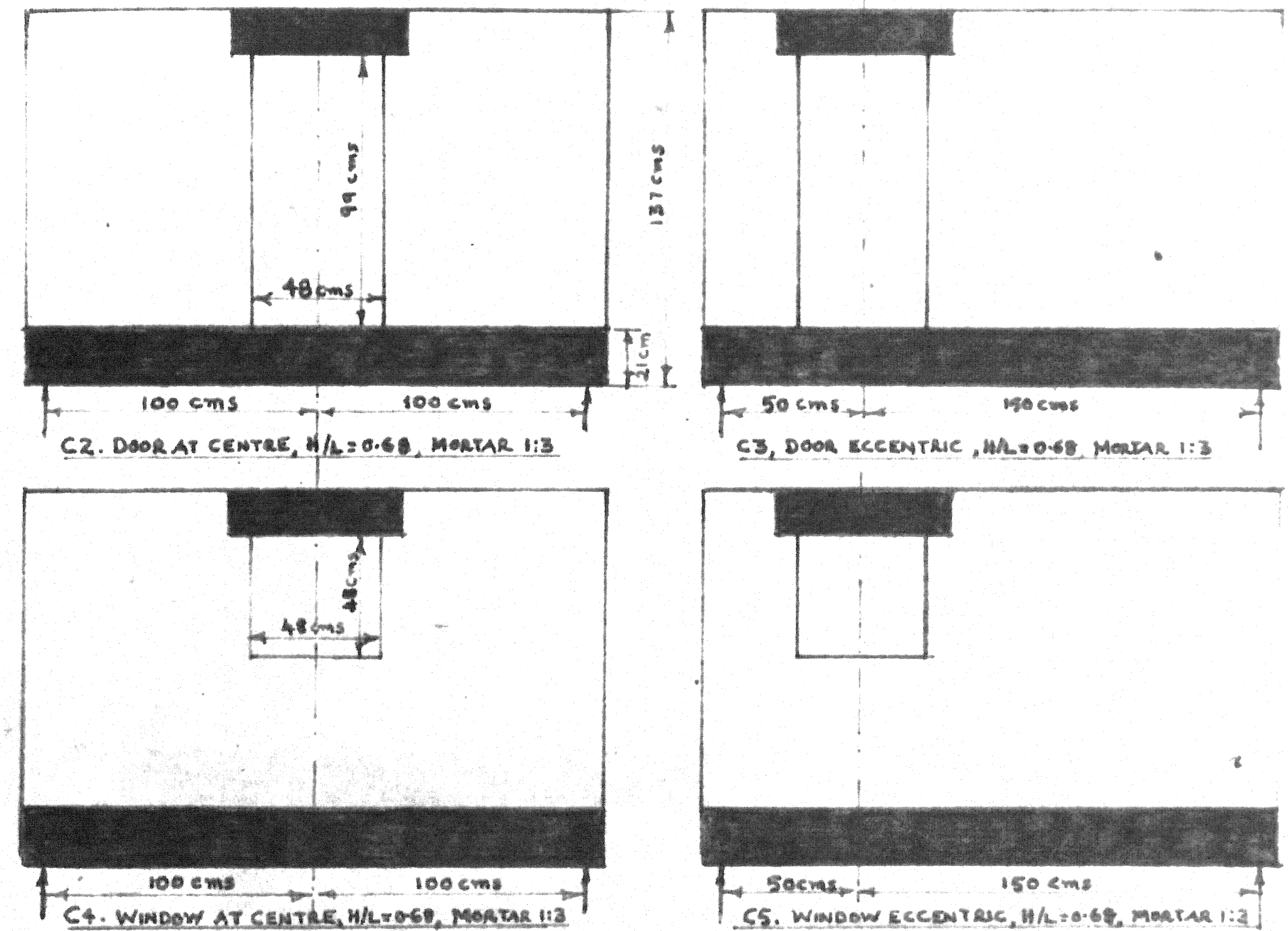


FIG. 5.3(b). DETAILS OF SPECIMENS WITH OPENINGS UNDER COMPRESSIVE LOADING.

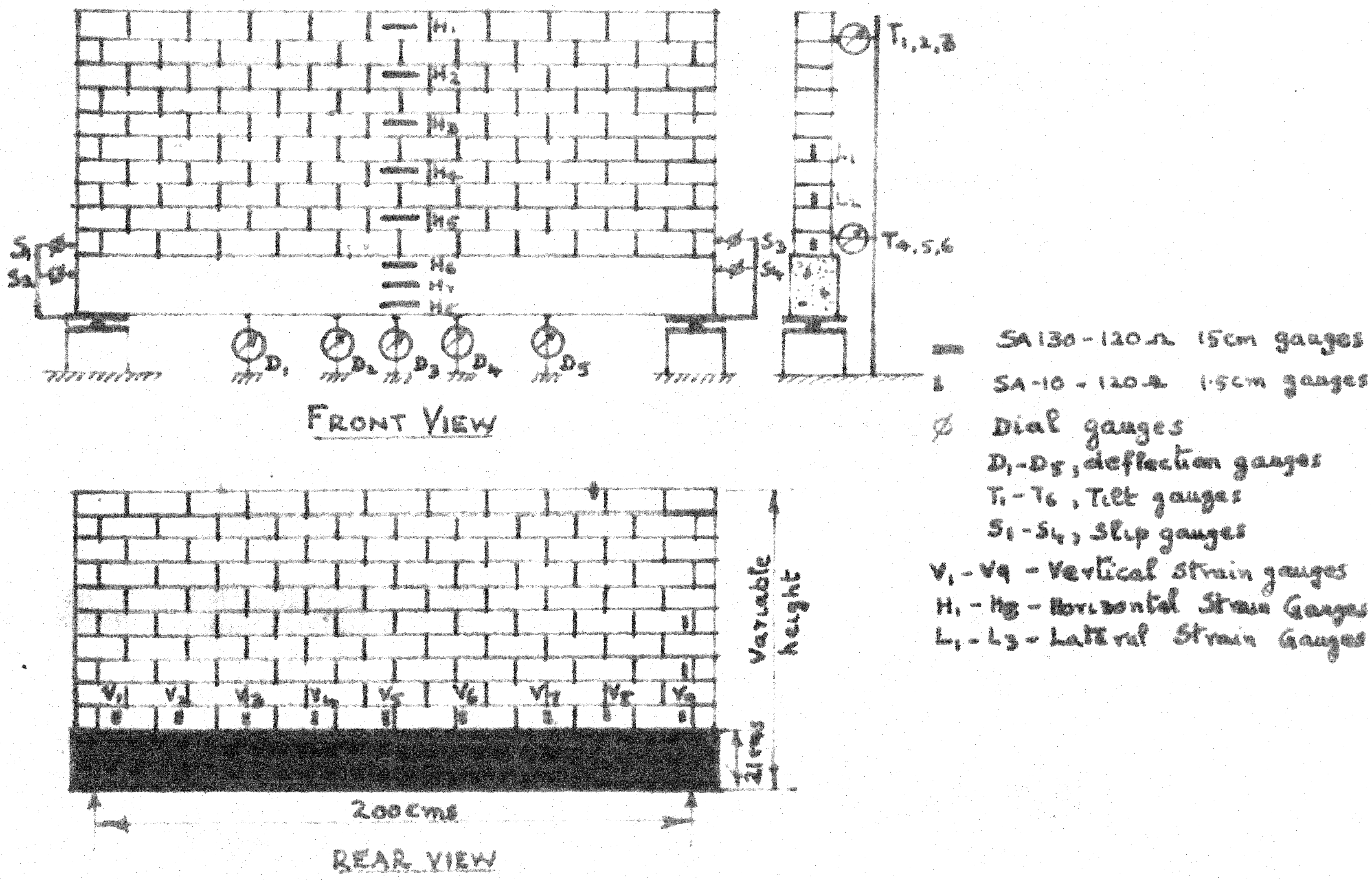


FIG. 5.4. INSTRUMENTATION FOR COMPRESSION SPECIMENS

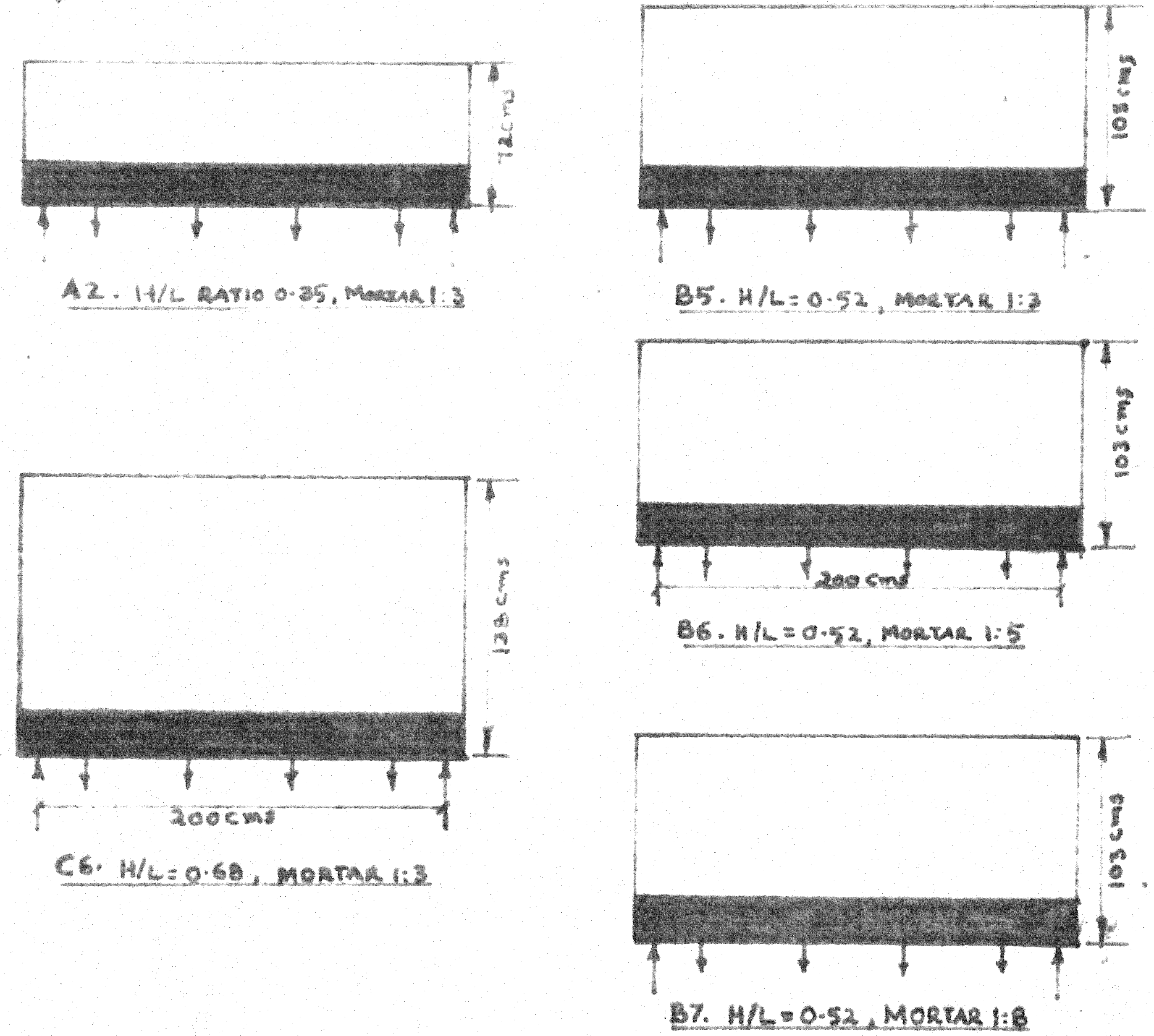


FIG. 5·5· DETAILS OF SPECIMENS IN TENSILE LOADING

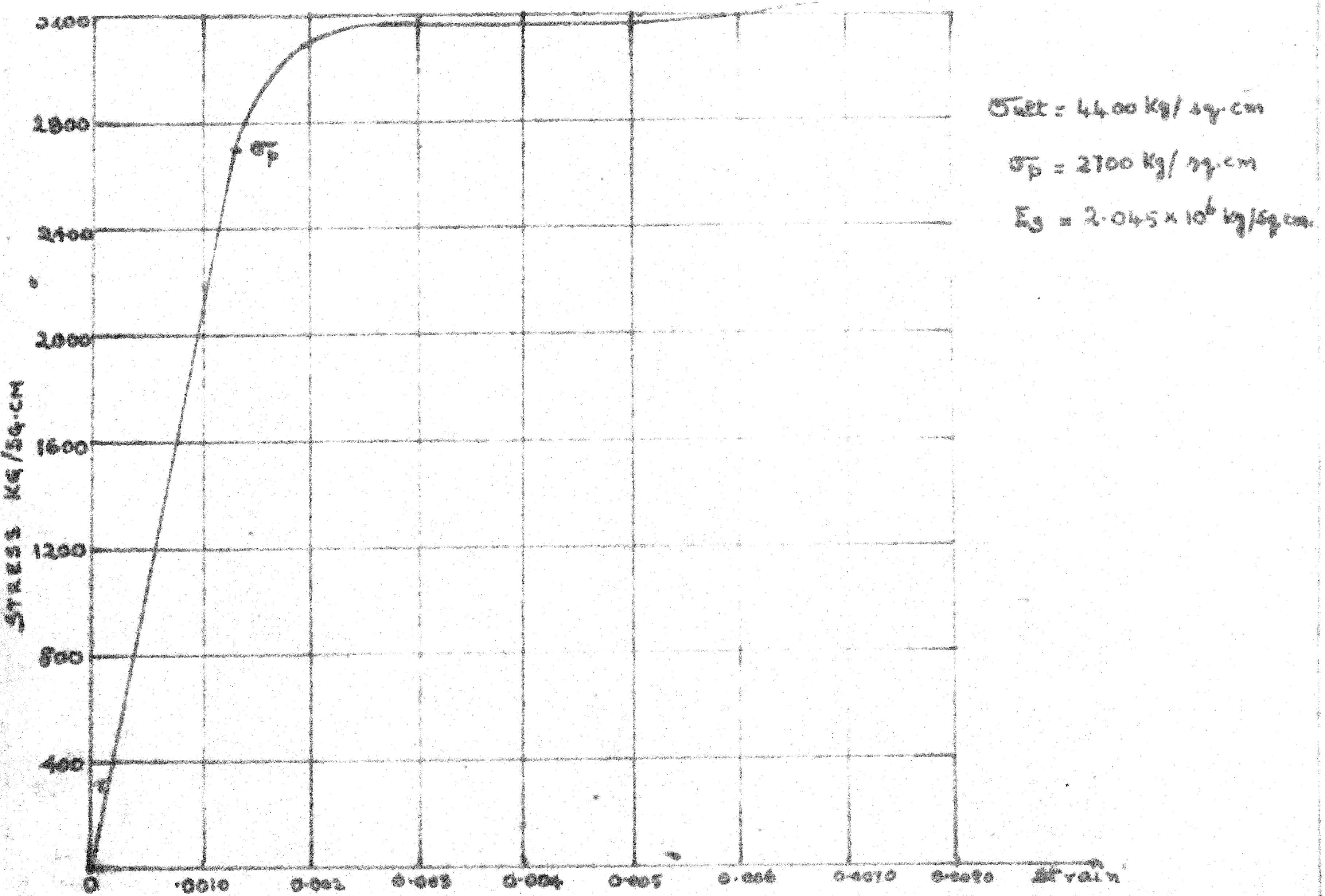


FIG. 5·6· STRESS - STRAIN CURVE FOR 10 mm ϕ, REINFORCEMENT

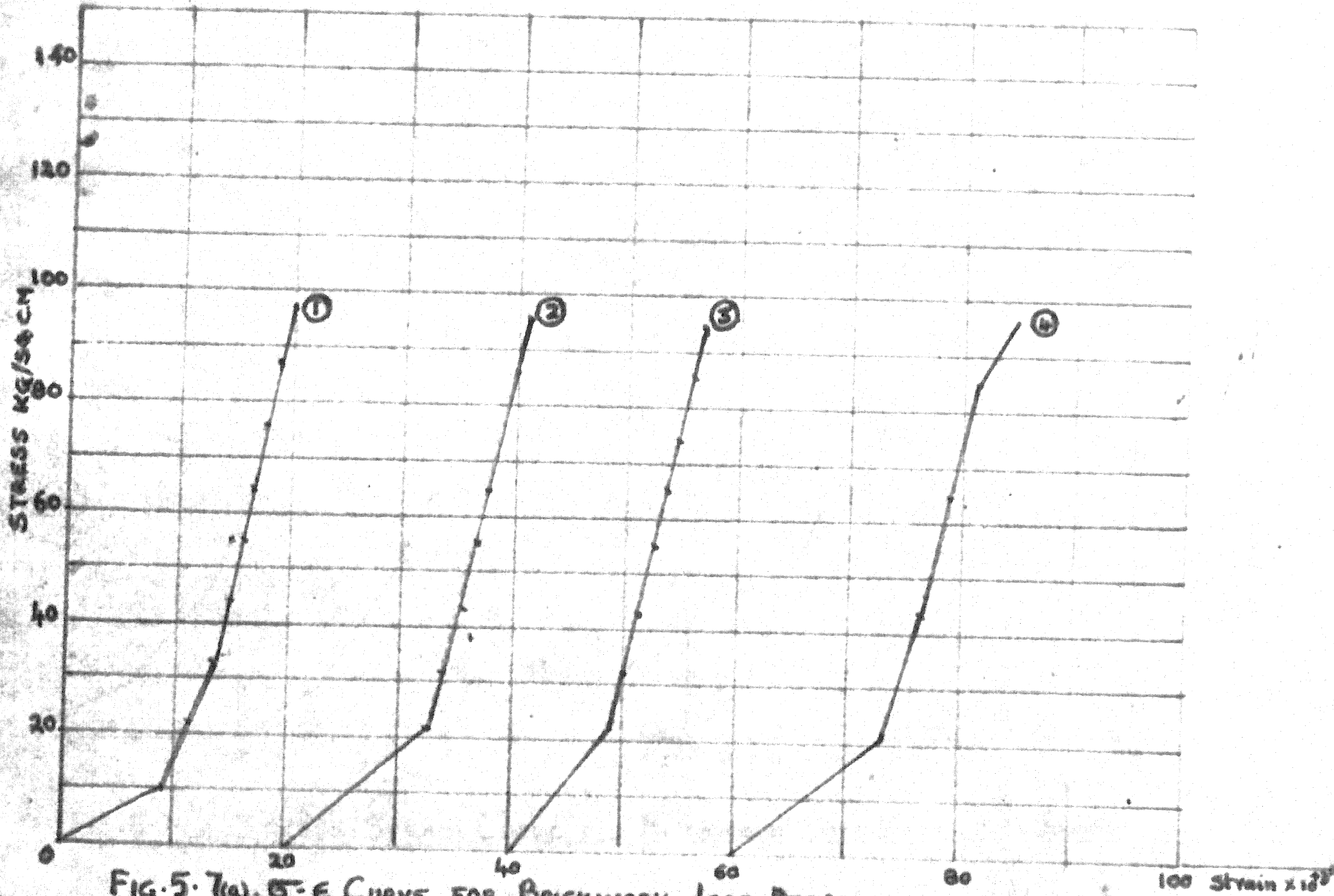


FIG. 5· 7(a). σ - ε CURVE FOR BRICKWORK, LOAD PERPENDICULAR TO JOINTS

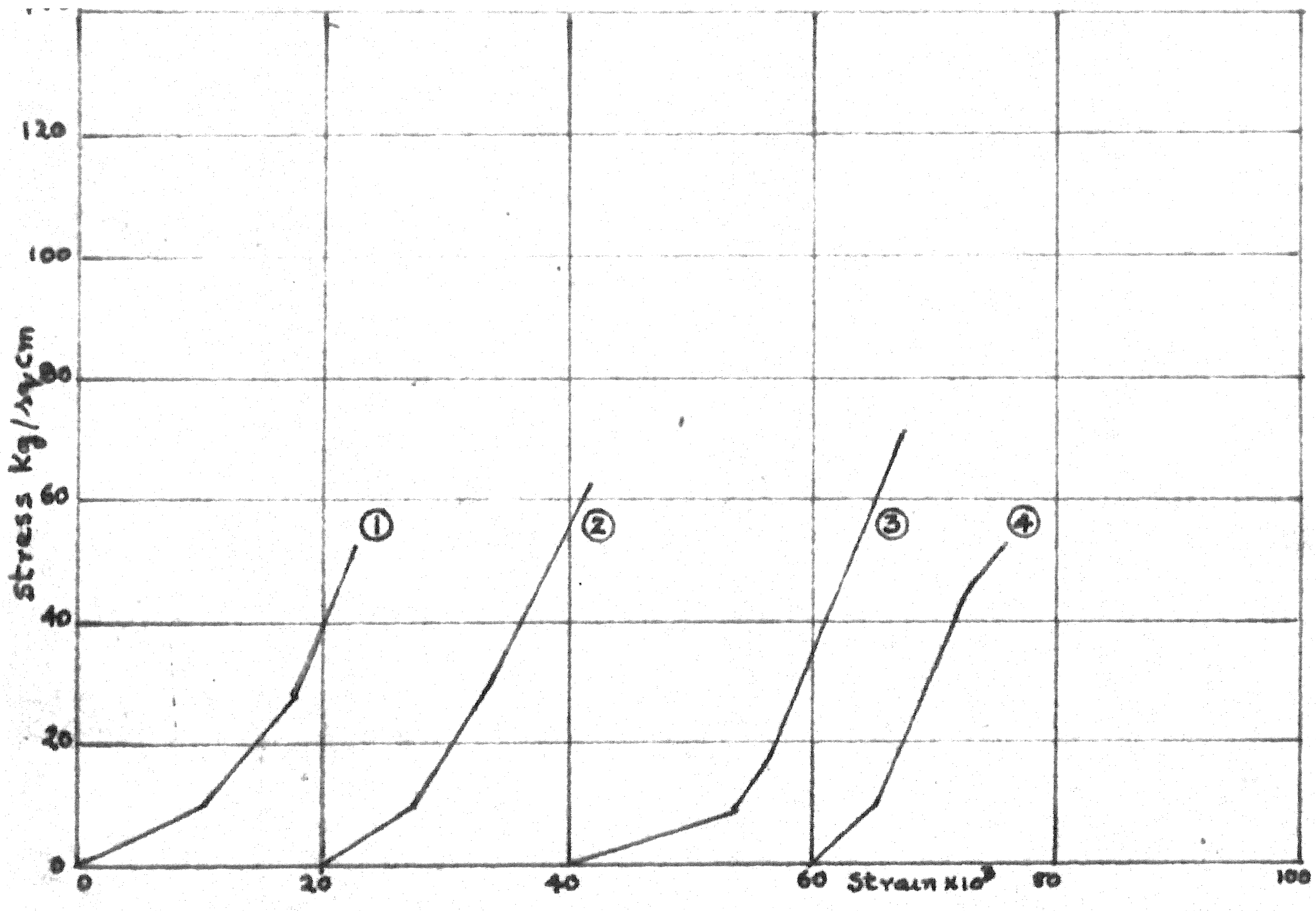


FIG. 5.7.(b) STRESS-STRAIN CURVE FOR BRICKWORK - LOAD PARALLEL TO JOINTS.

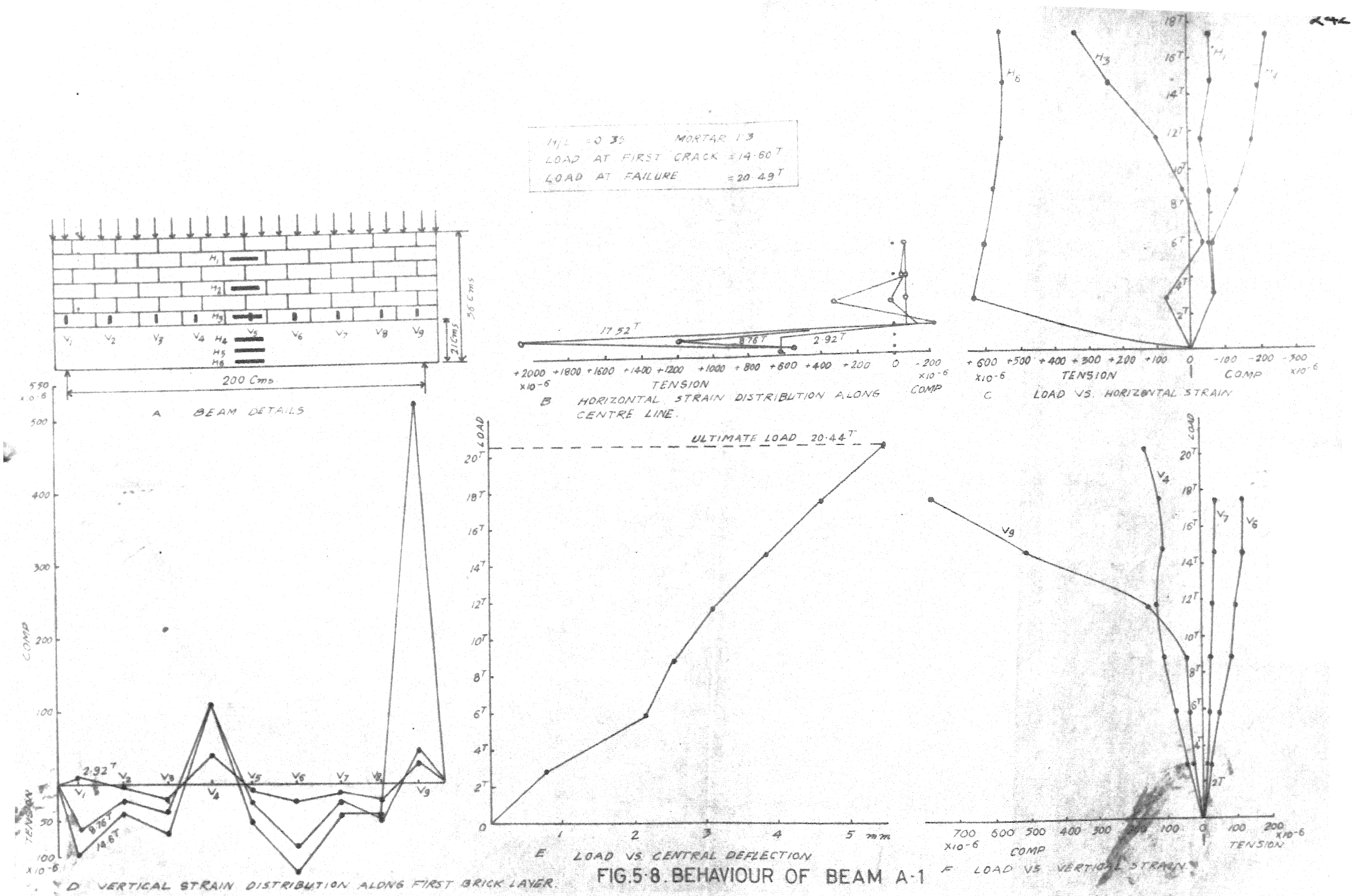


FIG.5.8. BEHAVIOUR OF BEAM A-1

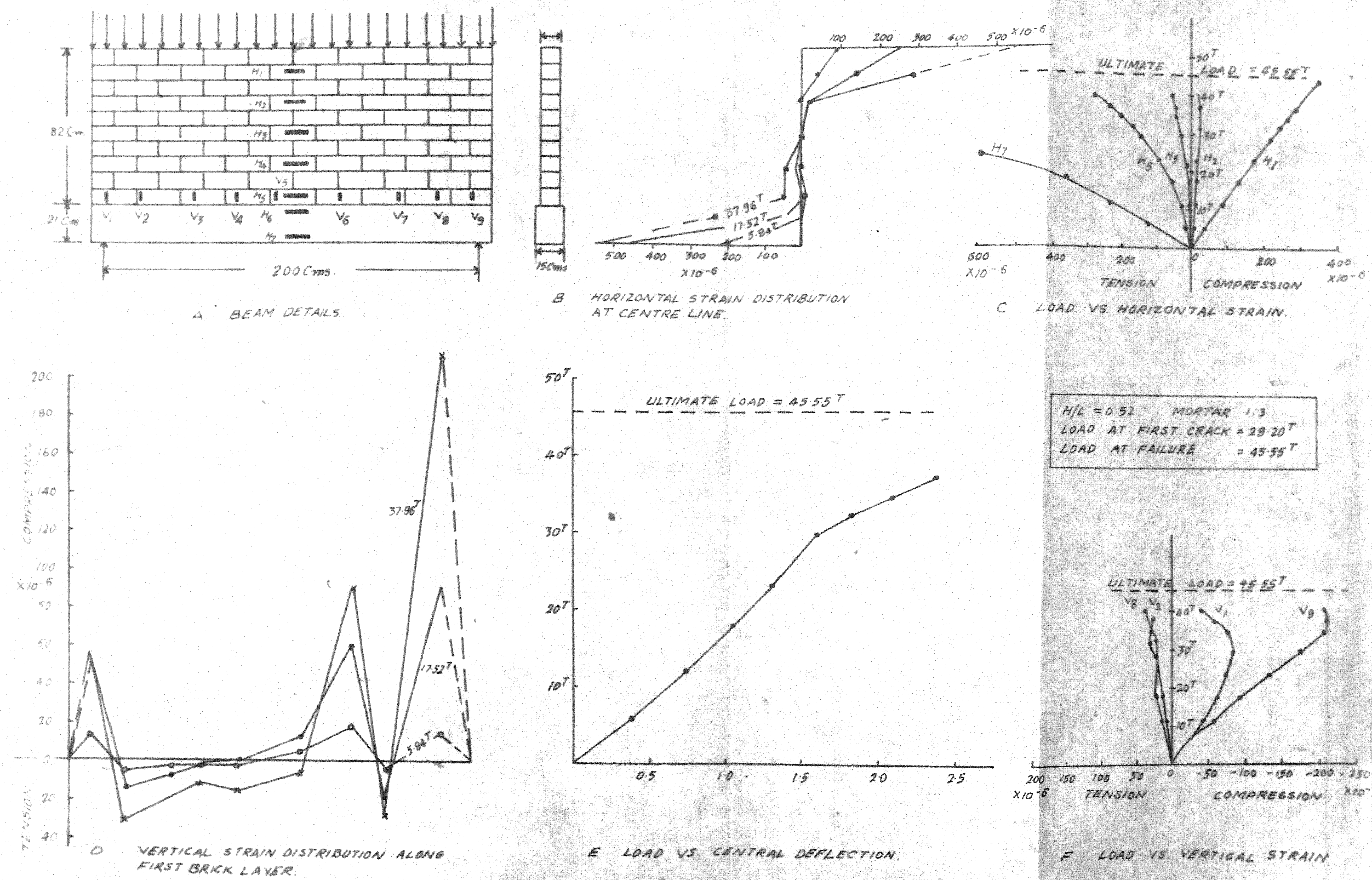


FIG. 5.9. BEHAVIOUR OF BEAM B·1.

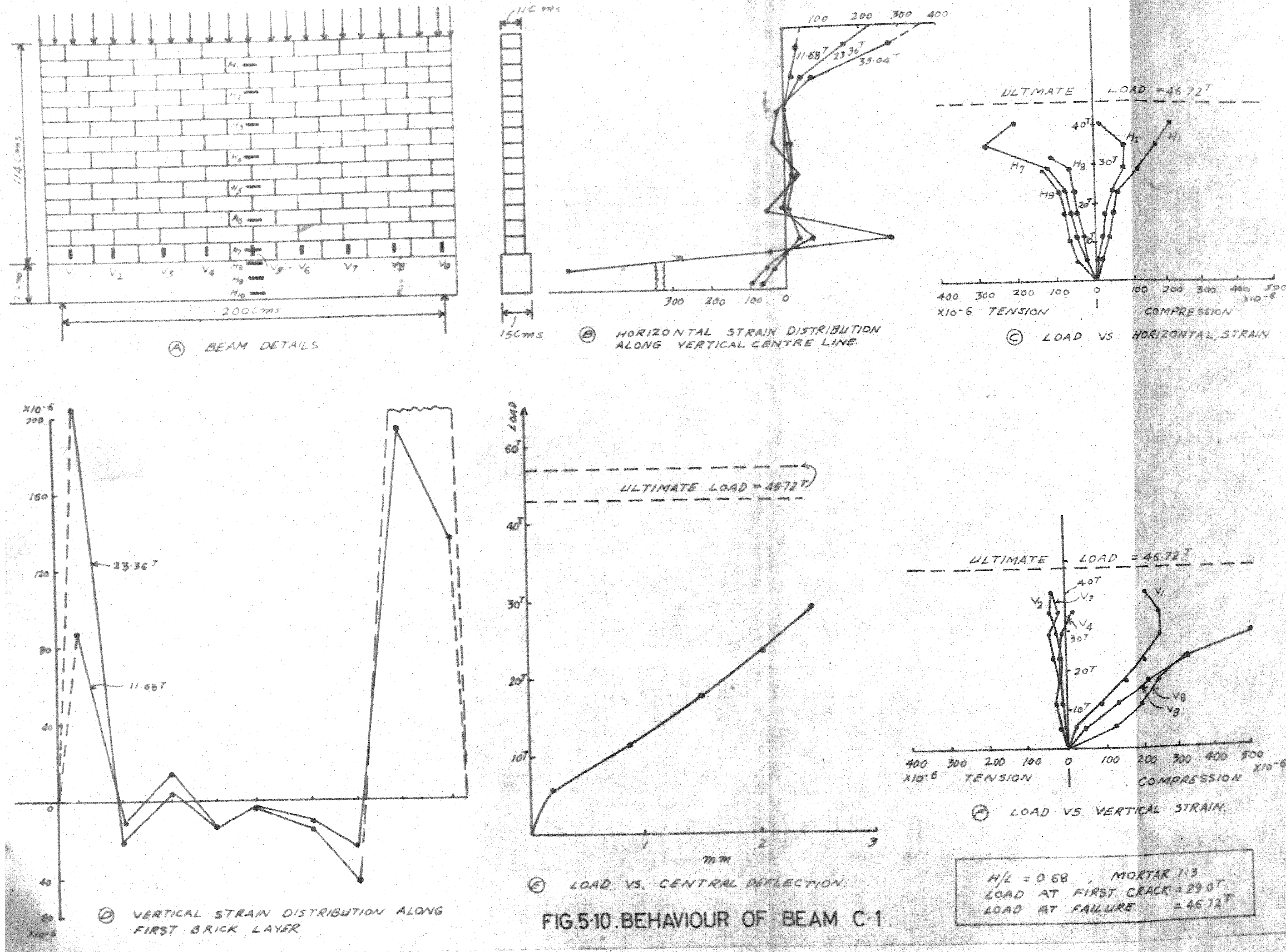


FIG.5·10. BEHAVIOUR OF BEAM C·1.

H/L = 0·68 , MORTAR 1:3
LOAD AT FIRST CRACK = 29·0T
LOAD AT FAILURE = 46·72T

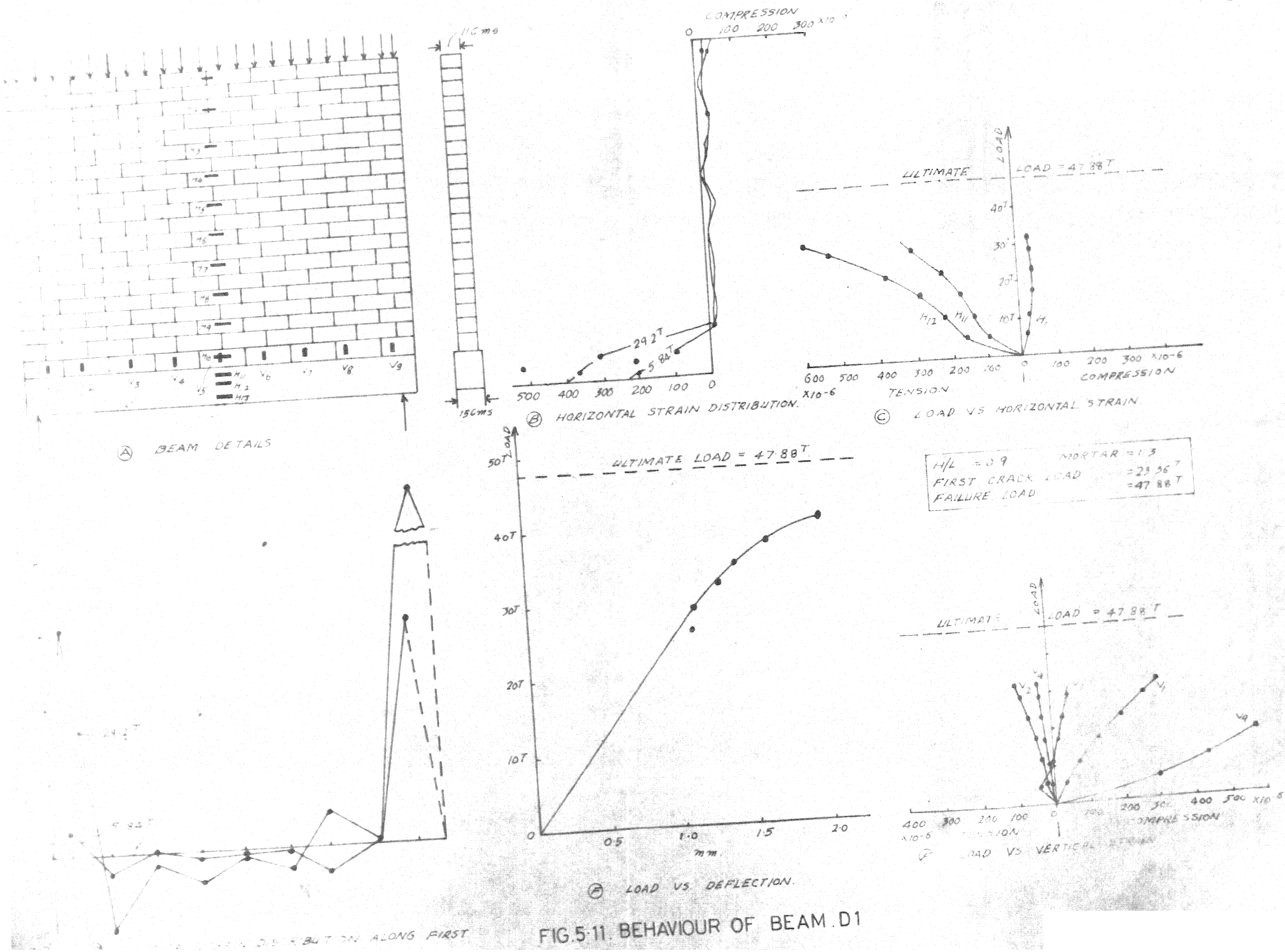


FIG.5.11. BEHAVIOUR OF BEAM.D1

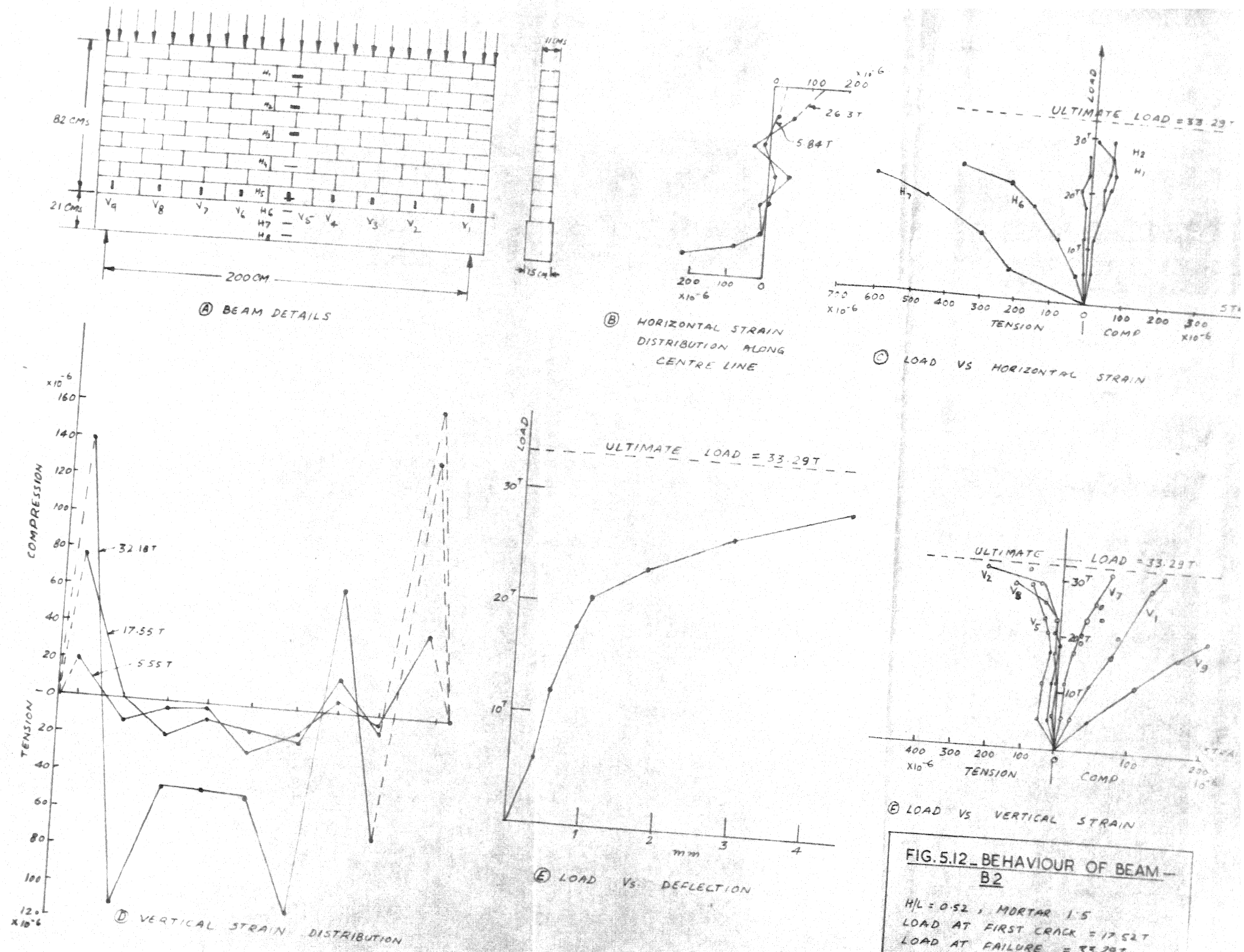
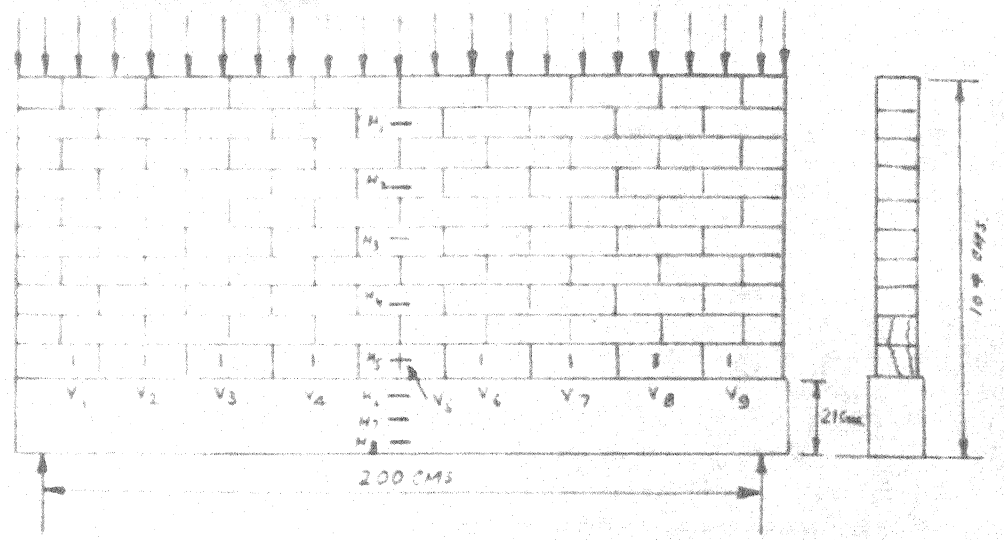


FIG. 5.12 _ BEHAVIOUR OF BEAM — B2

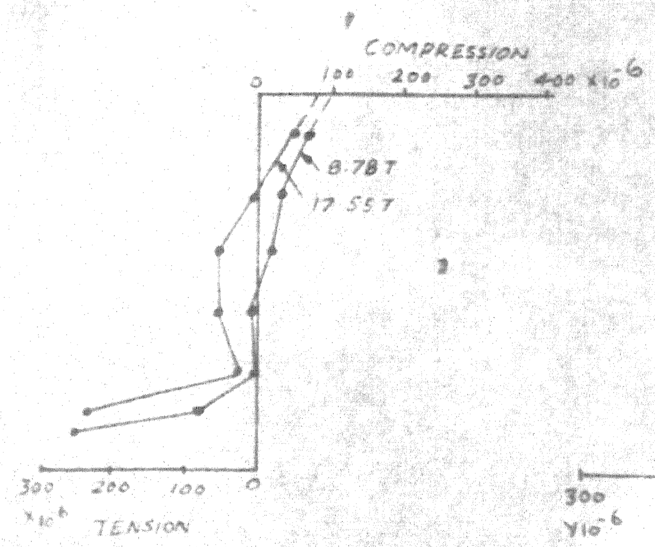
H/L = 0.52 , MORTAR 1:5

LOAD AT FIRST CRACK = 17.52 T

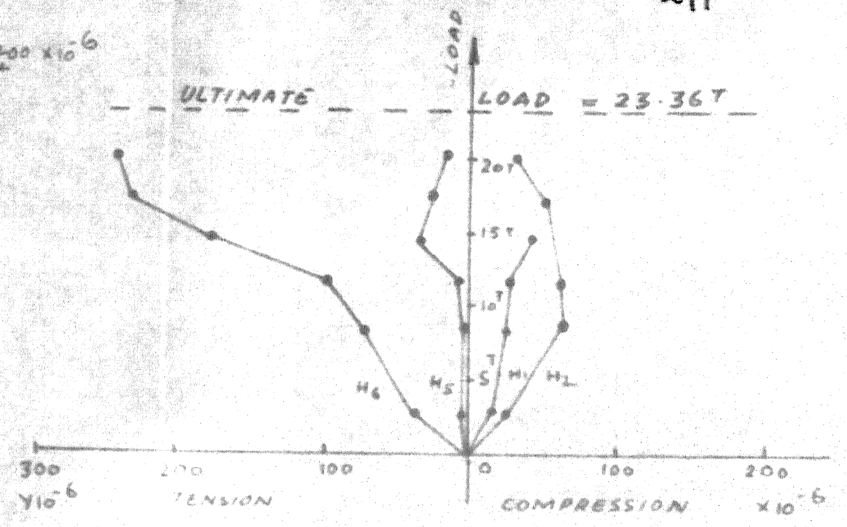
LOAD AT FAILURE = 33.29 T



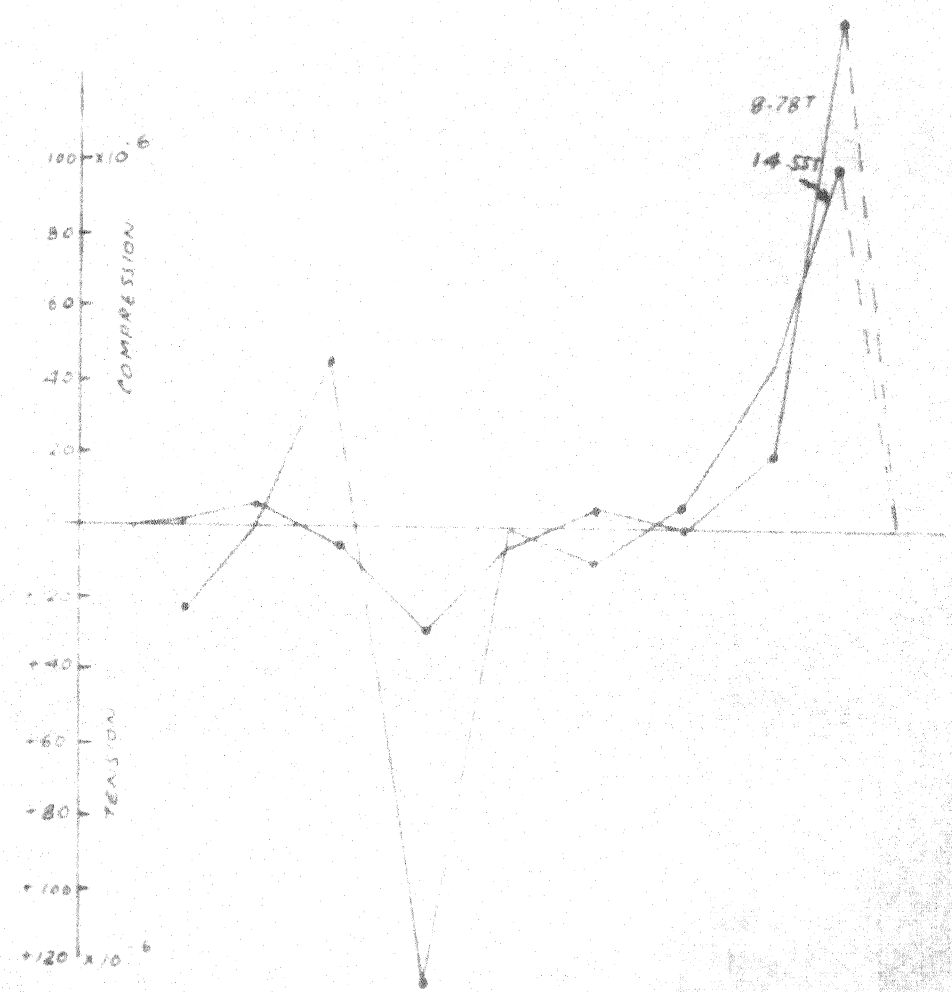
(A) BEAM DETAILS



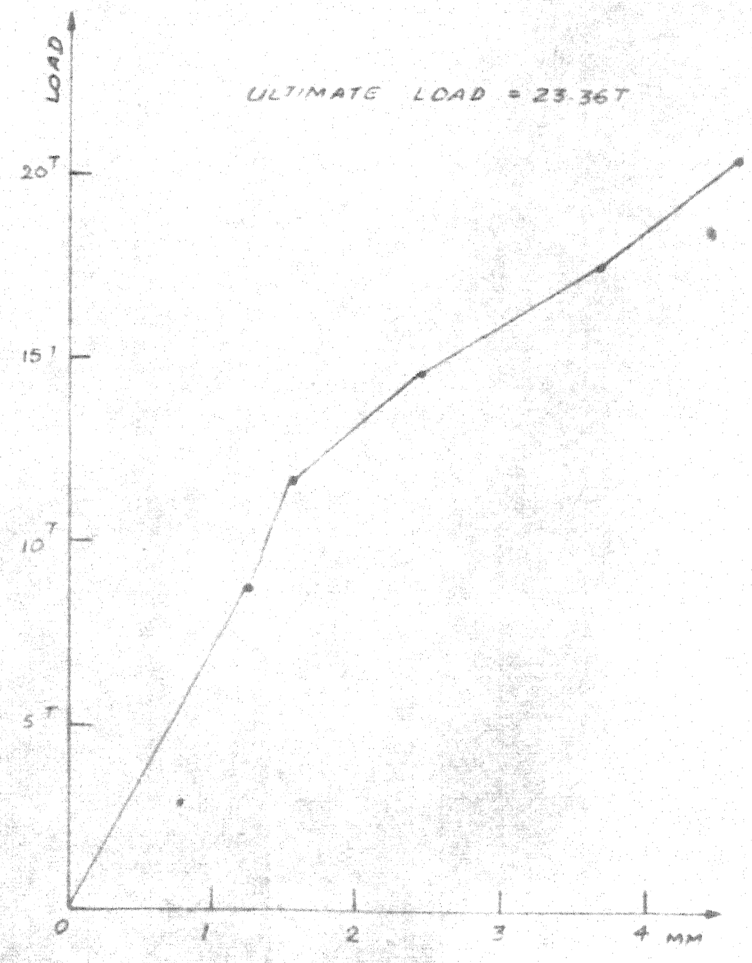
(B) HORIZONTAL STRAIN DISTRIBUTION ALONG CENTRE LINE



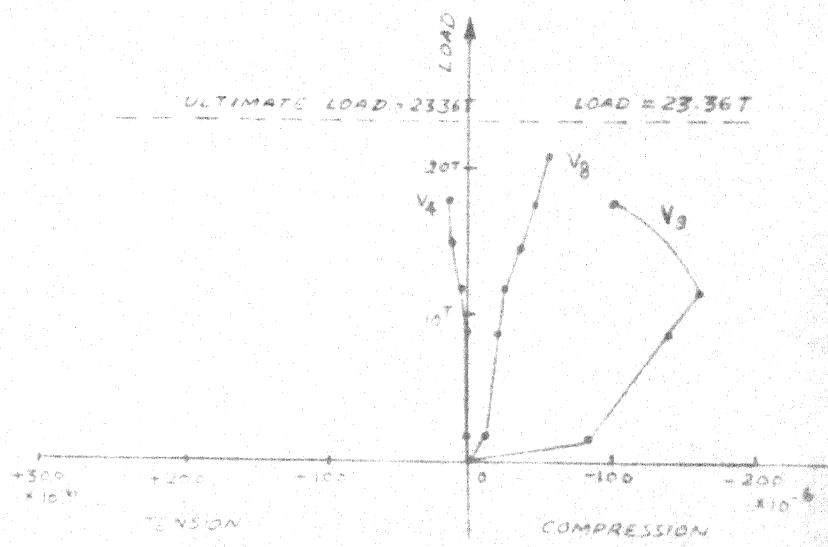
(C) LOAD Vs HORIZONTAL STRAIN



(D) VERTICAL STRAIN DISTRIBUTION ALONG FIRST BRICK LAYER



(E) LOAD Vs CENTRAL DEFLECTION



(F) LOAD Vs VERTICAL STRAIN

FIG 5.13_BEHAVIOUR OF BEAM—B 3

H/L = 0.52	MORTAR = 1:8
LOAD AT FIRST CRACK = 8.76 T	
LOAD AT FAILURE = 23.36T	

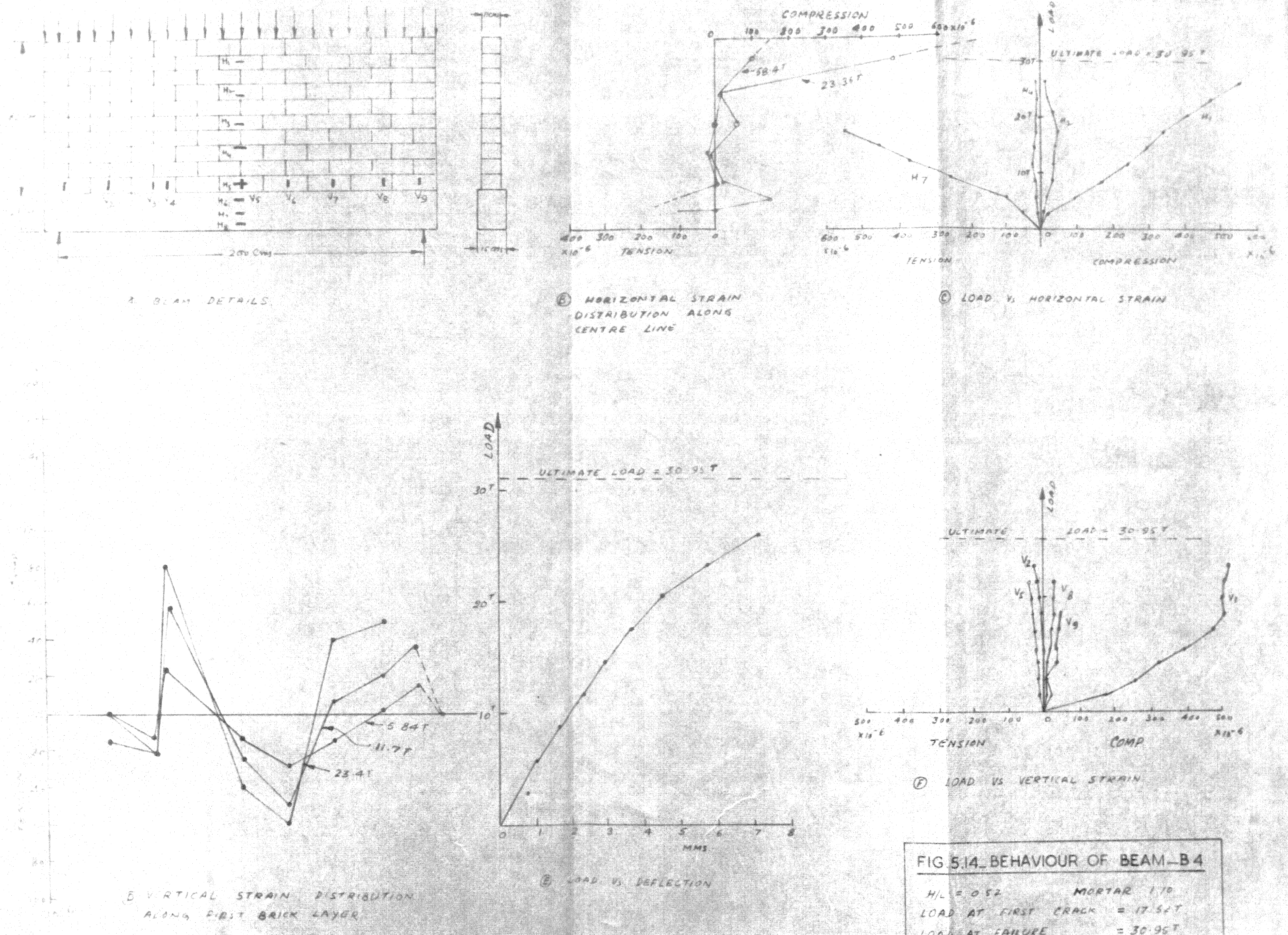
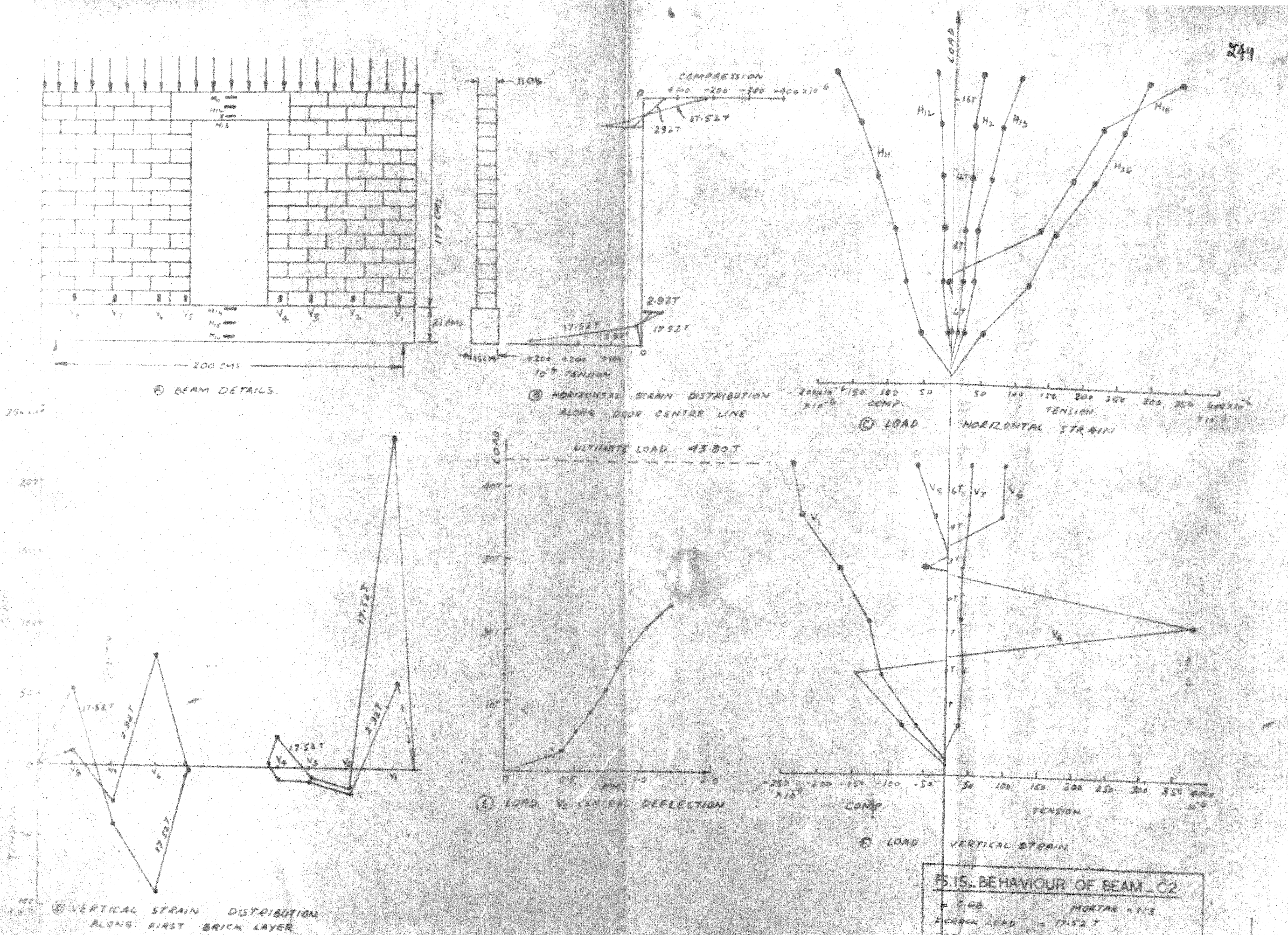


FIG 5.14_BEHAVIOUR OF BEAM-B4

H/L = 0.52 MORTAR 1:10

LOAD AT FIRST CRACK = 17.54T

LOAD AT FAILURE = 30.95T



F5.15_BEHAVIOUR OF BEAM_C2

= 0.68 MORTAR = 1:3
CRACK LOAD = 17.52 T
FRE LOAD = 43.80T

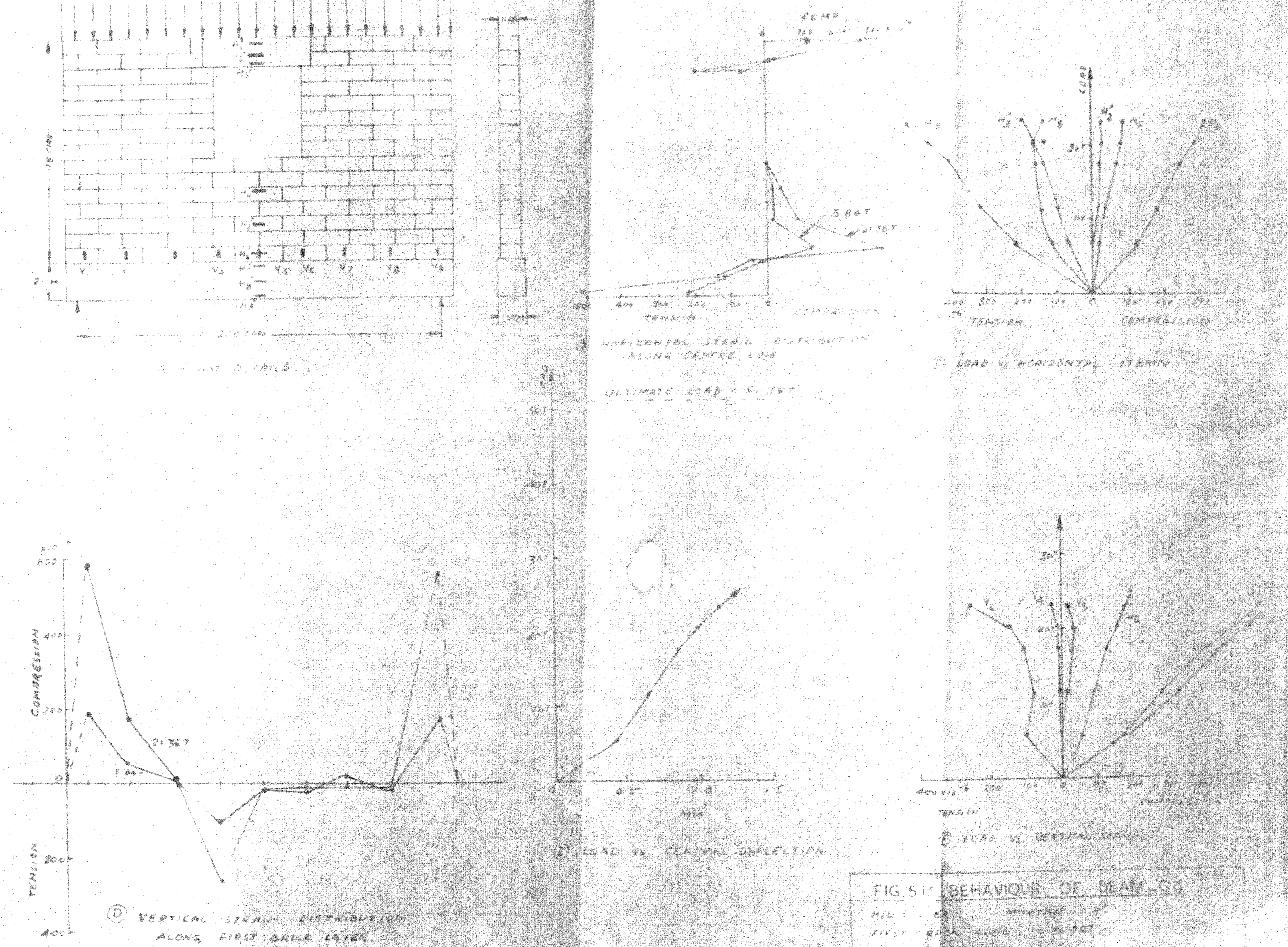


FIG. 5.1[illegible] BEHAVIOUR OF BEAM_C4

H/L = [illegible] 68 , MORTAR 1:3

FIRST CRACK LOAD = 30.79T

FAILURE LOAD = 51.39T

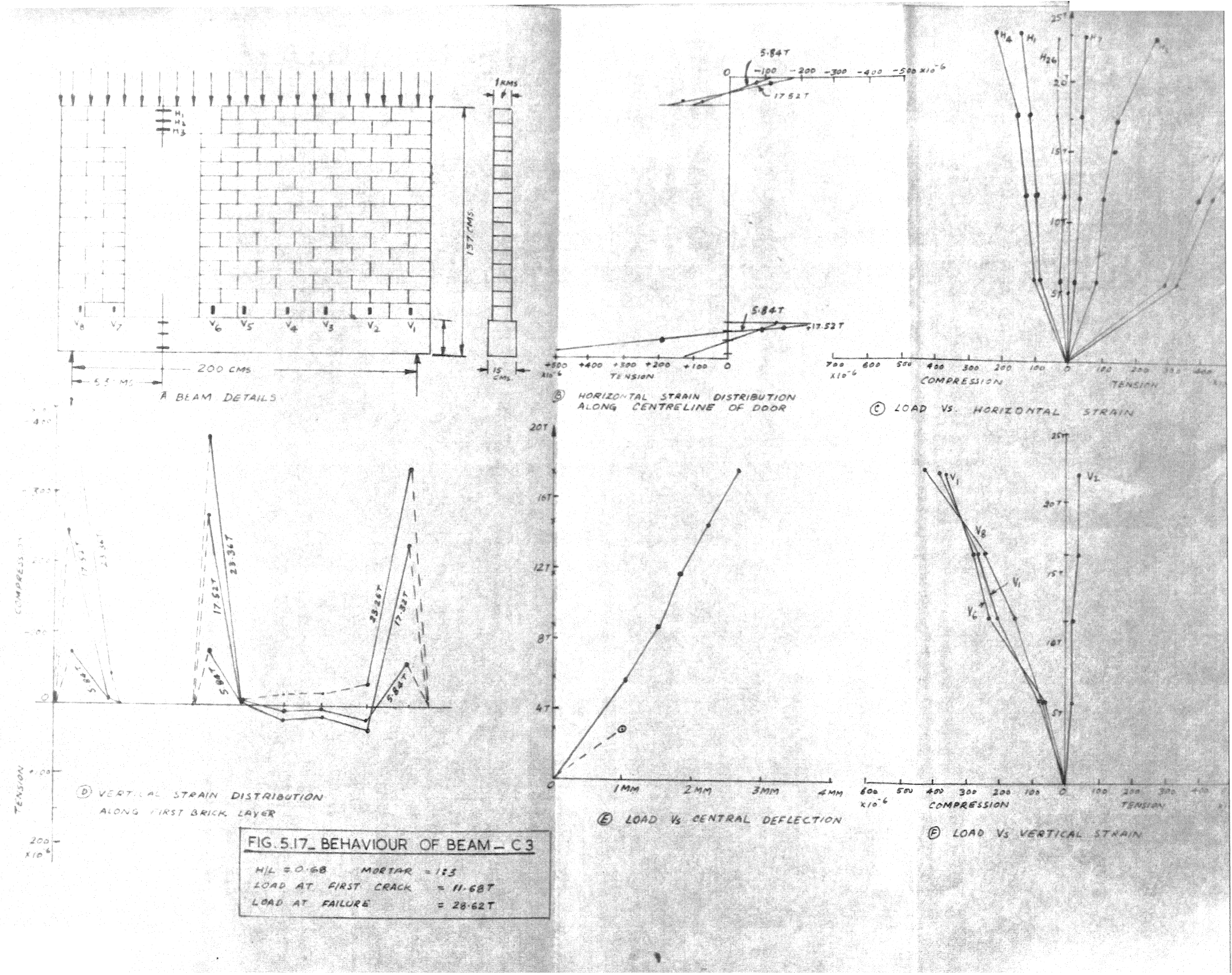


FIG. 5.17_ BEHAVIOUR OF BEAM – C3

H/L = 0.68 MORTAR = 1:3
LOAD AT FIRST CRACK = 11.68T
LOAD AT FAILURE = 28.62T

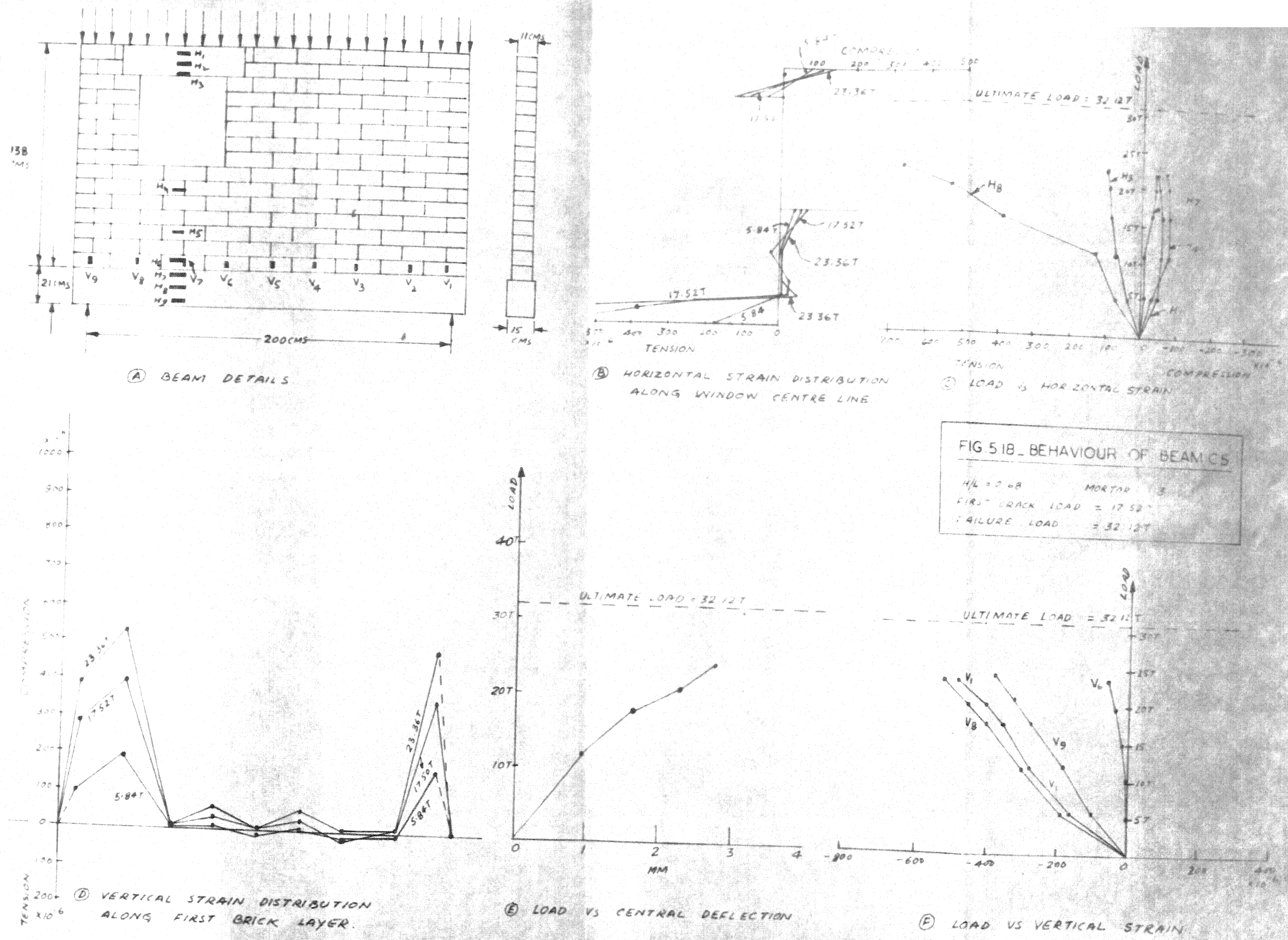


FIG. 5.18 _ BEHAVIOUR OF BEAM CS

H/L = 0.68 MORTAR 3

FIRST CRACK LOAD = 17.52T

FAILURE LOAD = 32.12T

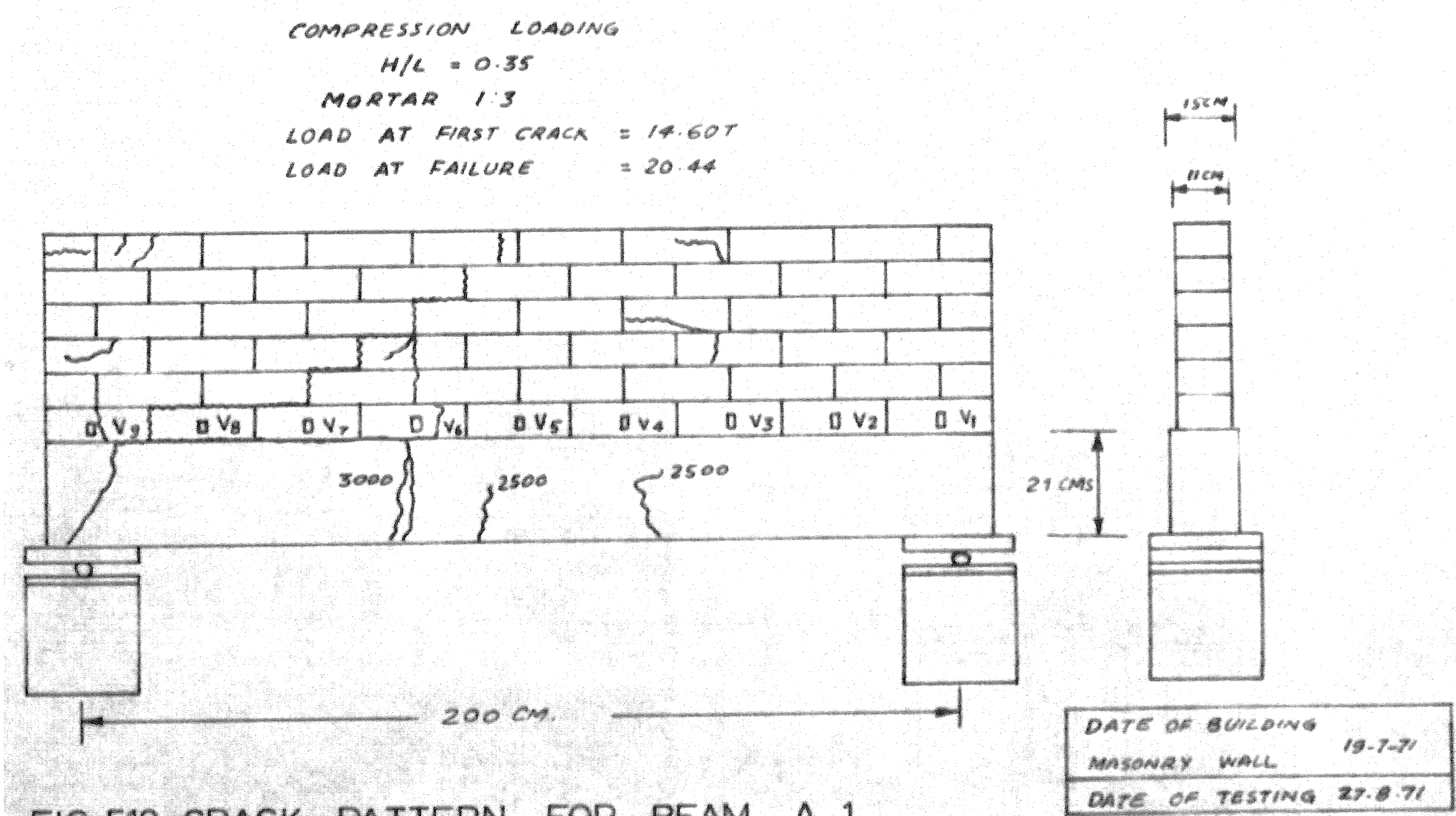
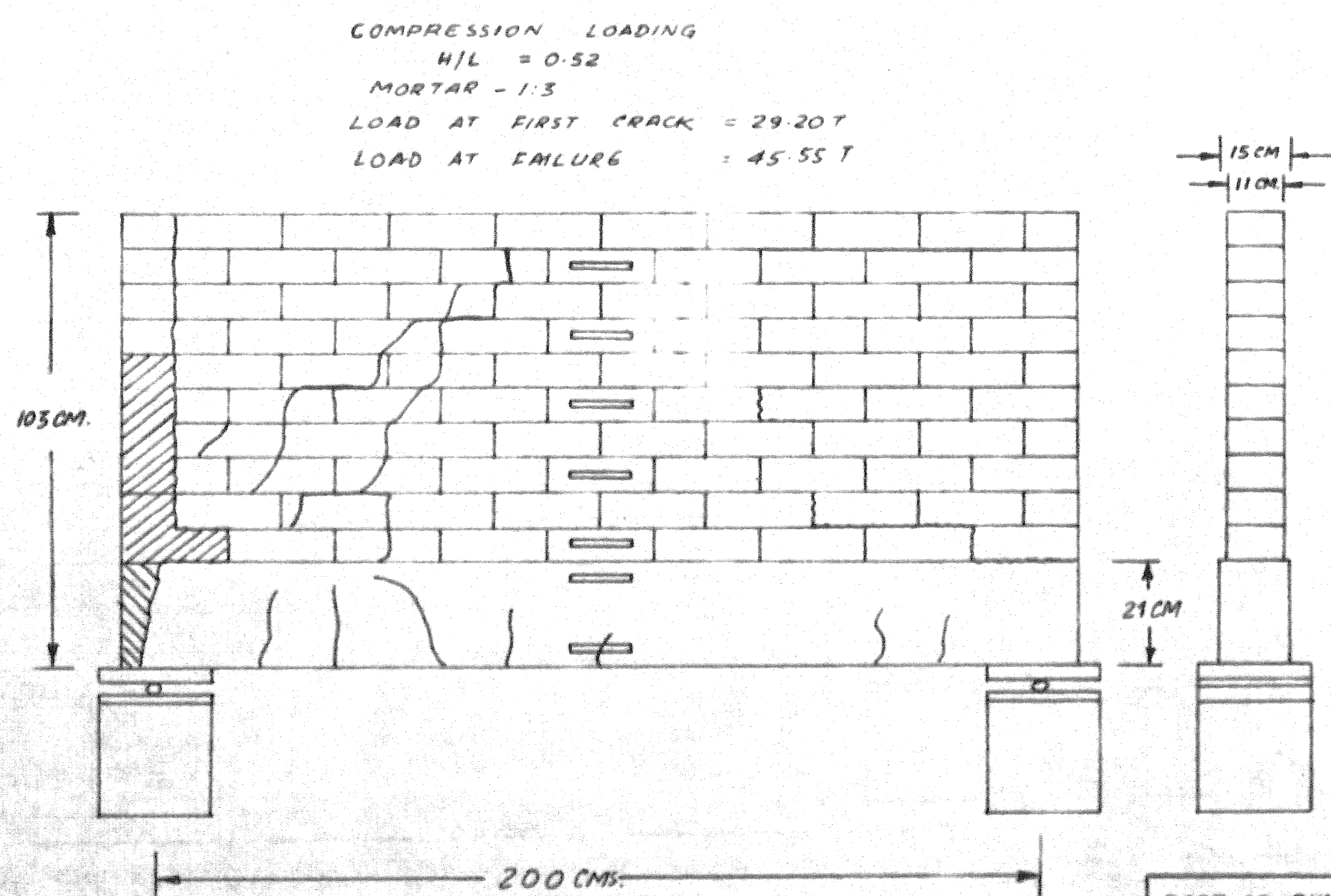


FIG. 5.19_CRACK PATTERN FOR BEAM A_1



DATE OF BUILDING MASONRY WALL	15.4.71
DATE OF TESTING	9.6.71

FIG. 5.20_CRACK PATTERN AT FAILURE - BEAM B.1

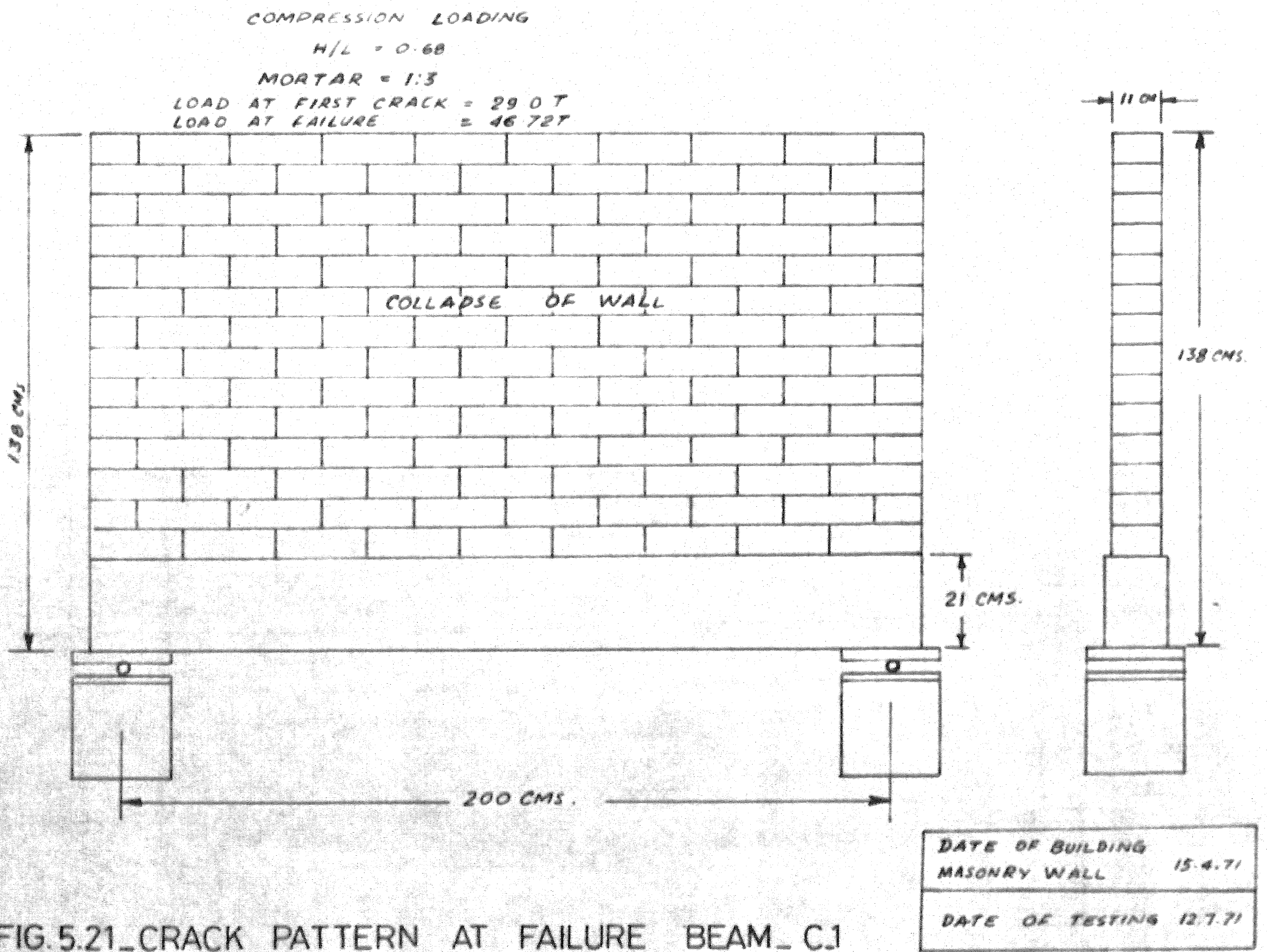


FIG. 5.21_CRACK PATTERN AT FAILURE BEAM_C.1
(WALL COLLAPSED TO PIECES)

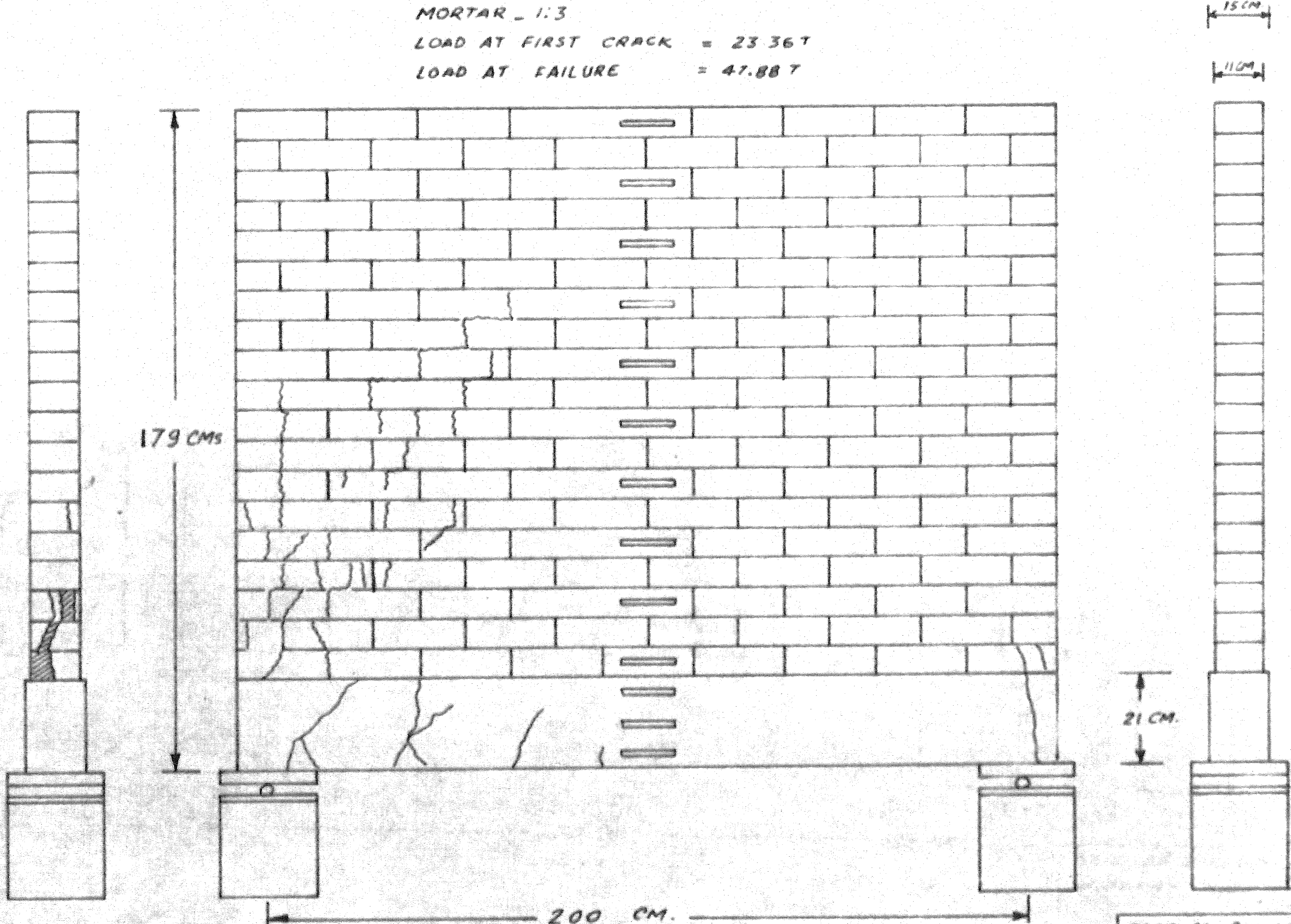
COMPRESSION LOADING

$H/L = 0.90$

MORTAR _ 1:3

LOAD AT FIRST CRACK = 23.36 T

LOAD AT FAILURE = 47.88 T



DATE OF BUILDING MASONRY WALL	8-8-71
DATE OF TESTING	20.9.71

FIG. 5.22_CRACK PATTERN AT FAILURE BEAM-D_1

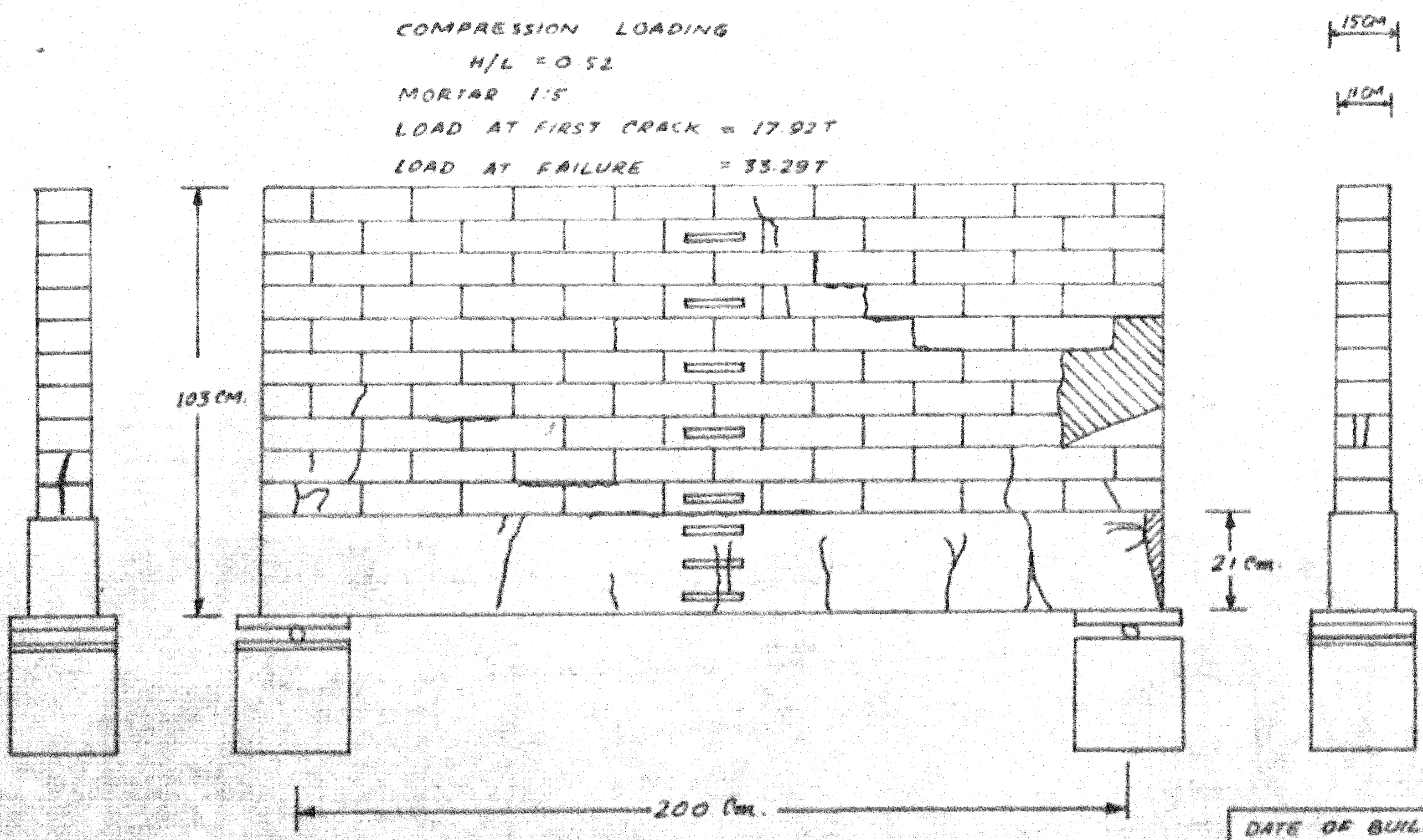
COMPRESSION LOADING

H/L = 0.52

MORTAR 1:5

LOAD AT FIRST CRACK = 17.92T

LOAD AT FAILURE = 33.29T



DATE OF BUILDING MASONRY WALL	7-7-71
DATE OF TESTING	5-8-71

FIG.5.23_CRACK PATTERN AT FAILURE - BEAM B-2

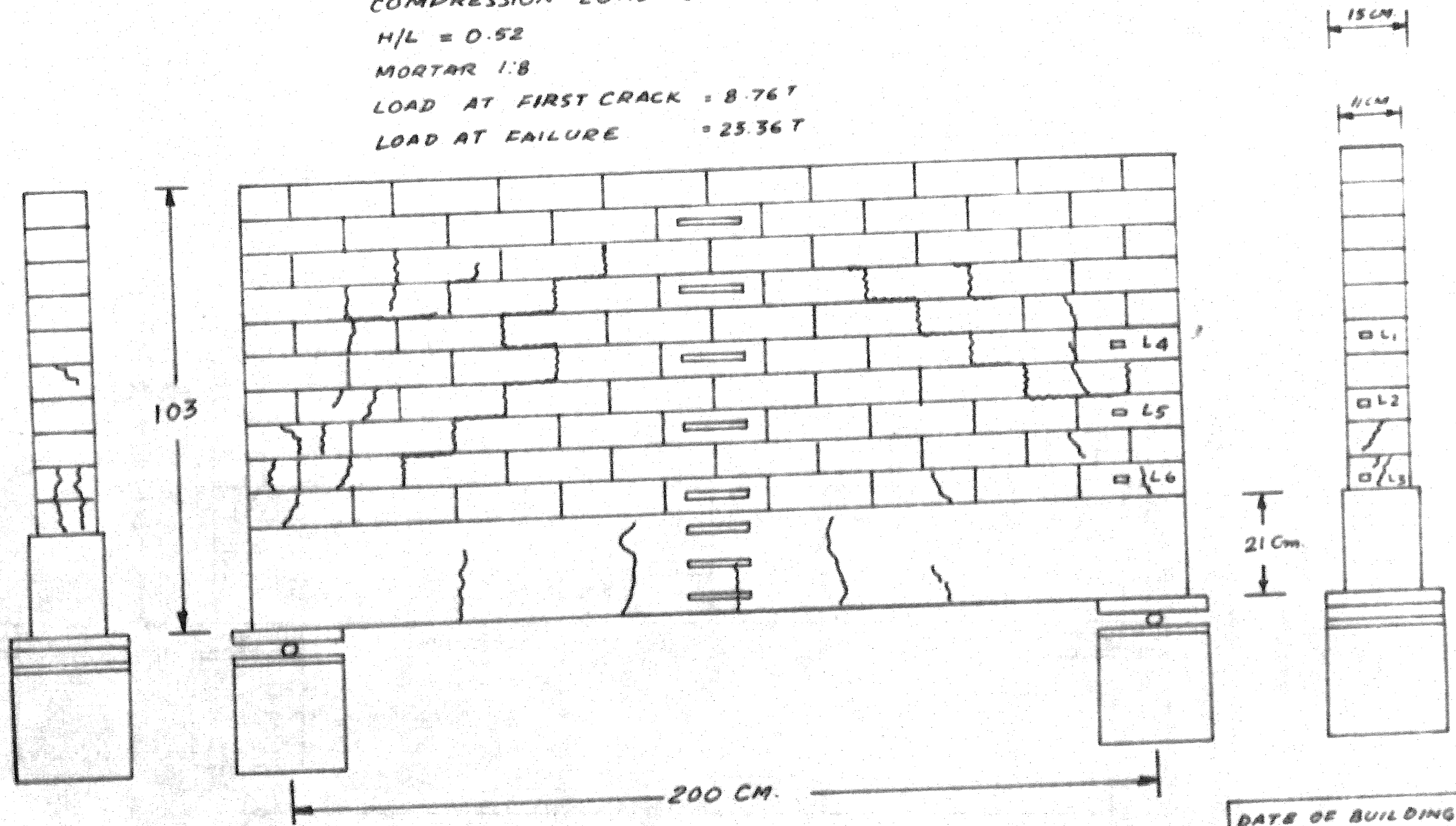
COMPRESSION LOADING

H/L = 0.52

MORTAR 1:8

LOAD AT FIRST CRACK = 8.76 T

LOAD AT FAILURE = 23.36 T



DATE OF BUILDING MASONRY WALL	18.7.71
DATE OF TESTING -	19.8.71

FIG. 5.24_CRACK PATTERN AT FAILURE - BEAM_B 3

COMPRESSION LOADING

H/L = 0.52

MORTAR _ 1:10

LOAD AT FIRST CRACK = 17.52 T

LOAD AT FAILURE = 30.952 T

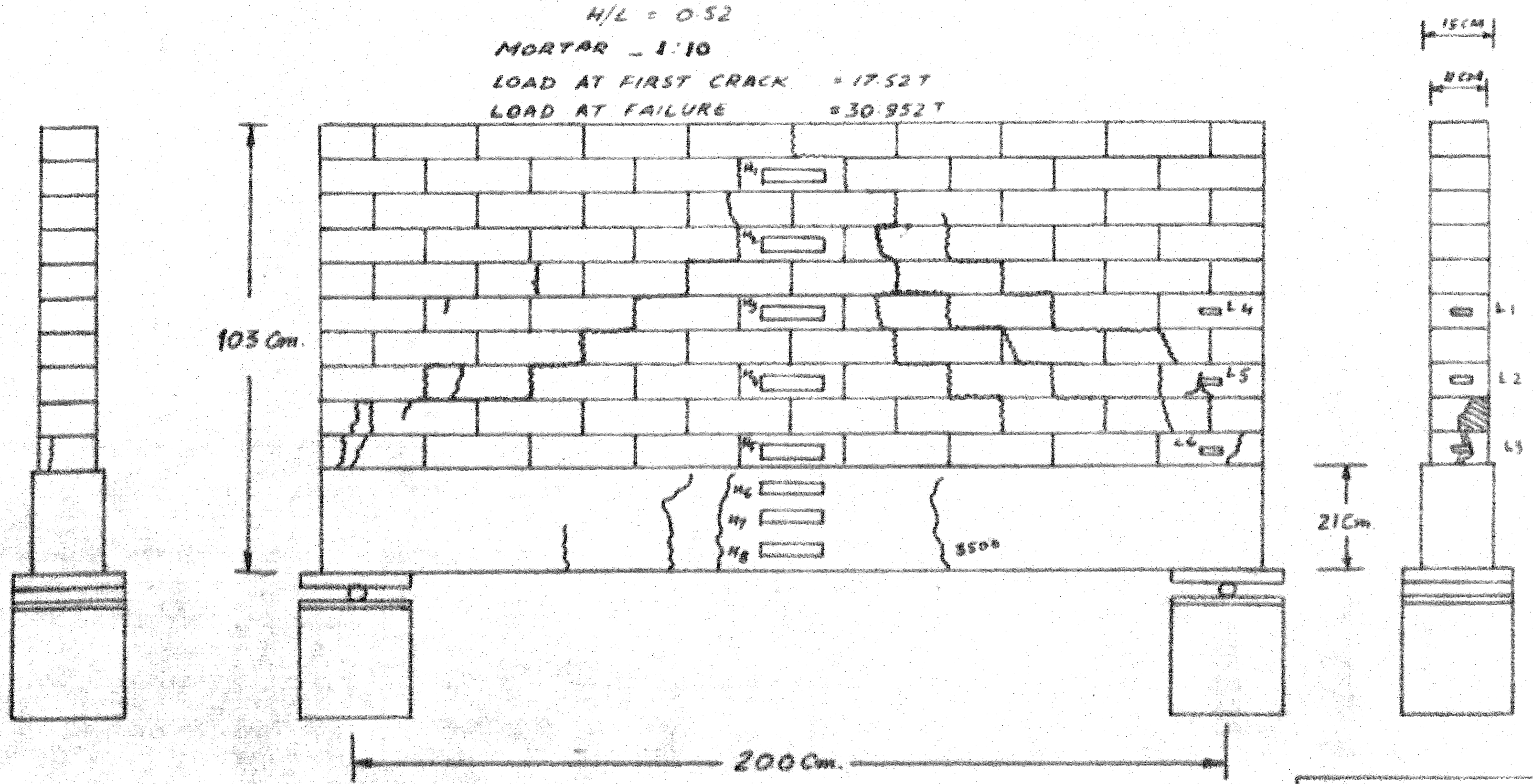


FIG. 5.25_CRACK PATTERN AT FAILURE BEAM _ B 4

DATE OF BUILDING MASONRY WALL	18.7.71
DATE OF TESTING	22.8.71

COMPRESSION LOADING
H/L = 0·68
MORTAR 1:3

LOAD AT FIRST CRACK = 17·52 T
LOAD AT FAILURE = 43·80 T

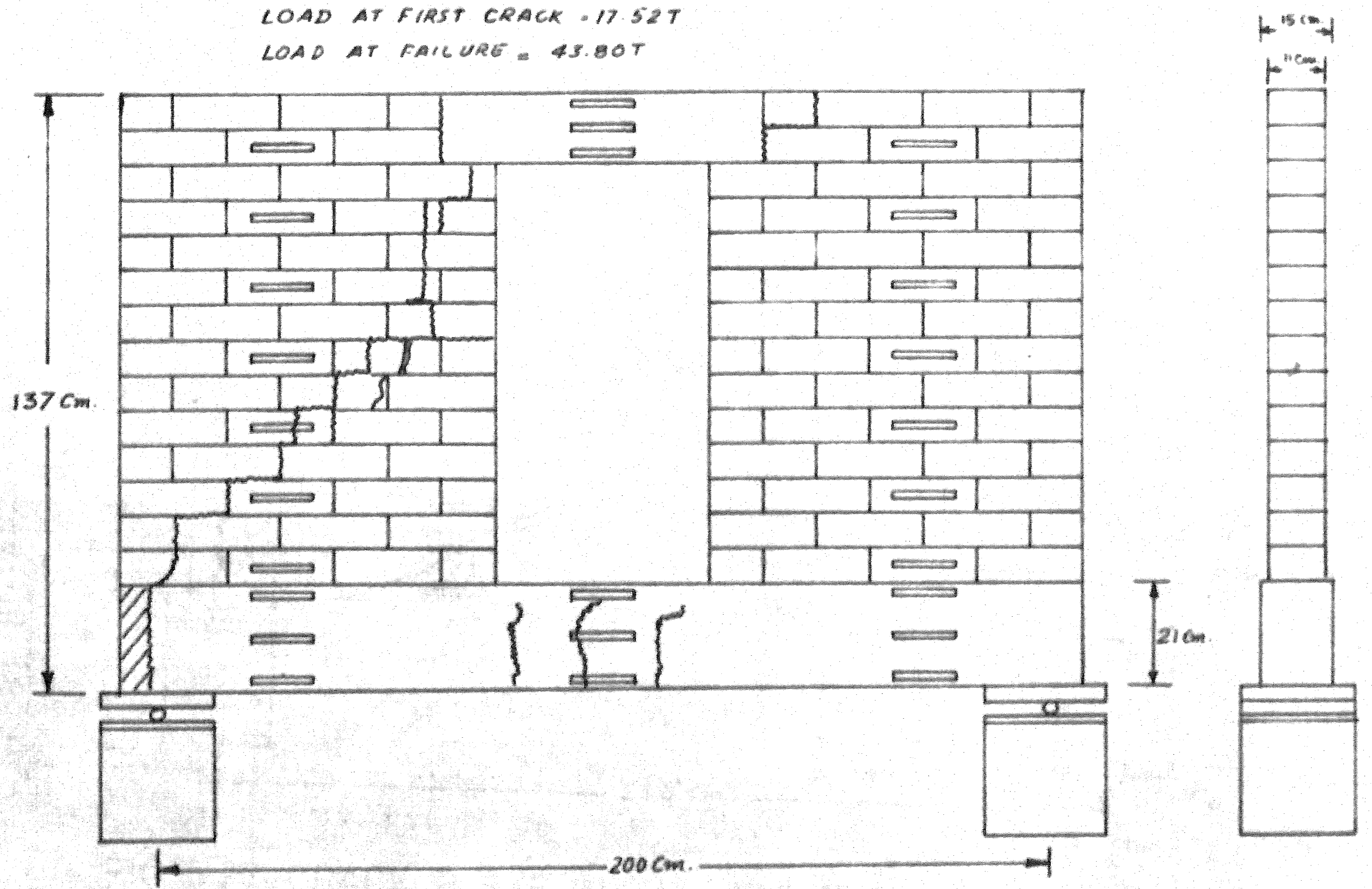


FIG. 5.26_CRACK PATTERN AT FAILURE - BEAM C.2

DATE OF BUILDING MASONRY WALL	22.8.71
DATES OF TESTING	23.10.71 24.10.71

COMPRESSION LOADING

H/L = 0·68

LOAD AT FIRST CRACK = 11·68T

LOAD AT FAILURE = 28·62T

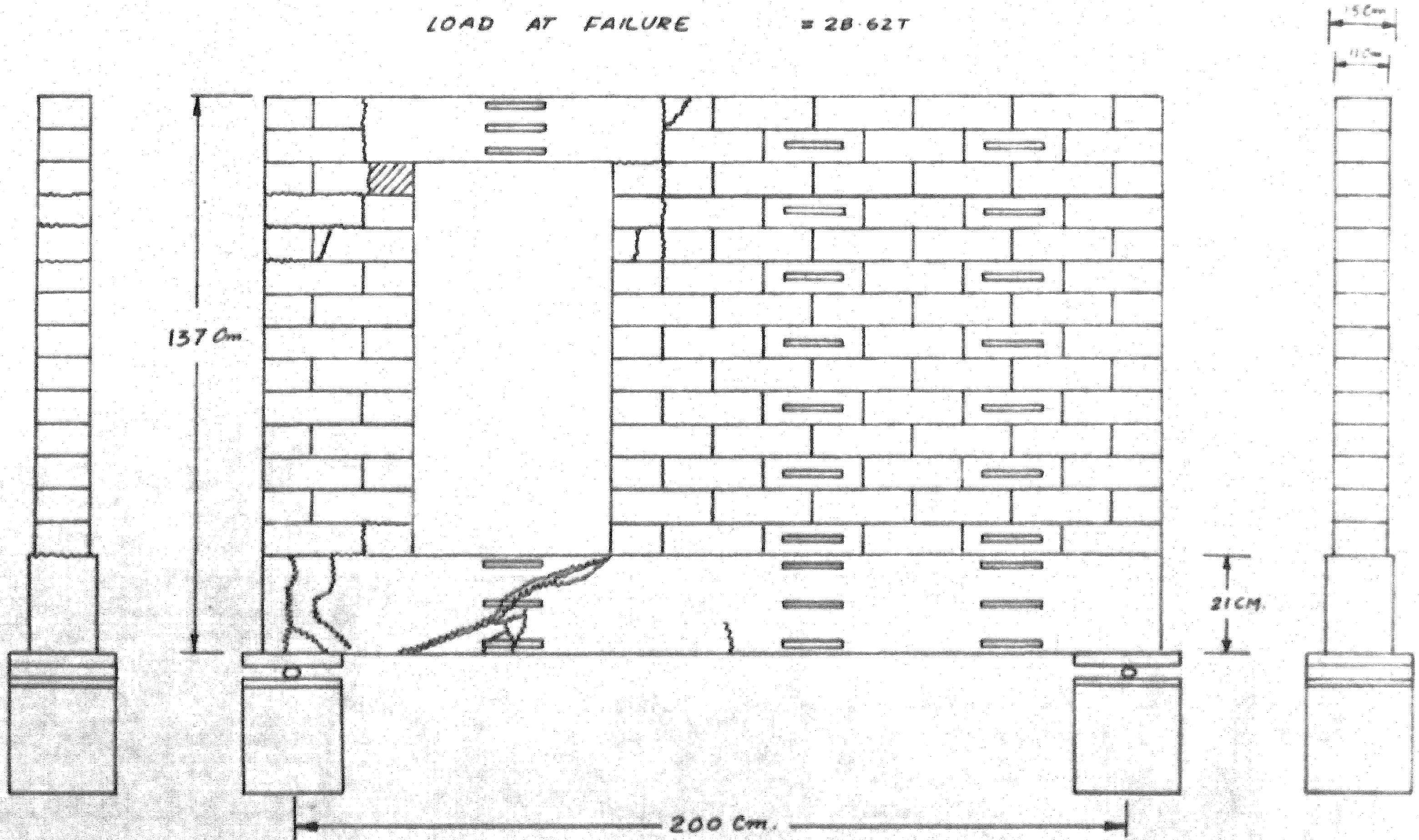


FIG_5.27_CRACK PATTERN AT FAILURE BEAM_C 3

DATE OF BUILDING MASONRY WALL	22.3.'
DATE OF TESTING	11.10.' 12.10.'

COMPRESSION LOADING

H/L = 0.68

MORTAR 1:3

LOAD AT FIRST CRACK = 36.79T

LOAD AT FAILURE = 51.39T

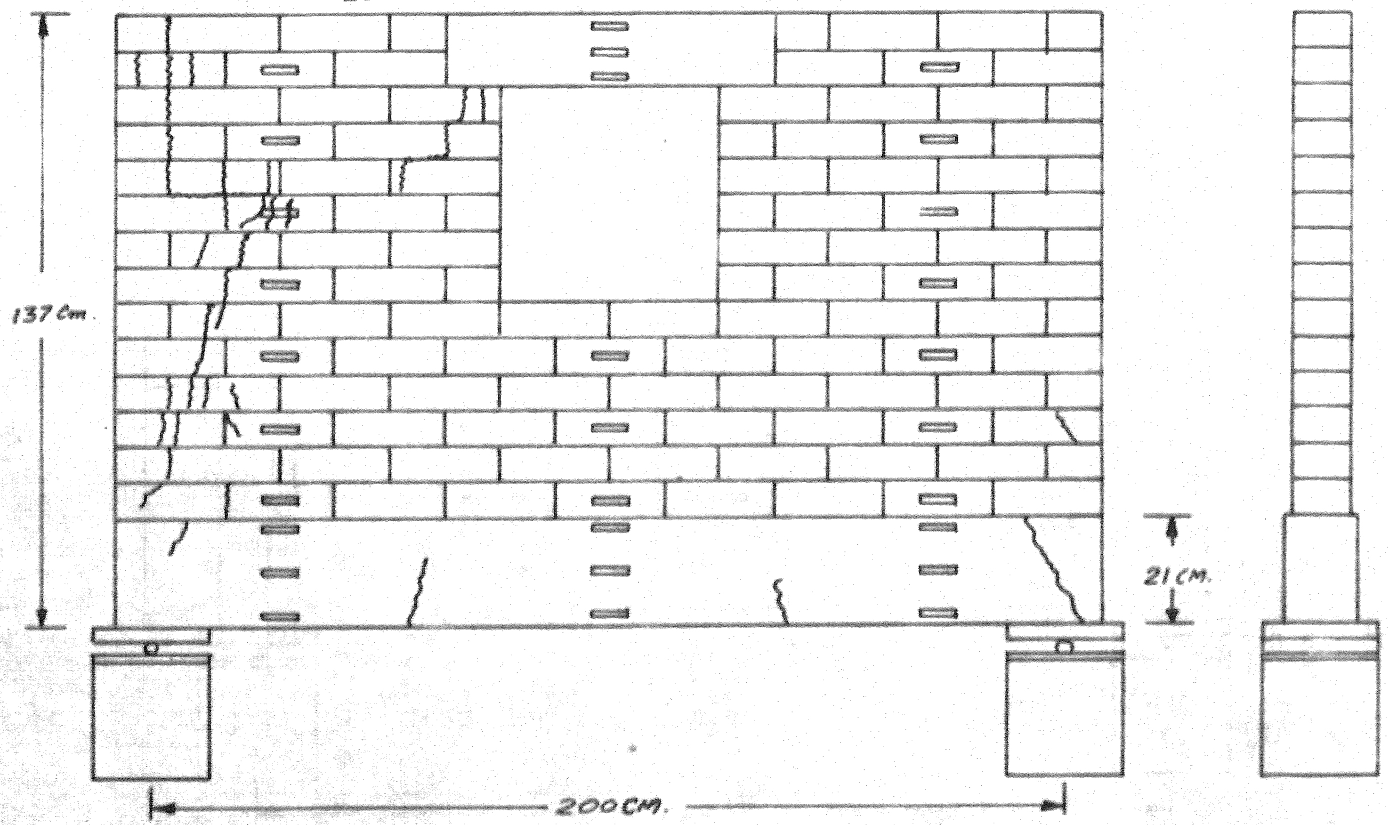


FIG. 5.28_CRACK PATTERN AT FAILURE – BEAM C4

DATE OF BUILDING MASONRY WALL	28.8.71
DATES OF TESTING	27.9.71 28.9.71

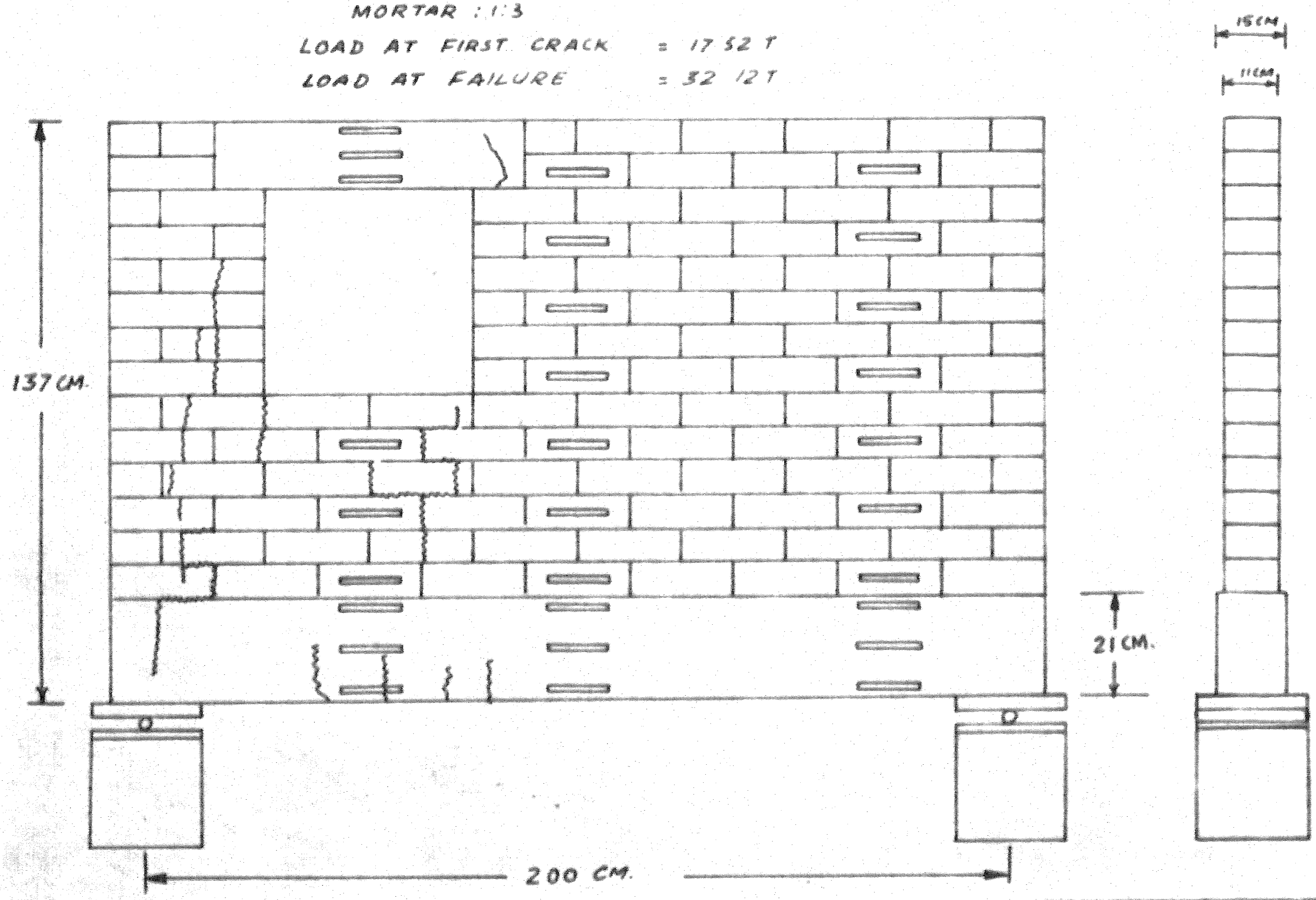
COMPRESSION LOADING

H/L = 0.68

MORTAR : 1:3

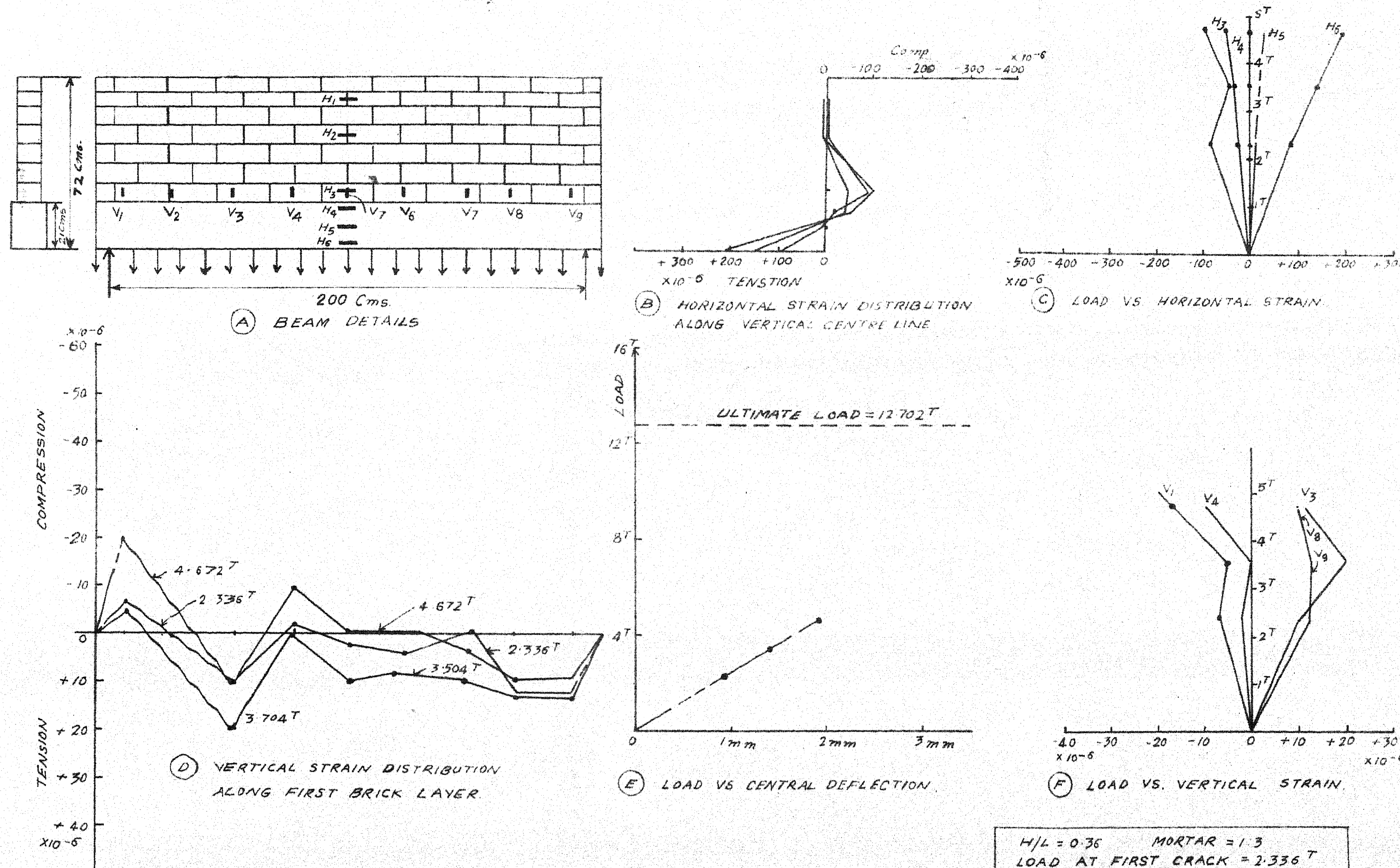
LOAD AT FIRST CRACK = 17.52 T

LOAD AT FAILURE = 32.12 T



DATE OF BUILDING MASONRY WALL	28.8.71
DATES OF TESTING	15-10-71 16-10-71

FIG. 5.29_ CRACK PATTERN AT FAILURE - BEAM C_5



H/L = 0.36 MORTAR = 1:3
LOAD AT FIRST CRACK = 2.336 T
LOAD AT FAILURE = 12.702 T

FIG. 5.30. BEHAVIOUR OF BEAM A.2

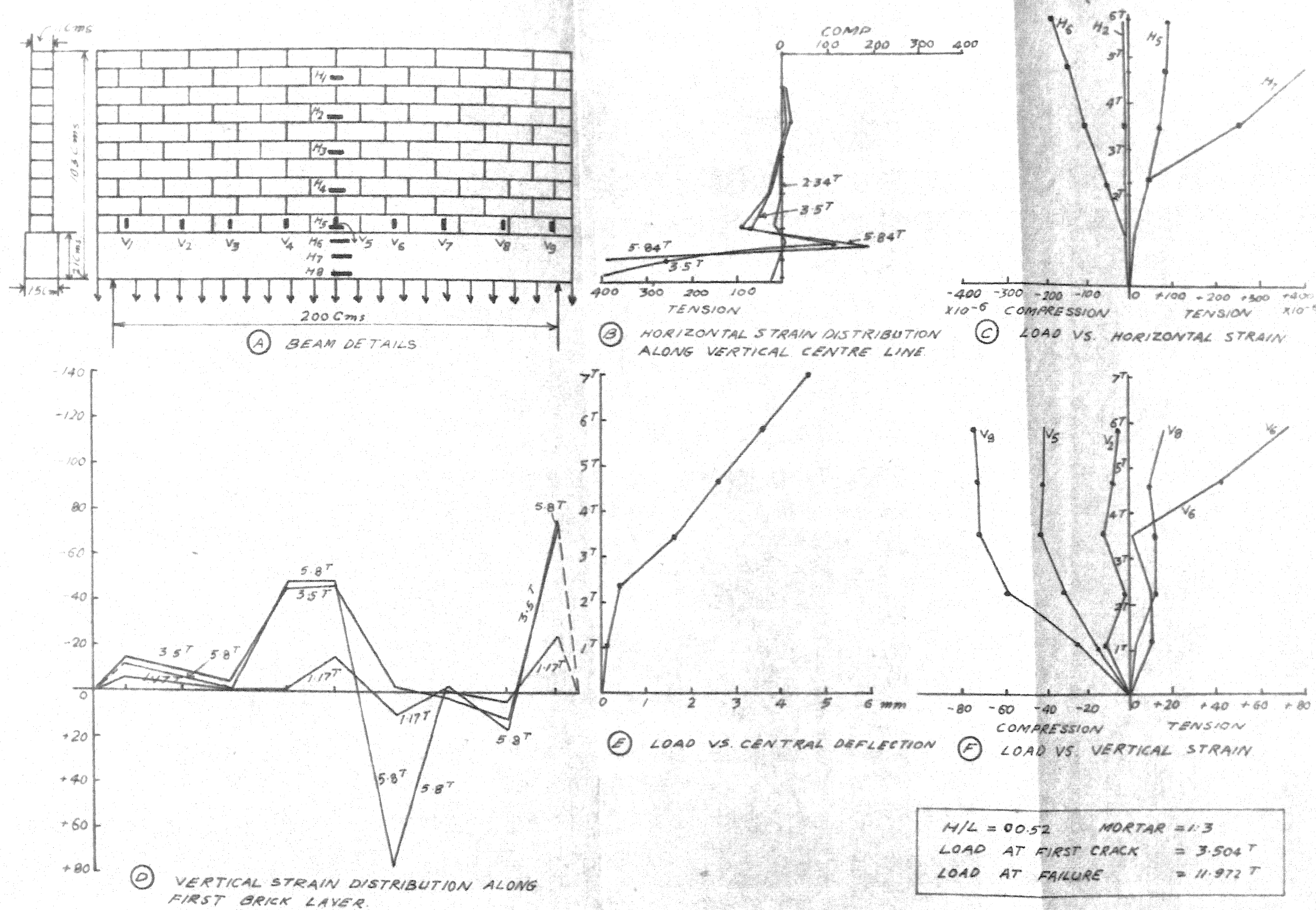


FIG.5·31. BEHAVIOUR OF BEAM B·8

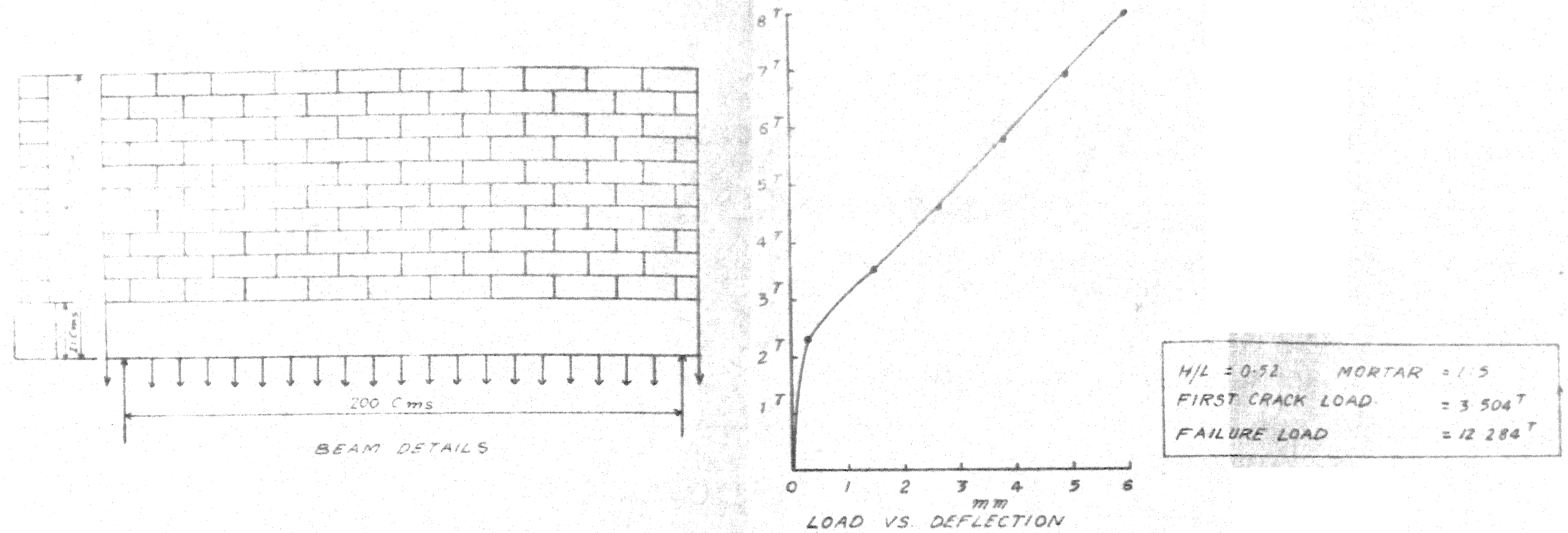


FIG. 5·32. BEHAVIOUR OF BEAM B·6

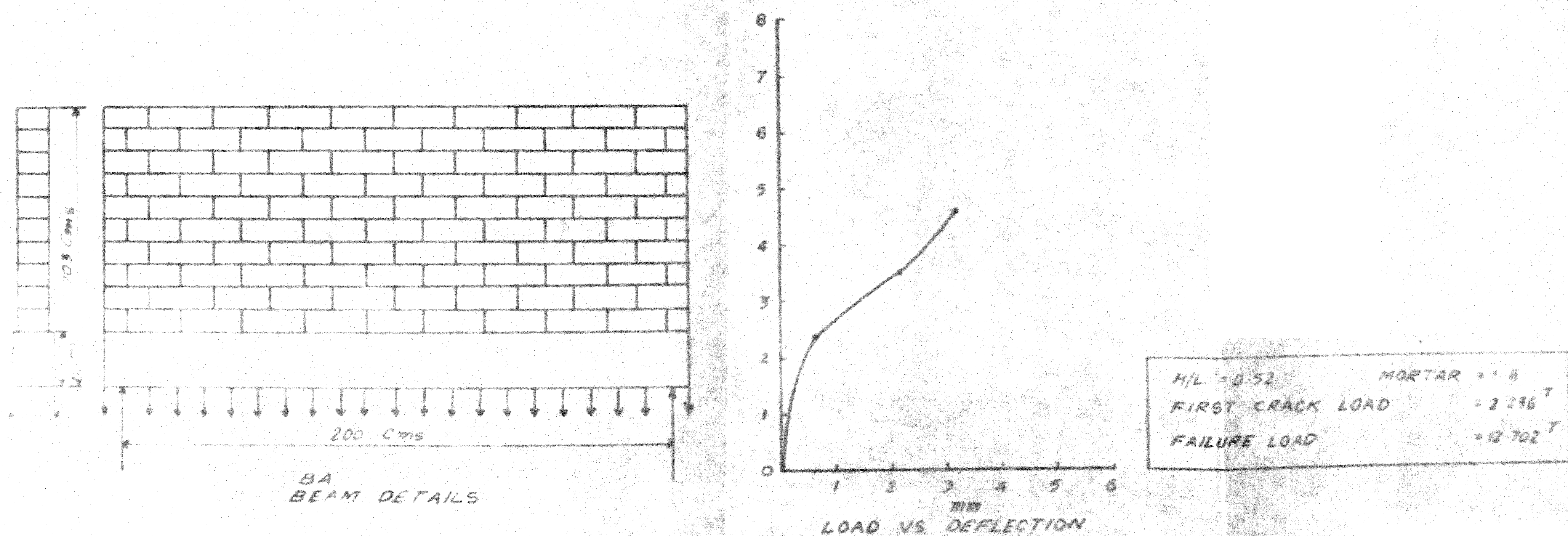


FIG. 5·33. BEHAVIOUR OF BEAM B·7

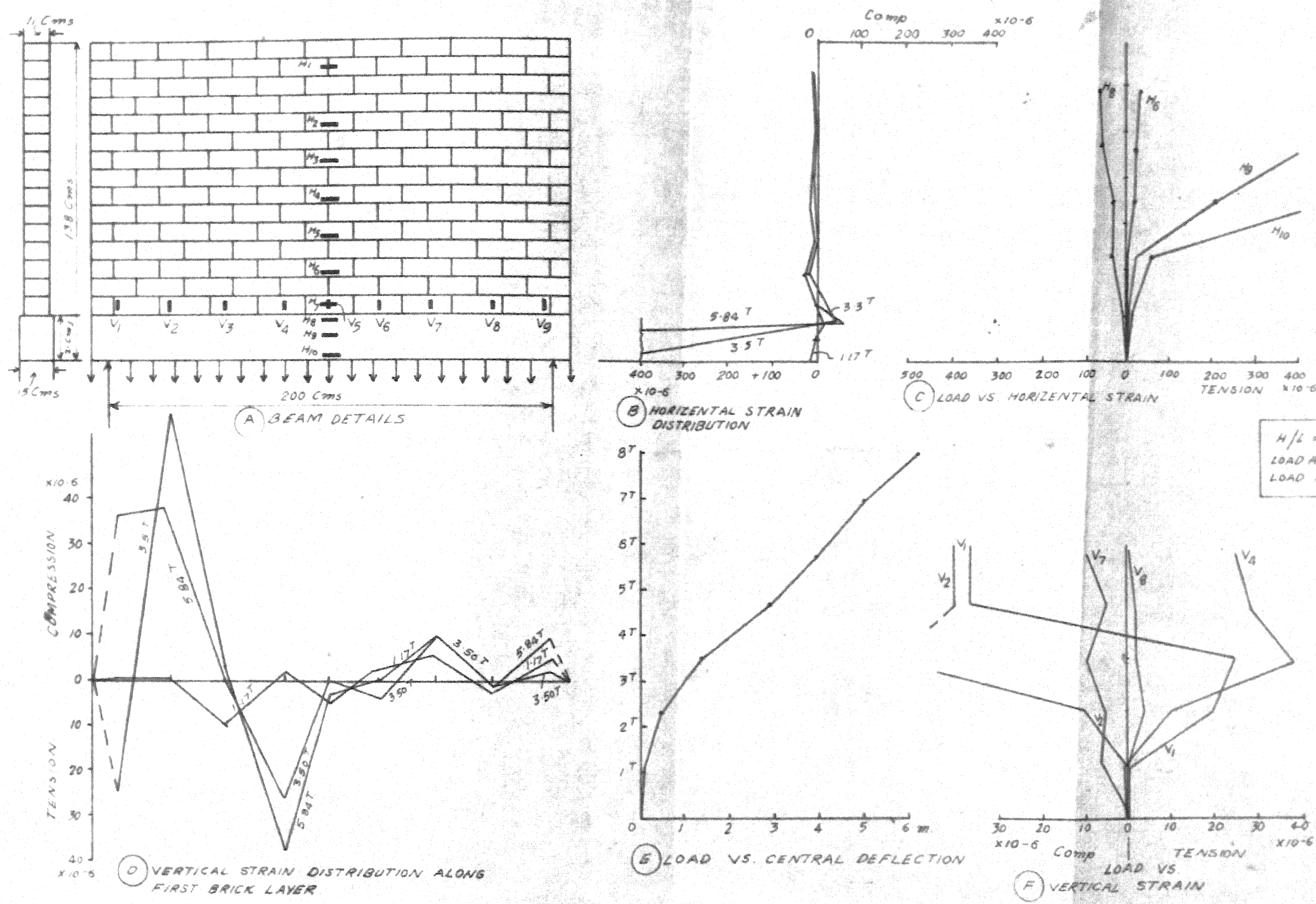


FIG. 5·34. BEHAVIOUR OF BEAM C·6

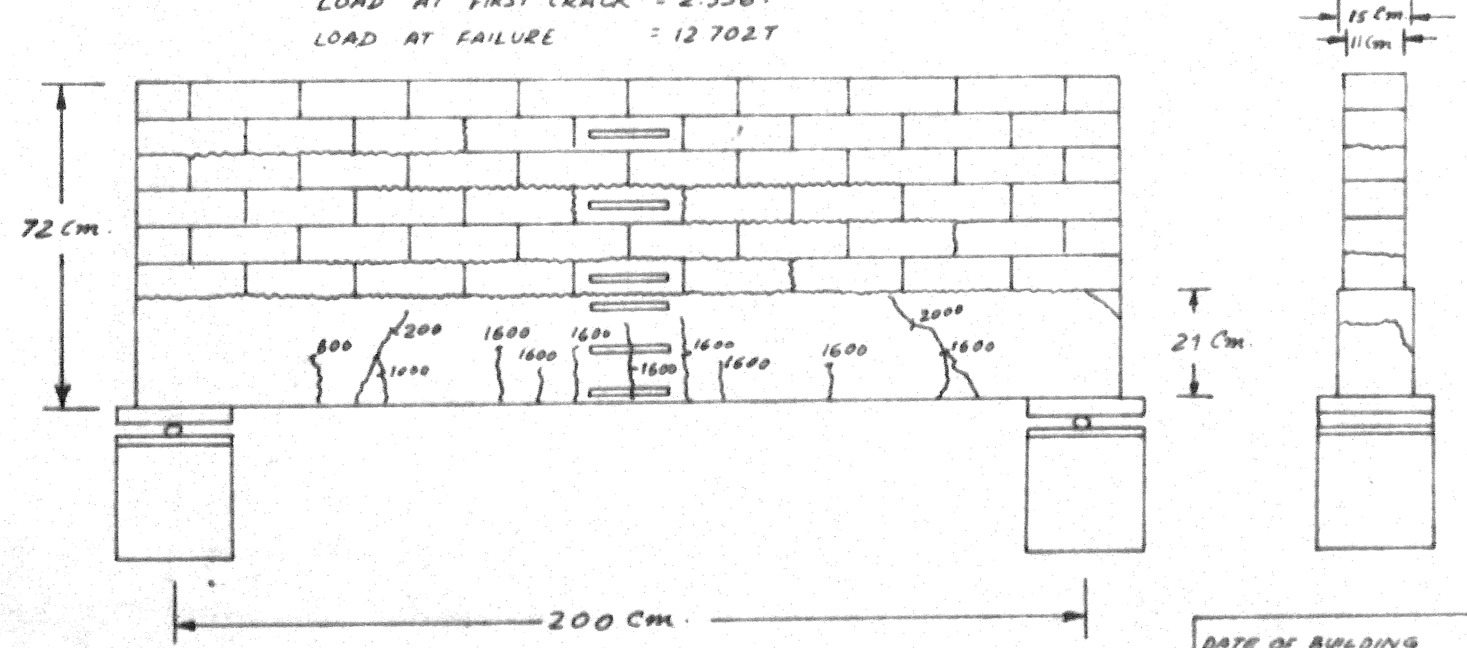
TENSION LOADING

H/L = 0.36

MORTAR = 1:3

LOAD AT FIRST CRACK = 2.336T

LOAD AT FAILURE = 12.702T



DATE OF BUILDING MASONRY WALL	19-9-71
DATES OF TESTING	4.11.71 5.11.71

FIG. 5.35_CRACK PATTERN AT FAILURE - BEAM A2

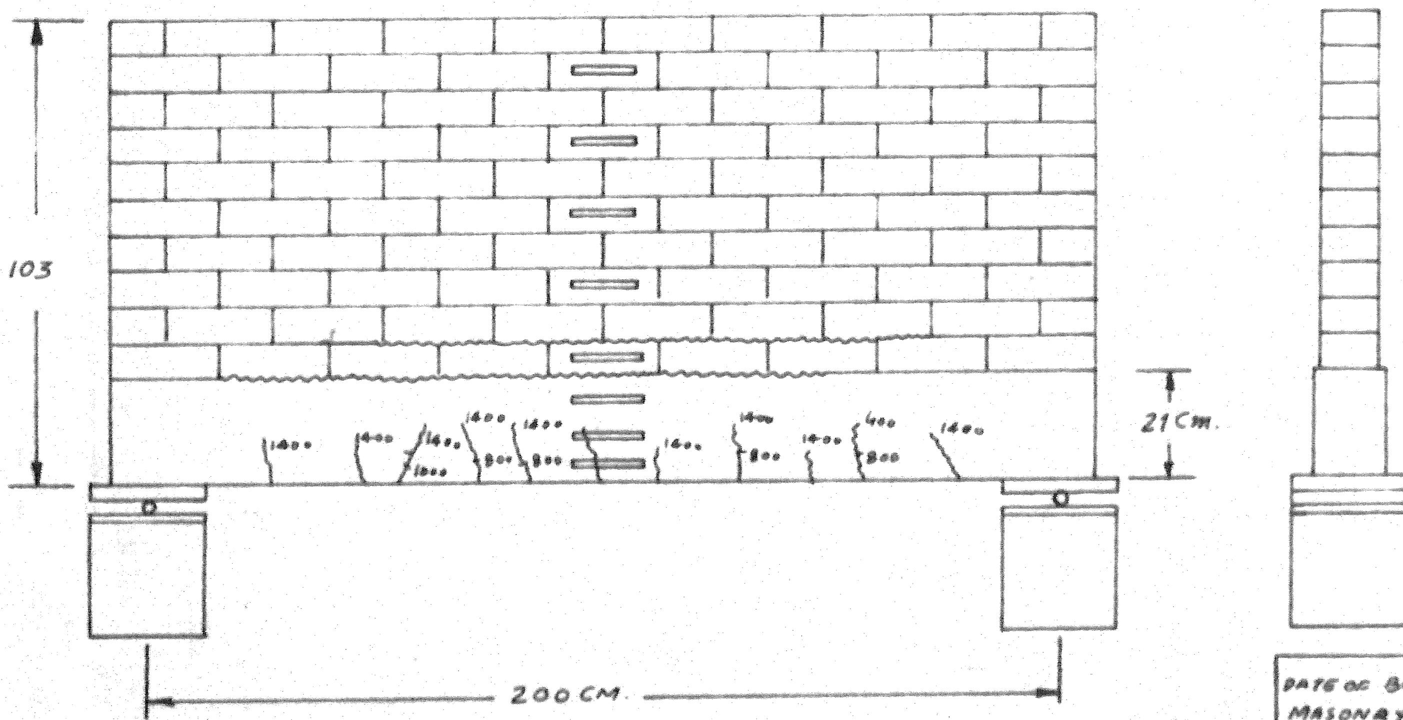
TENSION LOADING

H/L = 0.52

MORTAR = 1:3

LOAD AT FIRST CRACK = 3.504T

LOAD AT FAILURE = 11.972T



DATE OF BUILDING MASONRY WALL	12-9-71
DATE OF TESTING	6-11-71

FIG. 5.36 _ CRACK PATTERN AT FAILURE BEAM B-5

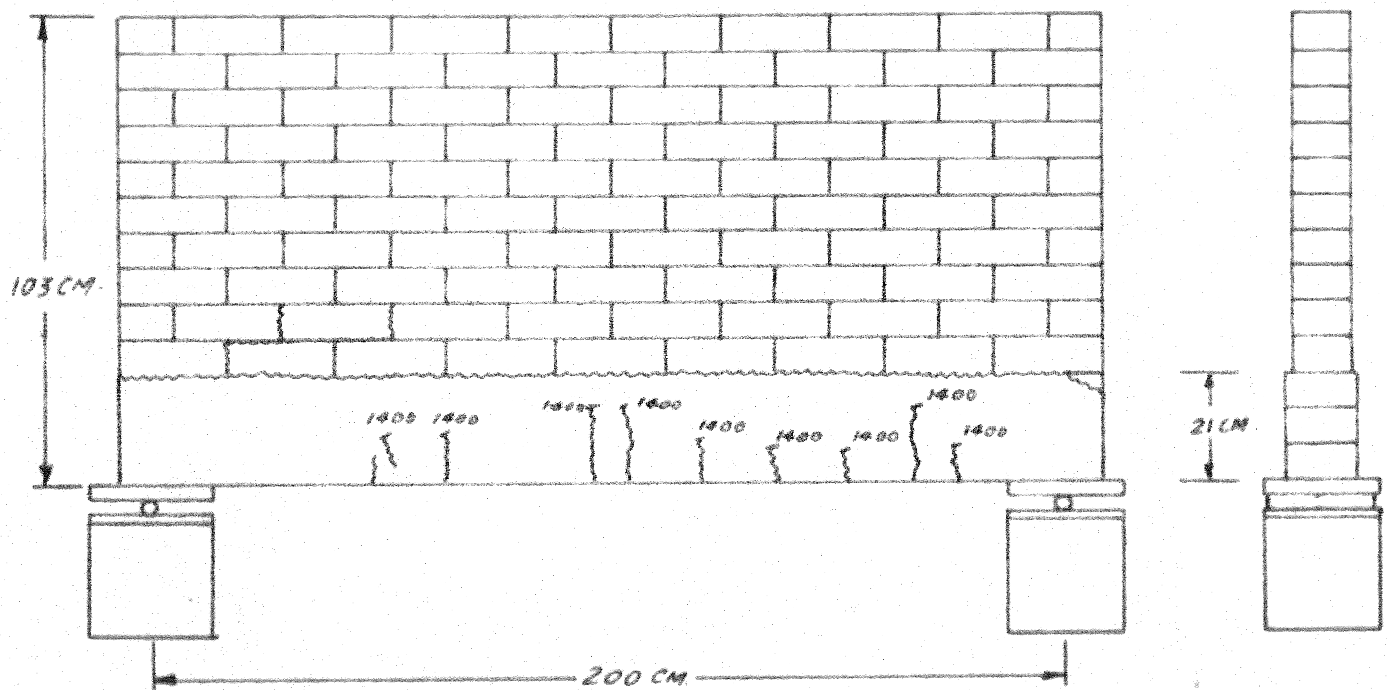
TENSION LOADING

$H/L = 0.52$

MORTAR 1:5

LOAD AT FIRST CRACK = 3.504 T

LOAD AT FAILURE = 12.264 T



DATE OF BUILDING MASONRY WALL	12.9.71
DATE OF TESTING	6.11.71

.5.37_CRACK PATTERN AT FAILURE BEAM _ B.6

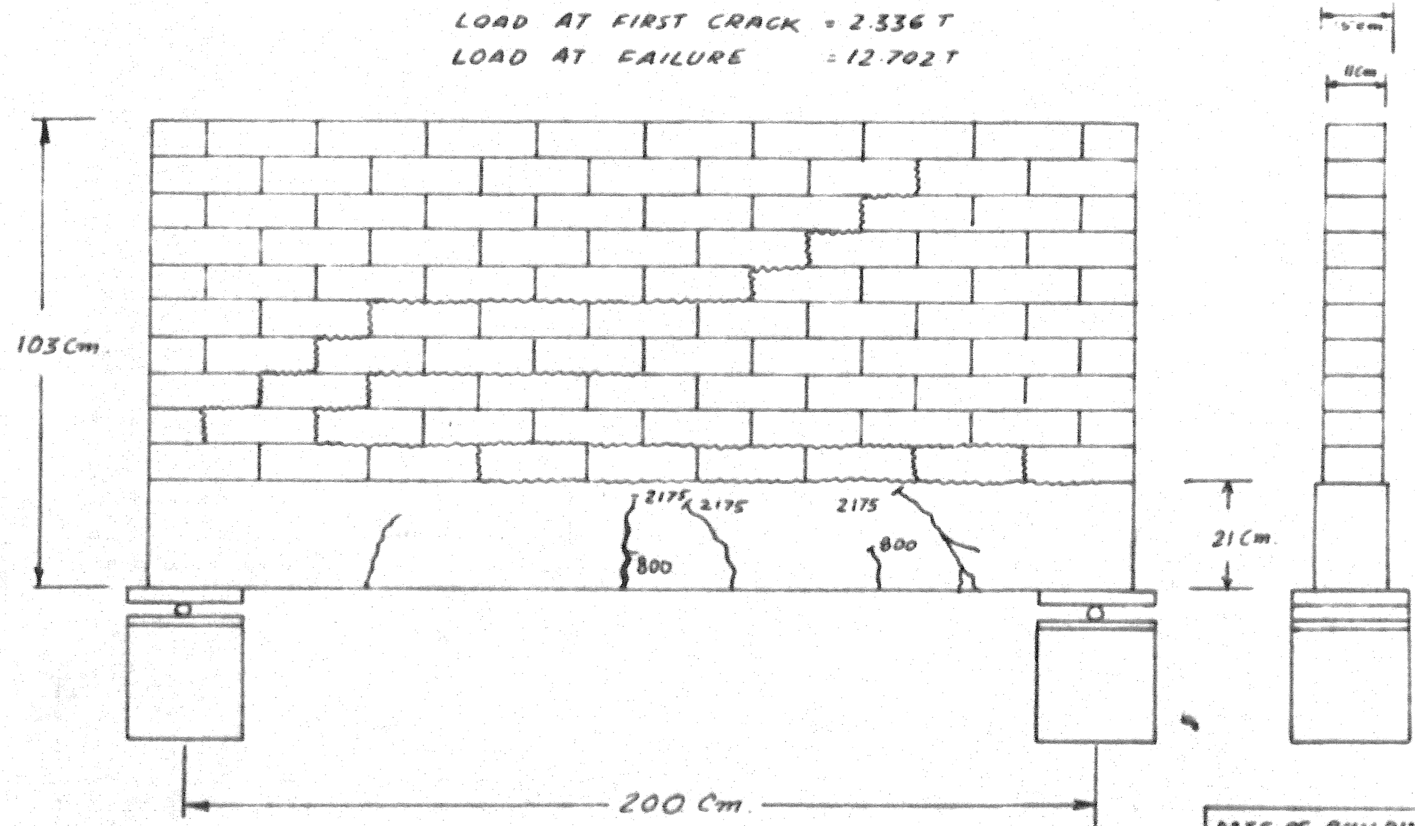
TENSION LOADING

H/L = 0.52

MORTAR _ 1:8

LOAD AT FIRST CRACK = 2.336 T

LOAD AT FAILURE = 12.702 T



DATE OF BUILDING MASONRY WALL	19.9.71
DATE OF TESTING	5-11-71

FIG. 5.38_CRACK PATTERN AT FAILURE _ BEAM B.7

TENSION LOADING

H/L = 0.69

MORTAR _ 1:3

LOAD AT FIRST CRACK = 2.336 T

LOAD AT FAILURE = 12.848 T

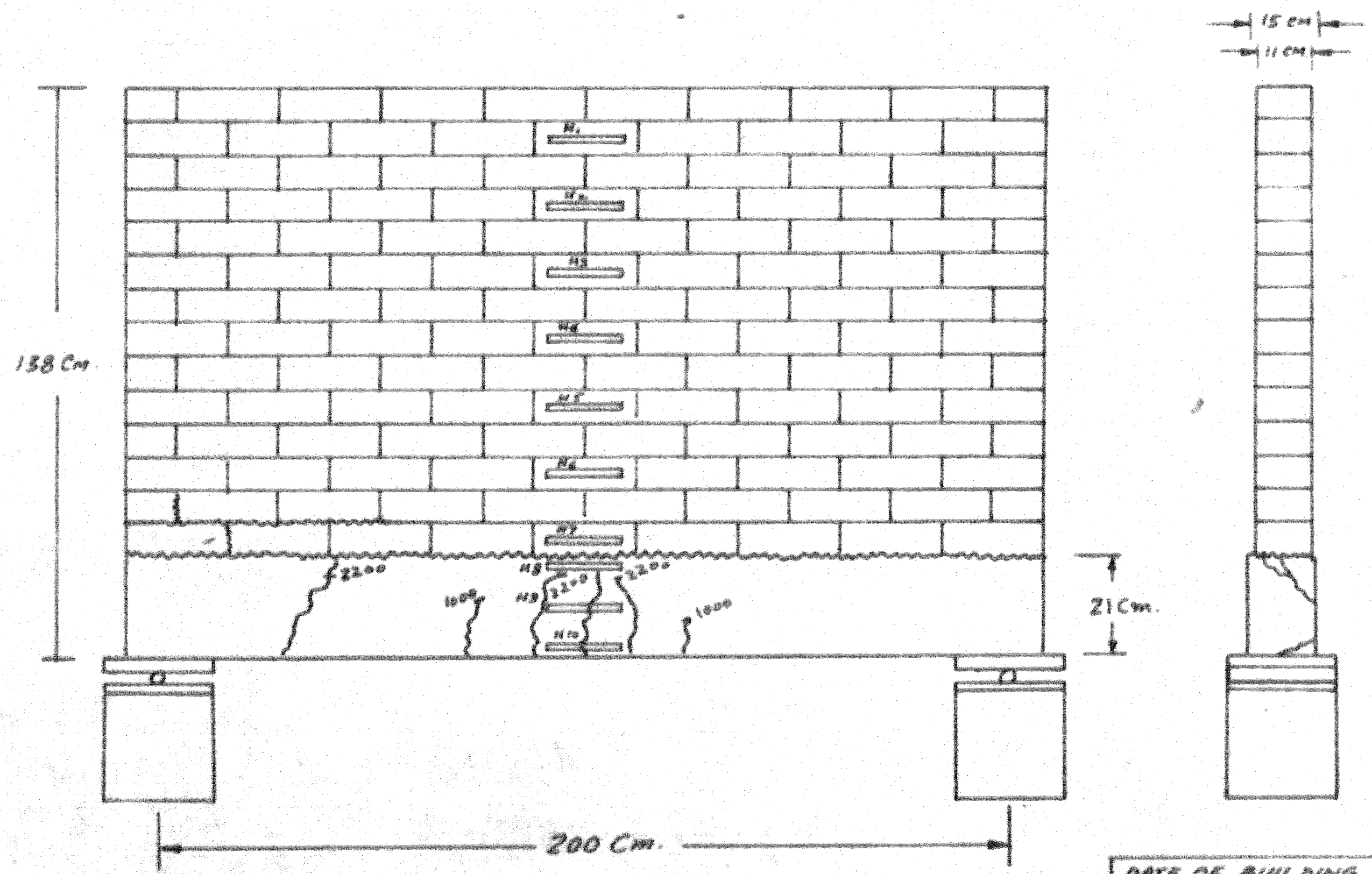


FIG.5.39_CRACK PATTERN AT FAILURE BEAM _ C_6

DATE OF BUILDING MASONRY WALL - 26-9-71
DATES OF TESTING 9-11-71

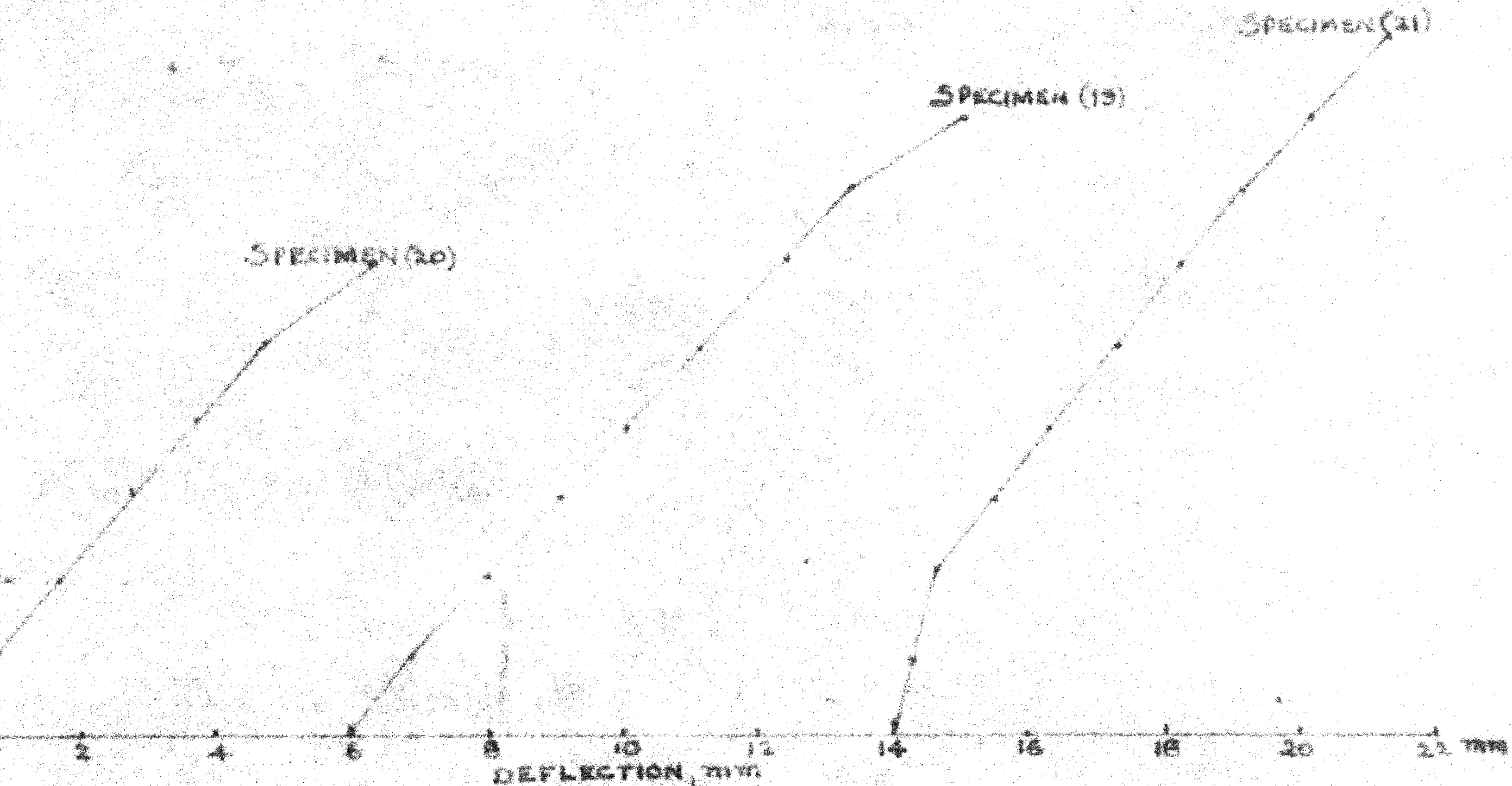
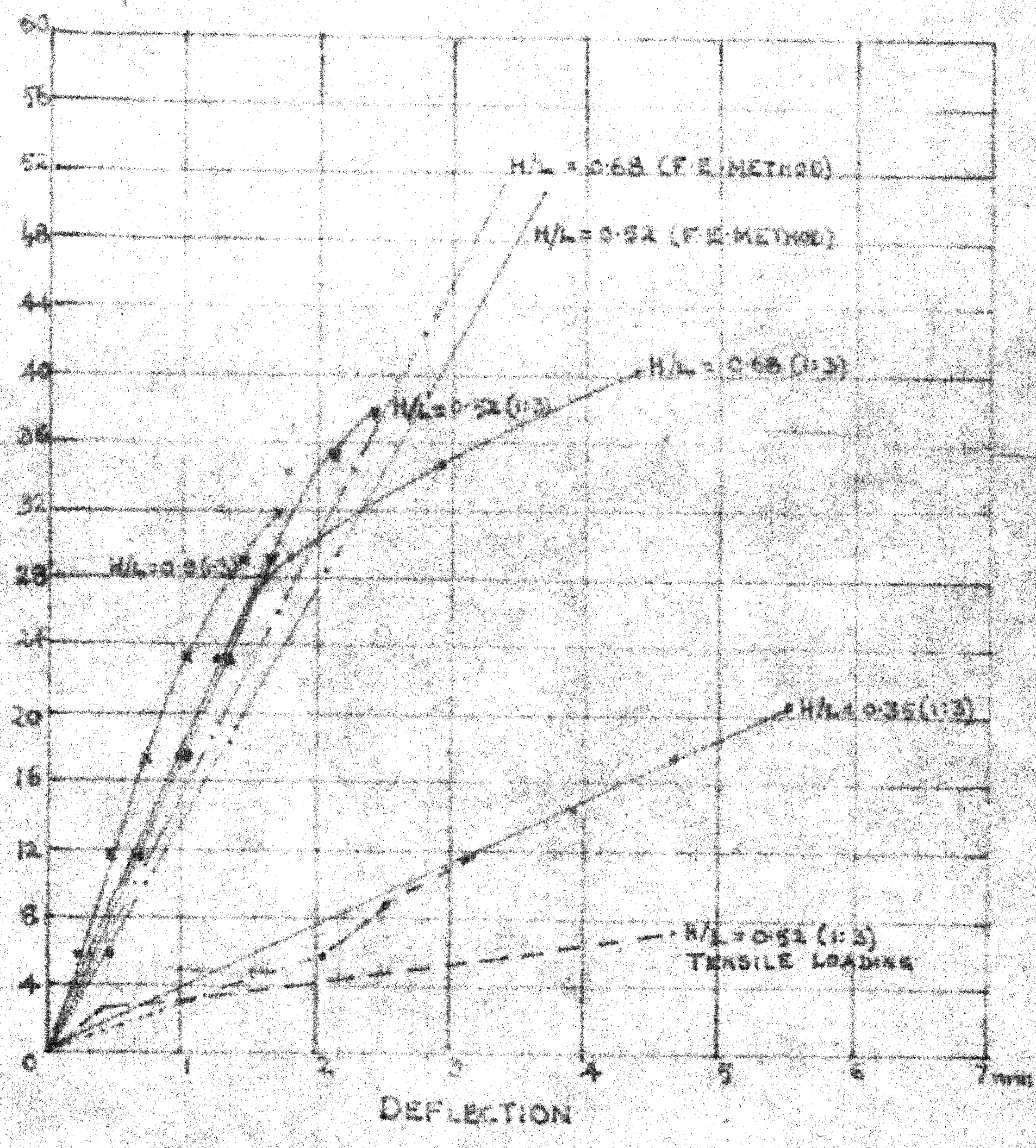


FIG. 5.40. LOAD-DEFLECTION CURVES OF SALVAGED R.C. BEAMS


H/L = 0·68 (F·E·METHOD)
H/L = 0·52 (F·E·METHOD)
H/L = 0·68 (1:3)
H/L = 0·52 (1:3)
H/L = 0·35 (1:3)
H/L = 0·52 (1:3)
TENSILE LOADING
DEFLECTION
7 mm

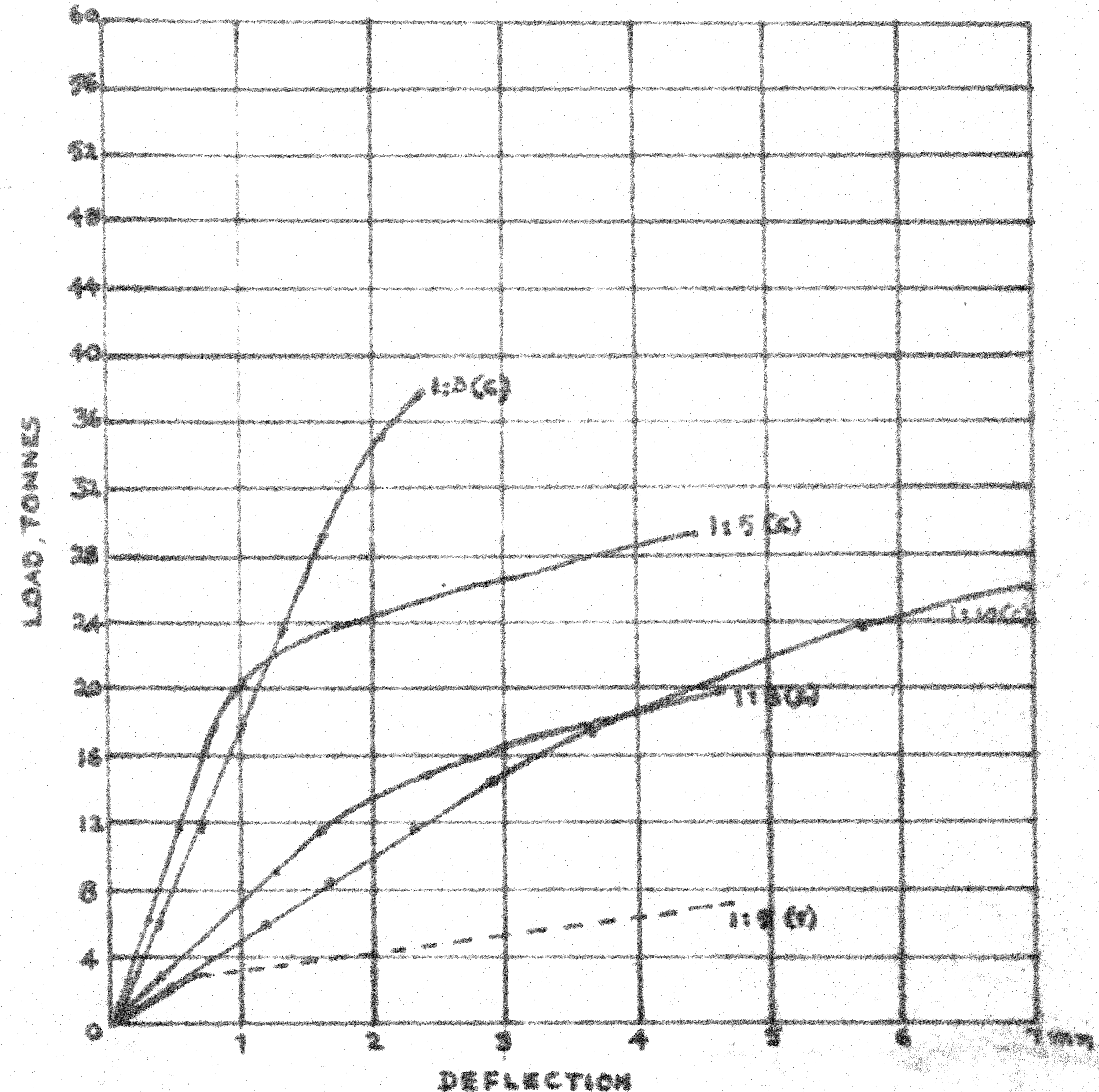


FIG. 5·42. EFFECT OF MORTAR ON LOAD-DEFLECTION (H/L = 0·52)

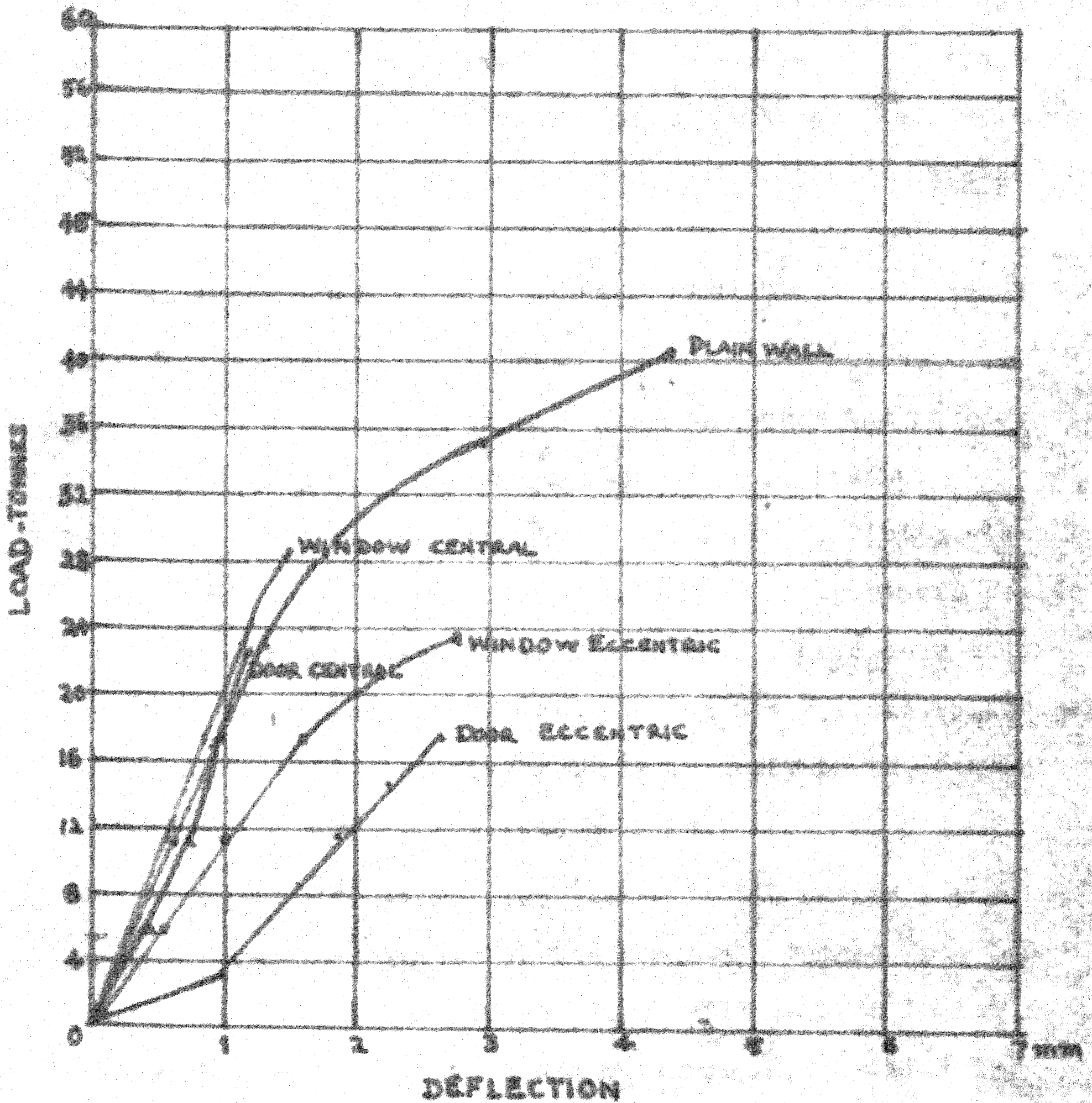


FIG. 5·43. EFFECT OF OPENINGS ON LOAD-DEFLECTION
H/L=0·68, MORTAR (1:3)

CHAPTER VI

SUMMARY OF RESULTS AND DISCUSSION

In this thesis, the load-response characteristics of brickwalls interacting with their supporting beams have been studied. Elastic, post-cracking and ultimate stages of loading have been covered. Besides laboratory testing, computer simulation studies have been undertaken to isolate the effects of several variables influencing the problem. Attention has been given to the behaviour of composite materials, such as concrete and brickwork. Non-homogeneity, anisotropy and randomness in material properties which characterise the behaviour of these materials, have been given special attention.

The primary aim of the thesis has been the development of realistic methods of analysis which could predict the behaviour patterns observed in the laboratory. Finite element methods of analysis have been chosen for this purpose. Besides simple elastic analysis, simulation studies have been undertaken to trace progressive cracking and other local failures. Using incremental-iterative methods of analysis the complete load-response curves have been obtained.

In an attempt to generate realistic design recommendations a critical review of the results obtained from laboratory tests and simulation studies, will be made in the following pages. Material and structural aspects will be

separated for convenience in presentation.

6.1 Computer Simulation Studies on Concrete and Brickwork

Concrete and brickwork are two phase composite materials and the assumption of homogeneity suppresses valuable information on the nature and magnitude of interaction between the constituents.

Concrete:

The material concrete, has been idealised as a two phase composite media with aggregates randomly embedded in a matrix of mortar. The behaviour of a concrete prism, (20 cms. width, 27.5 cms. depth and 10 cms. ~~width~~ thickness) under compression, shear and flexural loads has been studied. Material and geometric properties in the direction of thickness have been kept constant. Subject to these restrictions the following results have been obtained.

1) The elastic constants E, ν and G can be more accurately predicted through computer simulation than from the theory of composite materials, since the random arrangement of materials has influence on these constants besides the volume fraction.

2) Nonhomogeneity introduces interaction stresses of significant magnitude, sufficient to cause local microcracking and yielding, resulting in nonlinear load responses

under progressive loading.

3) The interaction stresses are self-balancing in nature and cause overall flexure even in uniformly compressed specimens.

4) These local interaction stresses may be viewed as perturbations on the stress distributions obtained from conventional theory of elasticity, assuming homogeneity.

5) These local stresses can be suppressed to approach the stresses in homogeneous media by the obvious procedure of matching the elastic constants of the constituents in a composite media. This may not be possible in practice, but the optimisation trend is clear.

Brickwork:

1) Even when brickwork is uniformly compressed local flexural stresses and lateral interaction stresses are developed. Brick is generally in biaxial tension and the mortar is in triaxial compression.

2) The parameter $E_m \nu_b / E_b \nu_m$ has been identified as an important parameter, which when it approaches unity suppresses critical tensile stresses in brickwork. Vertical splitting of brickwork, which has been noticed in several investigations, reduces its useful capacity. It has been thus shown that this vertical splitting under service loads can be controlled.

3) Corners of bricks in a brickwall are critically stressed. It has been shown that hard inclusions in a soft media are sources of stress concentration and cracking at corners, and brickwork is characteristic of several such inclusions.

4) Increase in thickness of mortar generally increases tension in critical regions. It is better to have thin bed-joints.

6.2 Computer Simulation Results on Walls on Beams

The problem of walls on beams has been studied as a plane stress problem in non-homogeneous, anisotropic media, using the finite element technique. Besides simple elastic solutions, simulation studies have been undertaken to obtain the complete load response curves upto failure. The following observations are valid:

1) The elastic responses are sensitive to the elastic constants of the materials under use. Realistic data are required.

2) The elastic responses predicted from computer simulation tally with experimental results.

3) Computer simulation can effectively identify progressive cracking and local compression failures and yielding of reinforcement.

4) Stress-redistribution procedures are necessary for realistic prediction of load-response curves upto failure.

5) Once a good computer programme has been developed, it can be repeatedly used for parameter studies and generation of design charts. This will help in reducing the expenditure in laboratory investigations which are generally time-consuming and costly.

6) The safety reserve due to interaction between brickwalls and their supporting beams ranges from 1.0 for the plain beam to 3.14 for H/L ratio of 0.9. This observation is based on elastic stresses in the mid-span of the supporting beam. This may be contrasted with the factor of 12.0 suggested by R.H. Wood (C 21). Since cracking controls the problem, elastic stresses form a sound basis for design.

7) Computer simulations show that the presence of reinforcement reduces the elastic deflection of supporting beams by 40 percent. Hence it is suggested that the benefit of interaction should not be used for reduction of reinforcement in the supporting beams, but it could be used to reduce the size of concrete cross section subject to shear control.

8) Computer simulation studies have shown that the presence of central door and window openings, in fact reduces the critical stresses in the supporting beams. Mid-

span bending moment is reduced due to direct transfer of load to the supporting regions.

9) Eccentric openings have not been simulated but it will be shown through experimental results that eccentricity of openings reduces the benefit of structural interaction between walls and their supporting beams.

10) It must be pointed out at this stage that stresses are not accurately predicted by the finite-element method and hence the above recommendations may be viewed with some caution. The suggestions of the author seem to be conservative. This aspect will be discussed later in conjunction with experimental results.

11) Furthermore, computer simulation studies do not reflect the concentration of vertical strains on either side of openings which have been noticed in experiments. Finer mesh sizes around openings could have helped.

6.3 Computer Time, Storage and Computational Effort

The computer system available to the author is the IBM 7044 system with 32 K memory. Five magnetic tapes could be used for storage during computation. The short-jobs which were easily permitted were of 8 minutes maximum duration and could accommodate the plain beam only. Thus solutions for walls on beams could be obtained from long-jobs only

which were available once a week. This limited the use of the computer significantly for studies on progressive loading and incremental iterative methods of analysis.

Furthermore the Fortran IV Programme under use could not suppress under-flows and a local sub-programme had been in use to suppress upto 32000 under-flows. Even when this routine was called inside a loop the programme often terminated without adequate message to enable correction. Often several weeks were required to enable the programme to continue execution with large underflows. The incremental-iterative methods of analysis had to be abondoned due to this peculiarity associated with the Fortran IV processor under use. Since the programme was developed sequentially, the author is confident that the version appended to this thesis will work satisfactorily. It has been tested for several incremental cycles and one iteration at which time the underflow could not be suppressed, even after several attempts.

6.4 Laboratory Test Results on Brickwork

The following observations are valid on the basis of laboratory tests on brickwork.

1) The large scatter in the strength of bricks (C_V = 22%) and the strength of mortar (C_V = 35.6%) are responsible for the poor quality of brickwork.

2) Increase in number of mortar layers affects the strength of brickwork. When the strength of brick is 345 Kg/sq.cm. and the strength of mortar used in the order of 200 Kg/sq.cm., the strength of brickwork with mortar joints reduces to approximately 100 Kg/sq.cm. Even 50 percent efficiency on the basis of mortar strength could not be obtained.

3) Porosity of mortar layers which are not consolidated in the brick-laying process is responsible for this poor performance.

4) The appreciating nature of stress-strain curves also indicate the consolidation of mortar under progressive increase in stresses.

5) The elastic modulus E, has been found to be 10 percent of the predictions based on the theory of composite materials, once again indicating poor quality of mortar joints.

6) It is thus suggested that efficiency of brickwork can be increased considerably by excercising control on the brick-laying process.

7) Vertical splitting of brickwork has been a common feature in the laboratory tests of the author, indicating the presence of lateral tensile interaction stresses. It has been pointed out earlier that the elastic constants of the constituents must be controlled for crackfree brickwork.

8) Elastic constants of brickwork deserve more attention. In particular the values of ν, have been given scant attention.

9) In particular the orthotropic nature of brickwork requires special attention since the values of E obtained from tests parallel and perpendicular to bed joints have been significantly different. Furthermore the results obtained by the author are in contradiction with the theory of composite materials.

6.5 Laboratory Test Results on Walls on Beams

As the number of variables in a problem increases, isolation of individual effects becomes a costly process since the number of specimens to be tested increases enormously. In the case of walls on beams sixteen full scale tests were conducted and a minimum of four specimens were used to isolate the effects of each variable.

Height-span ratio, mortar strength, size and location of openings and nature of loading, tensile and compressive, where the several aspects isolated. The following observations are of interest.

1) Vertical stresses concentrate towards supports and lateral tensile stresses concentrate in the supporting beams. The wall is generally in compression. There are large

vertical strains on either side of openings. There are vertical tensile strains at the mid-span of walls on beams and the beam and the wall separate at interface.

2) Cracking in the supporting beams and in the brickwork is quite extensive as the load is increased beyond 50 percent of ultimate load.

3) Interaction is marginal (if any) in the case of tensile loading, and direct loading of the supporting beams results in immediate separation of brickwork and the beam.

4) Interaction is poor in the case of 1:8 and 1:10 mortars used in compression specimens.

5) Interaction is to be neglected in the case of eccentric openings; central door and window openings are not detrimental to successful interaction.

6) The deflection curves show distinct non-linearity due to cracking.

7) As per the numerical evaluation of interaction attention must be restricted to compressive loading, in specimens made of 1:3 and 1:5 (volume proportion) mortar with with central openings. The safety reserve over and above that of the plain R.C. beam ranges from 1.0 for the plain beam to 3.4 times that value for walls on beams with H/L ratio of 0.90. This may be contrasted with a factor of

12.0 suggested by R.H. Wood (C 21).

8) Extensive cracking at ultimate loads prevents the full interaction from being utilised in practice. The service load must be about one third (33.1/3% of the ultimate load to prevent cracking at service loads.

9) H/L ratios of 0.52, 0.68 and 0.9 show more or less similar behaviour in compression loading and hence a minimum height-span ratio of 0.50 is suggested for interaction effects to be included in design.

6.3 Recommendations for Design

In spite of extensive experimental investigations conducted all over the world, design recommendations have emerged only from the Building Research Station, England. (C 23). The author of this thesis has been looking for a theoretical basis and the finite element method of analysis has been used with some success in the elastic range. The BRS recommendations are based on experiments.

Based on critical compressive stresses in the mid-span of supporting beams the author has been able to relate service loads with the height-span ratio for the problem studied by him. The concrete in the supporting beams has been assumed to have a cube strength of 200 Kg/sq.cm. Other variables have been eliminated on the basis of experimental

investigations and computer simulation, as below:

1) Any load coming directly on the supporting beams may be deemed to be tensile in nature on the brick-walls and in such cases experimental investigations have indicated negligible interaction (Table 5.18).

2) Weak mortars, 1:8 and 1:10 by volume are neglected from consideration on the basis of poor load-response, cracking and ultimate capacity (Figs. 5.42, 5.24, 5.25 and Table 5.15).

3) Eccentric openings have been eliminated because of poor interaction, cracking and load-response characteristics (Figs. 5.43, 5.27, 5.29 and Table 5.15).

4) Central openings have been included on the basis of finite element simulation (section 4.3.2). Central openings have a tendency to reduce mid-span moment and hence the stresses in supporting beams.

The service loads on walls on beams of the dimensions tested by the author are given in Fig. 6.1, based on allowable stresses in concrete in the supporting beam. The following observations are made:

1) The service loads are linearly related to the height-span ratios.

2) Cracking in a media such as brickwork arises due to local flaws, nonhomogeneity, poor construction etc.

and the first crack loads observed from experiments fall in a broad region. However, the proposed design loads are on the safe side with respect to cracking.

3) Since the first cracks are local in nature, mainly in the supporting beams, adequate safety against cracking in brickwork has been observed in the laboratory in all these cases without exception. The safety factor is in the order of 1.25 against cracking.

4) Ultimate loads rise sharply in the cases of H/L of 0.52, 0.68 and 0.90. It is clear that H/L ratios less than 0.5, should not be considered for benefits of interaction. In effect these are shallow beams.

5) Ultimate loads in the case of central door openings are not as large as in the case of plain walls and walls with central window openings.

6) Even 1:5 mortar (by volume) has poor performance at ultimate loads. It is suggested that 1 cement : 3 sand or equivalent cement/lime/sand mortar be used in walls on beams when interaction benefits are considered in design.

7) Subject to the above restrictions the author's recommendations for service loads, on the basis of critical stresses in the mid-span, have a safety reserve in the order of 3.14, 2.5, and 2.2 for H/L ratios of 0.50, 0.68 and 0.52 even with central openings.

It now remains to compare the author's recommendation with those of the Building Research Station, England (C 23). These recommendations are summarised below and should be read in conjunction with Fig. 6.2.

1) Since the support regions have large stress concentrations in walls on beams, a stress concentration factor 'C' has been defined where,

$$C = \frac{L}{2x}$$

2) The bending moment any where in the mid-span region is given by,

$$M = \frac{WL}{K} = \frac{W}{2}\frac{x}{2} = \frac{Wx}{4}$$

which leads to $\frac{1}{K} = \frac{1}{C} = \frac{x}{4L}$

or stress concentration factor $C = K/8$

bending moment factor $K = 8C$

stress block extent $x/L = 4/K$

This means that the bending moment factor and the stress block and the stress concentration factor have typical values such as is given in the following table:

K	x/L	C	
8	1/2	1	No composite action
12	1/3	1.1/2	

24	1/6	3	
48	1/12	6	
100	1/25	12.1/2	Maximum allowed composite action.

It can be clearly seen that to make use of maximum composite action the wall must be understressed as a whole.

Now these must be related to maximum allowable stresses in brickwork, for uniformly distributed loading and concentrated loading (girder bearings etc.). These are

f_b = a basic stress related to strength

f_p = an allowable uniformly distributed compressive stress

f_c = an allowable maximum stress for a combination of uniformly distributed load and concentrated load.

The code gives the following relationship:

f_c = 1.1/2 f_b for local concentrations

f_p = $F.f_b$, where F is a stress reduction factor for slenderness of wall.

Also in practice if the actual average wall stress f_w is less than the allowable f_p, by a reduction factor R, namely $f_w = R.F.f_b$.

Taking advantage of the local allowable stress concentration factor, then the stress concentration factor C must be given by

$$C.f_w \nless 1.1/2\ f_b$$

i.e. $\quad C.R.F.f_b \nless 1.1/2\ f_b$

$$C.R.F. \nless 1.1/2$$

leading to $R.F. \nless 12/K$

or $\quad K \nless \dfrac{(12)}{(R.F.)} \qquad (1)$

where the design bending moment is WL/K. The following examples illustrate the application of these rules:

Example	Type of wall	Required moment of resistance for beam design, WL/K
(a)	A 'squat' wall (slenderness ratio =6), and stressed to code limit, F = 1, R = 1	WL/12
(b)	A slender wall (slenderness ratio of 18) fully stressed F = 1/2, R = 1	WL/24
(c)	A slenderness ratio of 18 and only stressed to one half of allowable stress, F = 1/2, R = 1/2.	WL/48
(d)	Same as above but stressed only to 1/4 of allowable stress F = 1/2, R =1/4.	WL/96

Interpretation of the above* rules for the cases tested by the author requires the following computations :

H/L	Effective thickness	Effective height	Slenderness ratio	F	$F_p = F.f_b^*$
0.52	11 cm	154 cm	14	0.67	6.7 kg/sq.cm
0.68	11 cm	206 cm	19	0.49	4.9 kg/sq.cm
0.90	11 cm	281 cm	26	0.44	4.4 kg/sq.cm

f_b^* = 10 kg/sq.cm. for brickwork of 100 kg/sq.cm. ultimate strength. Assuming that the wall is fully stressed to its capacity, R ≐ 1. But author's proposed design loads are such as to yield the following values of R :

H/L	Allowable load	R	F	K = 12/(R.F)
0.52	7000 kg/metre	0.64	0.67	28.0
0.68	8250 kg/metre	0.75	0.49	32.5
0.90	10000 kg/metre	0.91	0.44	30.0

The interaction factors as per the British Practice and authors recommendations are as below:

H/L	Interaction Factor (K/8) British Code	Interaction Factor (K/8). Author's recommendations
0.52	3.5	2.2
0.68	4.0	2.5
0.90	3.75	3.1

It is necessary to point out that the author has not found the support stresses to be critical as assumed in British Practice, and on the other hand the mid-span stresses govern the design. It is thus concluded that author's recommendations are conservative as per British Practice. However, there is no empiricism whatsoever in the author's procedure. It simply uses the allowable stress concept which is rather familiar to engineers. Now that computers are being extensively used, brickwalls on supporting beams may be analysed with or without openings using the finite element method and the design of supporting beams can be done using the allowable stress concept. Such designs are bound to have adequate safety margin against cracking and ultimate strength as demonstrated by the laboratory tests of the author. The time has come when empiricism in design could be minimized considerably and this has been the aim of the author in undertaking this thesis work.

6.7 Proposed Extensions for Further Research

The following recommendations for further research are based on the findings of the author reported earlier in this thesis.

1) Brickwork itself has a large safety factor assigned to it, namely 10 as in British Code. The strength and porosity of mortar have been identified as the primary factors influencing the behaviour of brickwork. Furthermore

the brick laying process itself deserves attention in terms of quality control. The elastic constants of brickwork deserve further attention from research works. The beneficial role of computer simulation has been identified. The theory of composite materials is a good starting point. In addition, failure theories deserve greater attention. It should be recollected that the author has pointed out the importance of non-homogeneity at material level in generating perturbation stresses, which are no more secondary in nature.

2) Three dimensional tetrahedral elements should be used for the study of composite material such as concrete.

3) The computer programme developed by the author (which has been appended to this thesis), incorporating automatic generation of nodal connections, nodal coordinates, openings, reinforcements in the supporting beams, tracing of local failures and stress redistribution must be tested further in larger computers in the inelastic range and could then be used for generation of design charts and for further studies on the influence of various parameters.

4) Author's introduction of nonhomogeneity, anisotropy and randomness at material and structural levels could be used effectively for probabilistic studies and optimal design of walls on beams.

- - -

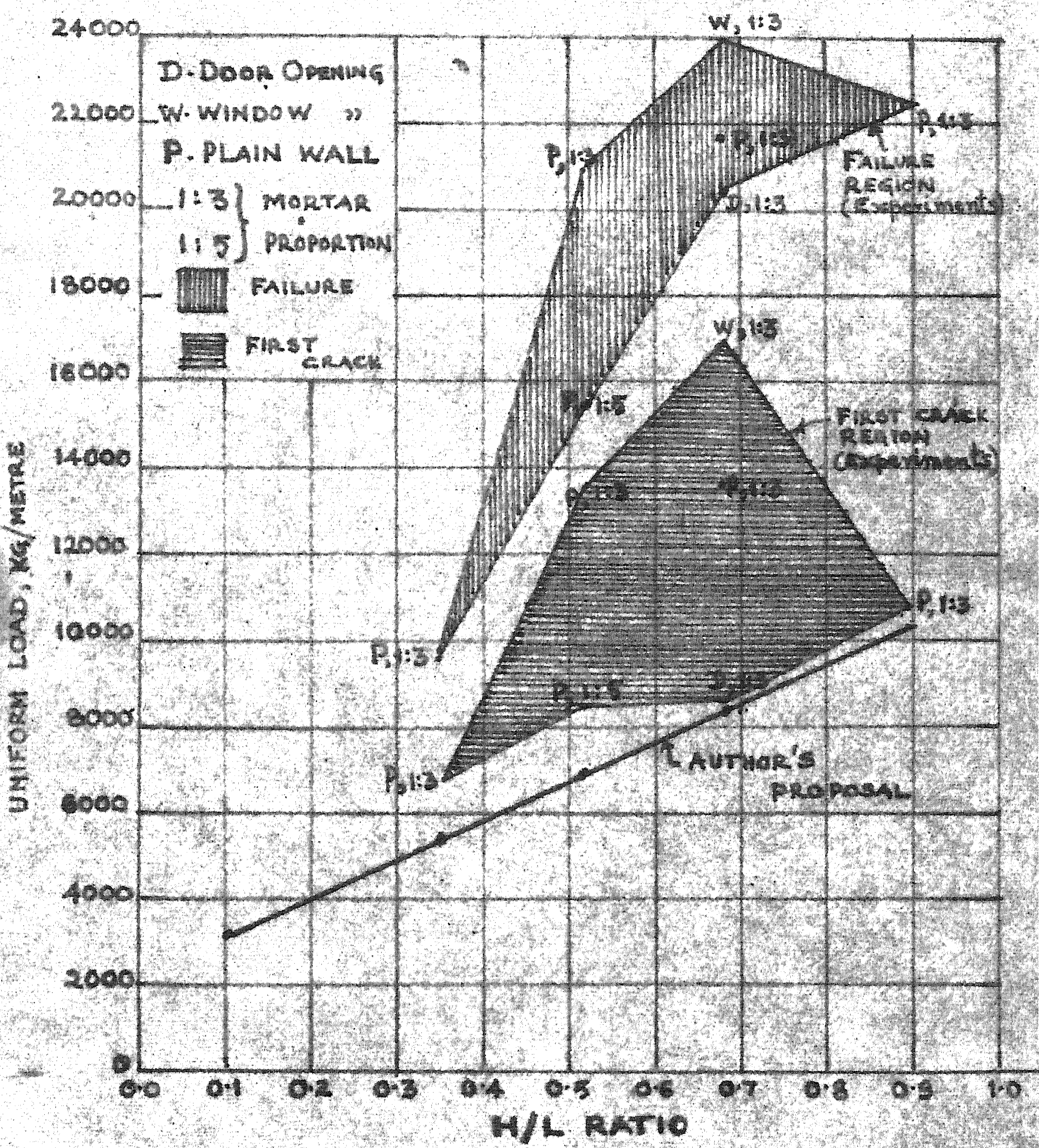
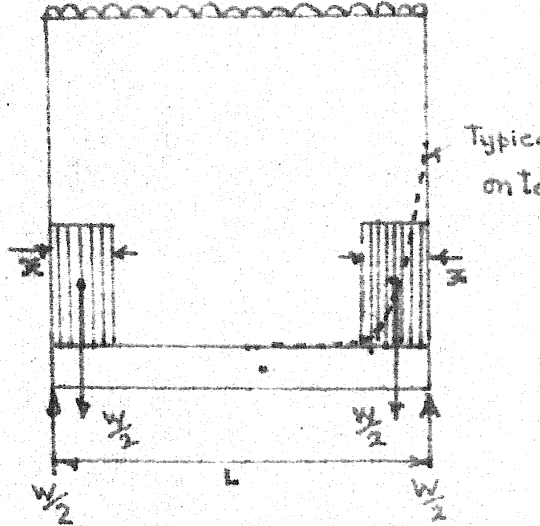
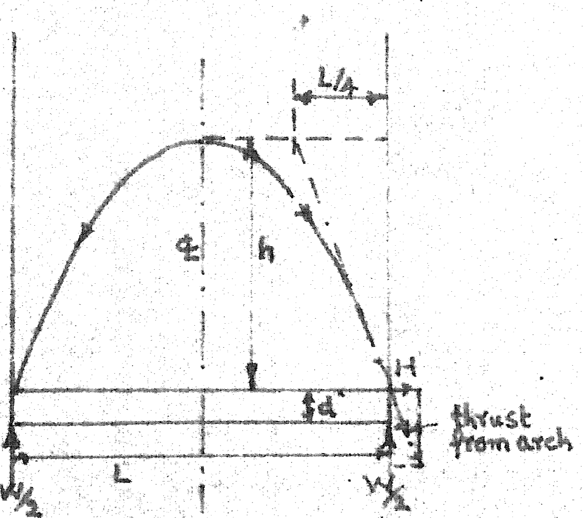


FIG. 6.1 SAFETY AGAINST FIRST CRACK AND ULTIMATE LOADS



A. ASSUMED EQUIVALENT BEAM LOADING AT FAILURE OF WALL



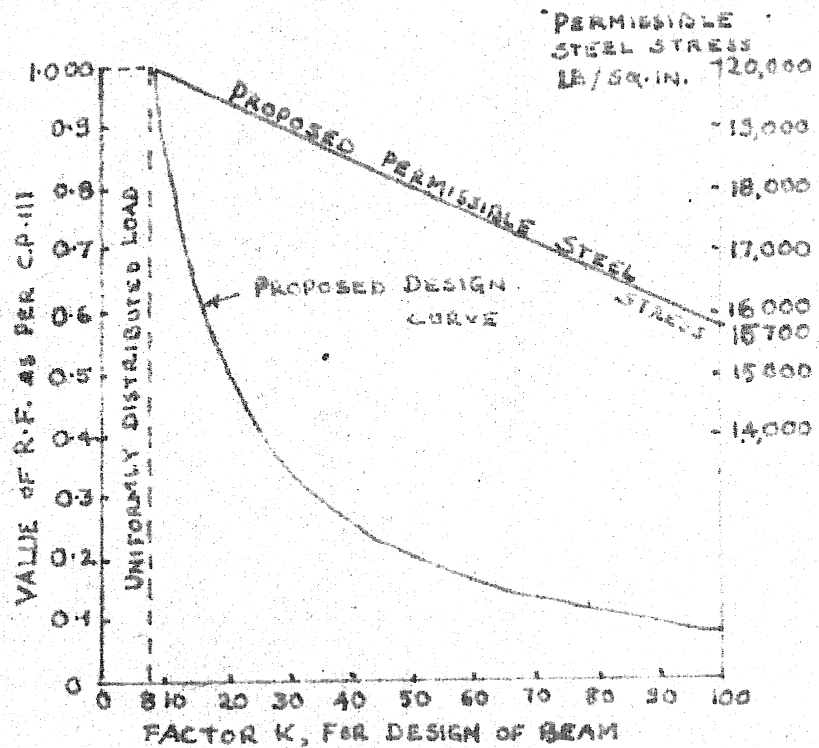
B. LINE OF THRUST AND EXTRA HORIZ. FORCES

F = STRESS REDUCTION FACTOR FOR SLENDERNESS

R = REDUCTION FACTOR FOR APPLIED LOAD

$f_w = R \cdot F \cdot f_b$

$Cf_w \ngtr 1\frac{1}{2} f_b$



C. COMPOSITE ACTION WITH LOAD BEARING WALLS

FIG. 6.2 BUILDING RESEARCH STATION (ENGLAND), RECCOMMENDATIONS

CHAPTER VII

CONCLUSIONS

The following conclusions may be read in conjunction with the critical evaluation of results presented in the previous chapter.

1) Beneficial interaction between brick walls and their supporting reinforced concrete beams exists under certa restrictions, as per the experimental investigations and computer simulation studies undertaken by the author.

2) Any load that comes directly on the supporting beams, does not benefit from interaction since the supporting beams and the brickwalls above, separate at the interface.

3) Door and window openings which are nearer to the supports generate large stress concentrations in masonry and result in premature failures.

4) Weak mortars result in separation of brick layer at mortar joints and thus prevent full interaction.

5) Full interaction exists in the case of walls on beams with compression loading on top of walls and with door and window openings centrally situated, provided they are built with relatively rich mortar of 1:3 proportion by volum

6) The primary variable which influences the inter action in walls on beams is the height-span ratio. A linear

relation exists between the allowable compression loads on such structures and their height/span ratios.

7) Computer simulation studies have shown that the flexural compressive stresses in the mid-span of supporting beams are critical rather than the stress concentrations at supports, in the elastic range.

8) Furthermore, the presence of central door and window openings have been shown to be beneficial in reducing the mid-span stresses in supporting beams, by the computer simulation programme. The loads are transmitted more towards the supports thus reducing bending moments in the mid-span region.

9) A finite element simulation programme has been developed to incorporate automatic generation of nodal connections and coordinates, reinforcement in the supports, openings in walls and to trace local failures such as cracking, crushing and yielding. The complete load-response curves could be traced realistically using incremental-iterative methods of analysis from zero-load to failure.

10) The beneficial role of computer simulation methods in isolating the effects of several variables, associated with walls on beams has been established. A substantial portion of the money now being spent in laboratory investigations can be profitably diverted to computer simulation studies.

11) At the material level, nonhomogeneity, anisotropy and randomness in material properties which characterise the behaviour of brickwalls before and after local failures have been identified by the author as important variables.

12) Based on finite element simulation at micromechanical level, author's simulation of a randomly arranged concrete like material has shown the validity and limitations of the theory of composite materials which are detailed elsewhere, in this thesis.

13) The parameter $E_m \nu_b / E_b \nu_m$, which relates the elastic constants of the constituents of brickwork has been identified by the author as important for rational design of brickwork. Through a proper choice of mortar, if this factor is kept close to unity, interaction stresses of critical nature (which generate vertical splitting of brickwork under compression) can be minimized, resulting in crack-free brickwork under service loads.

14) The interaction stresses which arise due to nonhomogeneity are essentially flexural in nature and may be viewed as perturbations on the solutions obtained from conventional theory of elasticity using the characteristic assumption of homogeneity. These local flexures arise due to mortar joints in brickwork and 'uniformly stressed brickwork' is a myth. Furthermore corners of bricks are critically stressed

due to stress concentration effects on a hard inclusion such as brick in a soft matrix such as mortar. These perturbations may be given a rational basis if studied through micro-polar elasticity (B_1).

15) While the author's studies ~~en~~ interaction between brickwalls and their supporting beams indicated a safety reserve of 2.2 to 3.1, the recommendations of the Building Research Station, England indicate reserves of 3.5 to 4.0.

16) This small reserve obtained through this broad-based study, may be compared with the safety reserve assigned to plain brickwork which is in the order of 10, because of the poor quality of brickwork itself. This certainly is a sad state of affairs, considering that a large volume of construction in India is based on brickwork. Perhaps brickwork, by its very familiarity, has been deemed to be unexotic for advanced research in various research organisations.

- - -

7.1 Recommendations for Further Research

Based on the knowledge gained by the author in his study of the problem of walls on beams, the following areas of research may be recommended for future investigations.

1) At the material level the author has investigated the effects of elastic constants for brickwork. Recently Sahlin (D 18) has come out with a comprehensive list of variables which characterise brickwork and these variables deserve experimental study and computer simulation. The variables are as below

x_1 = workmanship
x_2 = strength of bricks/blocks
x_3 = strength of mortar
x_4 = brick/block size
x_5 = workability of mortar
x_6 = water-retentivity of mortar
x_7 = suction
x_8 = gradation curve of sand
x_9 = density of mortar
x_{10} = density of blocks
x_{11} = load eccentricity
x_{12} = evenness of block/brick surface
x_{13} = curing
x_{14} = modulus of elasticity of blocks/bricks

x_{15} = modulus of elasticity of mortar

x_{16} = creep

x_{17} = shrinkage

x_{18} = tensile strength of blocks/bricks

x_{19} = tensile strength of mortar

x_{20} = bond strength

x_{21} = thermal expansion

x_{22} = Poisson's ratio of blocks/bricks

x_{23} = Poisson's ratio of mortar

x_{24} = coring

x_{25} = wetting

x_{26} = temperature

x_{27} = relative humidity

x_{28} = masonry pattern

x_{29} = additives

x_{30} = maximum particle size.

In addition, the settlement of mortar under progressive compression must be added.

2) An adequate failure theory for brickwork must be developed.

3) The author has recommended that the parameter $E_b \nu_m / E_m \nu_b$ must be kept close to unity to minimize critical interaction stresses. This must be verified by experiments

4) On optimal design of brickwork can be attempted on the basis of cost of brick and cost of mortar with a penalty cost added on the basis of detrimental interaction stresses in tension. Computer simulation studies can be made to evaluate these interaction stresses.

5) A three dimensional finite element programme with rectangular prism elements could be undertaken for the study of the behaviour of brickwork under various types of loading such as compression, flexure, shear and torsion.

6) Complete stress-strain curves for brickwork could be obtained using the incremental-iterative method of analysis programmed by the author.

7) At the structural level, combination loadings such as compression at the top of wall, direct tensile load at the bottom supporting beam and lateral loads in the plane of the wall should be simulated and interaction curves should be obtained.

8) Bond-slip between brick-wall and supporting beams and brick layers themselves should be included in the study of walls on beams, which the author has omitted. For this purpose the bond-slip element developed for jointed-rock (B 7) or the linkage elements developed for interaction between R.C. Frames and Infilled Panels (B 8, B 11, B 14, B 17) could be used.

by Girija Vallabhan (D 6).

16) Lateral loads perpendicular to the plane of the wall under static, dynamic and blast effects could be studied experimentally and through computer simulation (D 15, D 17).

17) Subject to the availability of larger and faster computers than the IBM 7044 computer used by the author, brick-work properties could be assigned in a random manner as programmed by the author and probabilistic studies could be undertaken. Since workmanship in brick-masonry is rather difficult to control, we must learn to live with the variation inherent in brickwork and all that could be done is to obtain bounds for the behaviour spectrum and their associated probabilities.

18) Besides conventional brick masonry, cellular block, sulphur mortars, cavity walls, hollow block masonry deserve further study.

19) Possibilities for precasting entire brick-panels with appropriate machinery to vibrate the mortar could be explored.

20) It has been shown by Gross and Dikkers (C 19) that there is very little agreement between various code formulating bodies on the assignment of allowable stresses for brickwork itself. Reexamination of various codes on basic brickwork must be undertaken.

20) The author is not aware of any code of practice for masonry walls interacting with their supporting beams and the existing state of knowledge is such, that guide lines could be given for the design of walls on beams as recommendations, if not in the form of codes.

21) Generation of design charts could be undertaken for elastic analysis and design on the lines indicated by the author.

- - -

LIST OF REFERENCES

A. MATERIALS

A 1. Buyukozturk, O., and Nilson, A.H. 'Finite Element Analysis of a Model of Plain Concrete with Biaxial Loads', Finite Element Methods in Civil Engineering, Speciality Conference, Mc Gill University, (June, 1972) pp.703-729.

A 2. Calcote, L.R. 'The Analysis of Laminated Composite Structures', von Nostrand Reinhold Co., (1969), pp. 13-38.

A 3. Ghosh, S.K. 'Some Aspects of the Non-linear Analysis of Reinforced Concrete Structures', Ph.D. Thesis, University of Waterloo, Canada (Sep., 1972).

A 4. Hilsdorf, H.K. 'Investigation into Failure Mechanisms of Brick Masonry Loaded in Axial Compression', International Conference on Masonry Structural Systems, University of Texas (Nov., 1967).

A 5. Jaennson, O.B., and Sundström, O.B. 'Determination of Youngs Modulus and Poisson's Ratio for WC-CO Alloys by the Finite Element Method', Material Science and Engineering, American Society for Metals, Vol.9, No.4 (April, 1972) pp.217-222.

LIST OF REFERENCES (CONTD.)

A 6. Kavanagh, K.T. 'Extension of Classical Experimental Techniques for Characterising Composite Material Behaviour', Experimental Mechanics, Journal of the Experimental Stress Analysis, Vol. 12, No.1 (Jan., 1972) pp. 50-56.

A 7. Kupfer, H., Hilsdorf, H.K., and Rush, H. 'Behaviour of Concrete Under Biaxial Stresses', Journal of the ACI, Vol.66, No.8 (Aug., 1969) pp. 656-666.

A 8. Miamoto, H., Ishijima, Y., and Homma, K. 'Interpretation of Mechanical Behaviour of Grinding Wheels in Terms of Micro-structure', Proceedings of the International Conference on Mechanical Behaviour of Materials, Kyoto, Japan, Vol.4 (Aug., 1971) pp. 484-494.

A 9. Mikkola, M.J., and Schnobrich, W.C. 'Material Behaviour Characteristics for Reinforced Concrete Shells Stressed Beyond the Elastic Range', Structural Research Series No. 367, University of Illinois (Aug., 1970).

A 10. Nakagawa, K., Kobayashi, S., and Niwa, Y. 'Study of Progressive Failure of Concrete Specimen in Uniaxial Compression', Proceedings of the International Conference on Mechanical Behaviour of Materials, Kyoto, Japan, Vol.4 (Aug. 1971) pp. 32-42.

LIST OF REFERENCES (CONTD.)

A 11. Popovics, S., 'Structural Model Approach to Two-phase Composite Materials - State of the Art', The American Ceramic Society Bulletin, Vol. 48, No.11 (Nov., 1969).

A 12. Smith B.S., and Carter, C. 'Hypothesis for Shear Failure of Brickwork', Journal of the Structural Division, ASCE, Vol. 97, No. ST4 (April, 1971) pp. 1055-62.

A 13. Sturman, G.M., Shah, S.P., and Winter, G. 'Effect of Flexural Strain Gradients on Micro-cracking and Stress-strain Behaviour of Concrete', Journal of the ACI, Vol. 62, No.7 (July, 1965) pp. 805-822.

A 14. Thomas, F.G. 'The Strength of Brickwork', The Structural Engineer, Vol. 33, No.2 (Feb., 1953).

B. CONTINUUM MECHANICS/FINITE ELEMENT METHODS

B 1. Baluch, M.H., Goldberg, J.E., and Koh, S.L. 'Micropolar Continua in a State of Plane-Stress- A Finite Element Approach' Finite Element Methods in Civil Engineering-Speciality Conference, Mc Gill University (June, 1972) pp. 287-302.

LIST OF REFERENCES (CONTD.)

B 2. Chattopadhyay, A. 'Application of Finite Element Method for Continuum Mechanics Problems', Ph.D. Thesis, Indian Institute of Technology, Kanpur, (Sep., 1970).

B 3. Desai, C.S., 'Overview Trends and Projections; Theory and Application of the Finite Element Methods in Geotechnical Engineering - State of the Art', Symposium on Applications of the Finite Element Methods in Civil Engineering, Vicksburg, Missisipi (May, 1972) pp.3-90.

B 4. Desai, C.S., and Abel, J.F. 'Introduction to the Finite Element Method for Engineering Analysis - A Numerical Method for Engineering Analysis', von Nostrand Reinhold Company, New York (1972).

B 5. Du Ching-Hua 'The Two-dimensional Problems of the Non-homogeneous, Isotropic Medium', Problems of Continuum Mechanics, Muskelisvile Anniversary Volume, Academy of Sciences, U.S.S.R. (1961).

B 6. Golecki, J. 'On the Foundations of the Theory of Elasticity of Plane Incompressible, Non-homogeneous Bodies', IUTAM Conference (1958) pp. 39-51.

LIST OF REFERENCES (CONTD.)

B 7. Goodman, R.E., Taylor, R.L., and Brekka, T.L. 'A Model for the Mechanics of Jointed Rock', Journal of the Soil Mechanics and Foundation Division, ASCE, No. SM 3 (May, 1968) pp. 637-659.

B 8. Kaldjian, M.J. 'Behaviour of Infilled Shear Walls for Various Bond and Shear Strengths', International Conference on Shear, Torsion, and Bond in Reinforced and Pre-stressed Concrete, PSG College of Technology, Combatore, India (Jan. 1969).

B 9. Lekhnitskii, S.G., 'Theory of Elasticity of an Anisotropic Elastic Body', Translation from Russian by P.Fern, Holden Day, San Francisco (1963) pp. 15-25.

B 10. Marcal, P.V. 'Finite Element Analysis with Material Non-linearities - Theory and Practice', Finite Element Methods in Civil Engineering, Speciality Conference, Mc Gill University, (June, 1972) pp. 35-70.

B 11. Mallick, D.V., and Severn, R.T. 'The Behaviour of Infilled Frames Under Static Loading', Proceedings of the Institution of Civil Engineers, London, Vol. 38 (Dec. 1967) pp. 639-656.

LIST OF REFERENCES (CONTD.)

B 12. Melosh, R.J. — 'Basis for Derivation of Matrices for the Direct Stiffness Method', Journal of AIAA, Vol.1 (1963) pp. 1631-37.

B 13. Ngo, D., and Scordelis, A.C. — 'Finite Element Analysis of Reinforced Concrete Beams', Journal of the ACI, Vol. 64, No.3 (March, 1967) pp.152-163.

B 14. Ono, K., and Mills, R.H. — 'Crack Propagation of Reinforced Concrete Members and Modelling of Concrete by the F.E. Method', International Conference on Mechanical Behaviour of Materials, Kyoto, Japan, Vol. 4 (Aug., 1971) pp. 43-52.

B 15. Sanyasi Raju, I. — 'Finite Element Analysis of Stress Concentration and Singularities', Ph.D. Thesis, Indian Institute of Science, Bangalore, India (1972).

B 16. Sethuratnam, A. — 'Design Interaction Force Coefficients of Shear Wall-Frame Systems', Ph.D. Thesis, Indian Institute of Technology, Kanpur, India (1971).

B 17. Yuzugullu, O., and Schnobrich, W.C. — 'Finite Element Approach for the Prediction of Inelastic Behaviour of Shear Wall-Frame Systems', Structural Research Series, No.386, University of Illinois (May, 1972).

LIST OF REFERENCES (CONTD.)

B 18. Zienkiewicz, O.C., and King, I.P. 'Elastic-Plastic Solutions for Engineering Problems, Initial Stress, Finite Element Approach', International Journal of Numerical Methods in Engineering, Vol. 1 (1969) pp. 75-100.

B 19. Zienkiewicz, O.C. 'The Finite Element Method in Engineering Science', Mc Graw Hill, London (1972) pp. 48-72 (1968 version also used).

B 20 ———— 'Stress and Strain Concentrations', Chap. 11, Mechanical Behaviour of Materials, Ed. Mc Clintock, F.A., and Argon, S.A. (1966) pp. 393-415.

C. WALLS ON BEAMS/INFILLED FRAMES

C 1. Benjamin, J.R. and Williams, H.A. 'The Behaviour of One-Storey Brick Shear Walls', Journal of the Structural Division, ASCE, Vol. 84, No.ST 4 (July, 1958) pp. 1-30.

C 2. Burhouse, P. 'Composite Action Between Brick Panel Walls on their Supporting Beams', Proceedings of the Institution of Civil Engineers, London, Vol.43 (June, 1969) pp. 175-194.

LIST OF REFERENCES (CONTD.)

C 3. Carter, C., and Smith, B.S. 'Structural Behaviour of Masonry Infilled Frames Subjected to Raking Loads', International Conference on Masonry Structural Systems, University of Texas, (Nov., 1967).

C 4. Colbourne, J.R. 'Studies in Composite Construction - an Elastic Analysis of Wall Beam Structures', CP 15/69, Building Research Station, England (May, 1969)

C 5. Coull, A. 'Composite Action of Walls Supported on Beams', Building Science, Vol.1, (1966) pp. 259-270.

C 6. Fiorato, A.E., Sozen, M.A., and Gamble, W.L. 'An Investigation of the Interaction of Reinforced Concrete Frames with Masonry Filler Walls', Structural Research Series, No. 370, University of Illinois (Nov., 1970).

C 7. Hinkley, T.A. 'Analysis of Shear Wall Supported by a Beam', Journal of the Structural Division, ASCE, Vol. 92, No. ST1 (Feb., 1966) pp. 121-130.

C 8. Mainstone, R.S. 'On the Stiffness and Strength of Infilled Frames', Building Research Station, England, Current Paper CP2 (1972).

LIST OF REFERENCES (CONTD.)

C 9. Murthy, C.K. and Hendry, A.W. 'Model Experiments in Load Bearing Brickwork', Building Science, Vol.1 (1966) pp. 289-294.

C 10. Ockleston, A.J. 'Load Tests on a Three Storey Reinforced Concrete Building in Johannesberg', The Structural Engineer, London, Vol. 33, No.10 (Oct., 1955) pp. 304-322.

C 11. Parghi, P.N., and Sridhar Rao, J.K. 'Reinforced Brick Masonry Structures', Technical Information Manual 1/68, Indian Institute of Technology, Kanpur, India (Feb., 1968).

C 12. Prasada Rao, N.V., and Mallick, S.K. 'Strength of Brick Masonry Walls, Supported on Reinforced Concrete Beams', Cement and Concrete, Vol.9 (April-June, 1968) pp. 14-27.

C 13. Rosenhaupt, S. 'Experimental Study of Masonry Walls on Beams', Journal of the Structural Division, ASCE, Vol. 88, No.ST3 (June, 1962) pp. 137-166.

C 14. Rosenhaupt, S. 'Stresses in Point Supported Composite Walls', Journal of the American Concrete Institute, Vol.61 (July, 1964) pp. 796-810.

LIST OF REFERENCES (CONTD.)

C 15. Rosenhaupt, S., Beresford, F.O., and Blakey, R.A. 'Tests of a Post-Tensioned Concrete Masonry Wall', Journal of the American Concrete Institute, Vol.64 (Dec., 1967) pp. 829-837.

C 16. Rosenhaupt, S., and Muller, G. 'Openings in Masonry Walls on Settling Supports', Journal of the Structural Division, ASCE, Vol.89, No. ST3, (June, 1963) pp. 107-132.

C 17. Rosenhaupt, S., and Sokal, Y. 'Masonry Walls on Continuous Beams', Journal of the Structural Division, ASCE, Vol. 91, No. ST1 (Feb., 1965) pp.155-171.

C 18. Sahlin, S., and Hellers, B.G. 'Load Bearing Capacity of 3M Brick-work', Building Research Station, England, Library Communication No. 1557 (May, 1970).

C 19. Gross, J.G., and Dikkers, R.D. 'Building Code Requirements Relating to Load Bearing Brick Masonry', International Conference on Masonry Structural.Systems, University of Texas (Nov., 1967).

C 20. SCPI 'Recommended Practice for Engineered Brick Masonry', Structural Clay Products Institute, Mc Lean, Virginia, (Nov., 1969) pp. 246-254.

LIST OF REFERENCES (CONTD.)

C 21. Wood, R.H. — 'Studies in Composite Construction – Part 1', National Building Studies, Research Paper No. 13, HMSO, London (1952).

C 22. Wood, R.H. — 'Studies in Composite Construction – Part II, The Interaction of Floors and Beams in Multistoreyed Buildings', National Building Studies, Research Paper No.22, HMSO, London (1955).

C 23. Wood, R.H., and Simms, L.G. — 'A Tentative Design Method for the Composite Action of Heavily Loaded Brick Panel Walls, Supported on Reinforced Concrete Beams', Building Research Station, England, Current Papers 26/69 (July, 1969).

C 24. Ramesh, C.K., Dravid, P.S., and Anjaneyulu, E. — A Study of Composite Action in Brick Panel Wall Supported on Reinforced Concrete, The Indian Concrete Journal, Vol. 44, No.10 (Oct., 1970) pp. 442-448.

– – –

LIST OF REFERENCES (CONTD.)

D - ADDITIONAL SELECTED BIBLIOGRAPHY

D 1. Lenczner, D., 'Strength and Elastic Properties of the 9 inch Brickwork Cube', Transactions of British Ceramics Society, Vol. 65, No.6 (June, 1966), pp. 363-382.

D 2. Yettram, A.L., and Hirst, M.J.S. 'An Elastic Analysis for the Composite Action of Walls Supported on Single Beams', Building Science, Vol. 65, No.3, (Sept., 1971), pp. 151-161.

D 3. Sven Sahlin 'Structural Masonry', Prentice-Hall Inc., Englewood Cliffs, U.S.A., (1971).

D 4. Mallick, D.V. 'Determination of Lateral Stiffness of Infilled Frames by Finite Element Method', Third Symposium on Earthquake Engineering held at University of Roorkee, Roorkee, India (Nov., 1966).

D 5. Yokel, F.Y., and Dikkers, R.D. 'Strength of Load Bearing Masonry Walls', Journal of the Structural Division, ASCE, Vol. 97, No. ST5, (May, 1971), pp. 1593-1609.

LIST OF REFERENCES (CONTD.)

D 6. Girijavallabhan, C.V. 'Analysis of Shear Walls by Finite Element Method', Proc. of the Symposium on Application of Finite Element Method in Civil Engineering, Vanderbilt University and American Society of Civil Engineers, (1969), pp. 631-641.

D 7. Oakberg, R.G., and Weaver, Jr.W. 'Analysis of Frames with Shear Walls by Finite Elements', Proceedings of the Symposium on Application of Finite Element Methods in Civil Engineering, Vanderbilt University and American Society of Civil Engineers, (1969), pp. 567-607.

D 8. Bhagwat, R.S. 'Analysis of Shear Walls and Infilled Frames by Finite Element Method', M.Tech. Thesis, Indian Institute of Technology, Kanpur (1968).

D 9. Paramasivam, P. 'Discrete Methods for the Analysis of Some Complex Plate Problems', Ph.D. Thesis, Indian Institute of Technology, Kanpur (1969).

D 10. Smith, B.S., and Carter, C. 'Hypothesis for Shear Failure of Brickwork', Journal of the Structural Division, ASCE, Vol. 97, No. ST4, (April, 1971), pp. 1055-1062.

LIST OF REFERENCES (CONTD.)

D 11. Mainstone, R.J. 'On the Stiffnesses and Strength of Infilled Frames', Proceedings of the Institution of Civil Engineers, 1971 Supplement (iv), Paper 7360S (1971), pp. 57-90.

D 12. Smith, B.S., Carter, C., and Chowdry, J.R. 'The Diagonal Tensile Strength of Brickwork', The Structural Engineer, Vol.48, No.6, (June, 1970) pp.219-225.

D 13. West, H.W.H., Hodgkinson, H.R., Beech, D.G. and Davenport, S.T.E. 'The Performance of Walls Built of Wirecut Bricks with and without Perforations', The Structural Engineer, Vol. 48, No.12 (Dec., 1970) pp. 481-488.

D 14. Jofriet, J.C., and Mc Neice, G.M. 'Finite Element Analysis of Reinforced Concrete Slabs', Journal of the Structural Division, ASCE, Vol. 97, No. ST3, (March, 1971), pp.785-806.

D 15. Sinha, B.P., and Hendry, A.W. 'Raking Tests on Storey Height Shear Wall Structures with Openings, Subjected to Precompression', Paper 23, Proceedings 1st International Conference on Masonry Structural Systems, Johnson, ed., University of Texas, 1969.

LIST OF REFERENCES (CONTD.)

D 16. Purushothaman, P., Sridhar Rao, J.K. and Murthy, P.N. 'Studies on Two Phase Composite Materials Through Computer Simulation', Paper Accepted for Presentation in Tokyo Seminar on Finite Element Analysis, 1973.

D 17. Sinha, B.P., and Hendry, A.W. 'The Stability of a Five-Storey Brickwork Cross-wall Structure Following the Removal of a Section of a Main Load-bearing Wall', The Structural Engineer, Vol.49, No.10, (Oct., 1971), pp. 467-474.

D 18. Sven Sahlin 'High Quality Structural Masonry Based on Controlled Parameters', Discussion Volume, Conference on Planning and Design of Tall Buildings, Lehigh University, (1972).

D 19. Pfeffermann, O. 'The Behaviour of Brick Partition with Deformation on Their Supports', Second International Brick Masonry Conference, Proceedings, Stoke-on-Trent, (1970), p. 137.

D 20. Mallet, R.J. 'Structural Behaviour of Masonry Elements', State of the Art Report No. 3, Technical Committee No.27, JCIB, Conference on Planning and Design of Tall Buildings, Lehigh University (1972).

LIST OF REFERENCES (CONTD.)

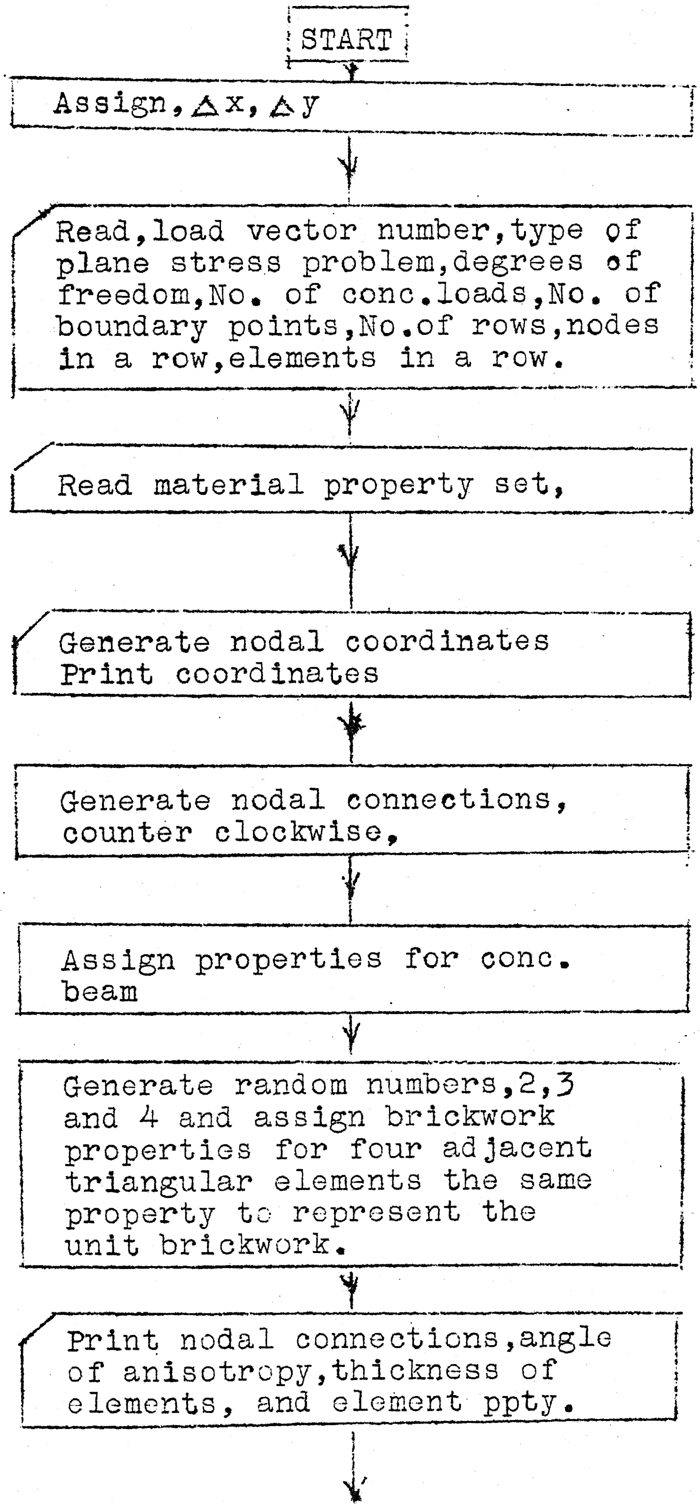
D 21. Hilsdorf, H.K. 'Masonry Materials and Their Physical Properties', Stated the Art, Report No. 2, Technical Committee No.27, JCTB, Conference on Planning and Design of Tall Buildings, Lehigh University, (1972).

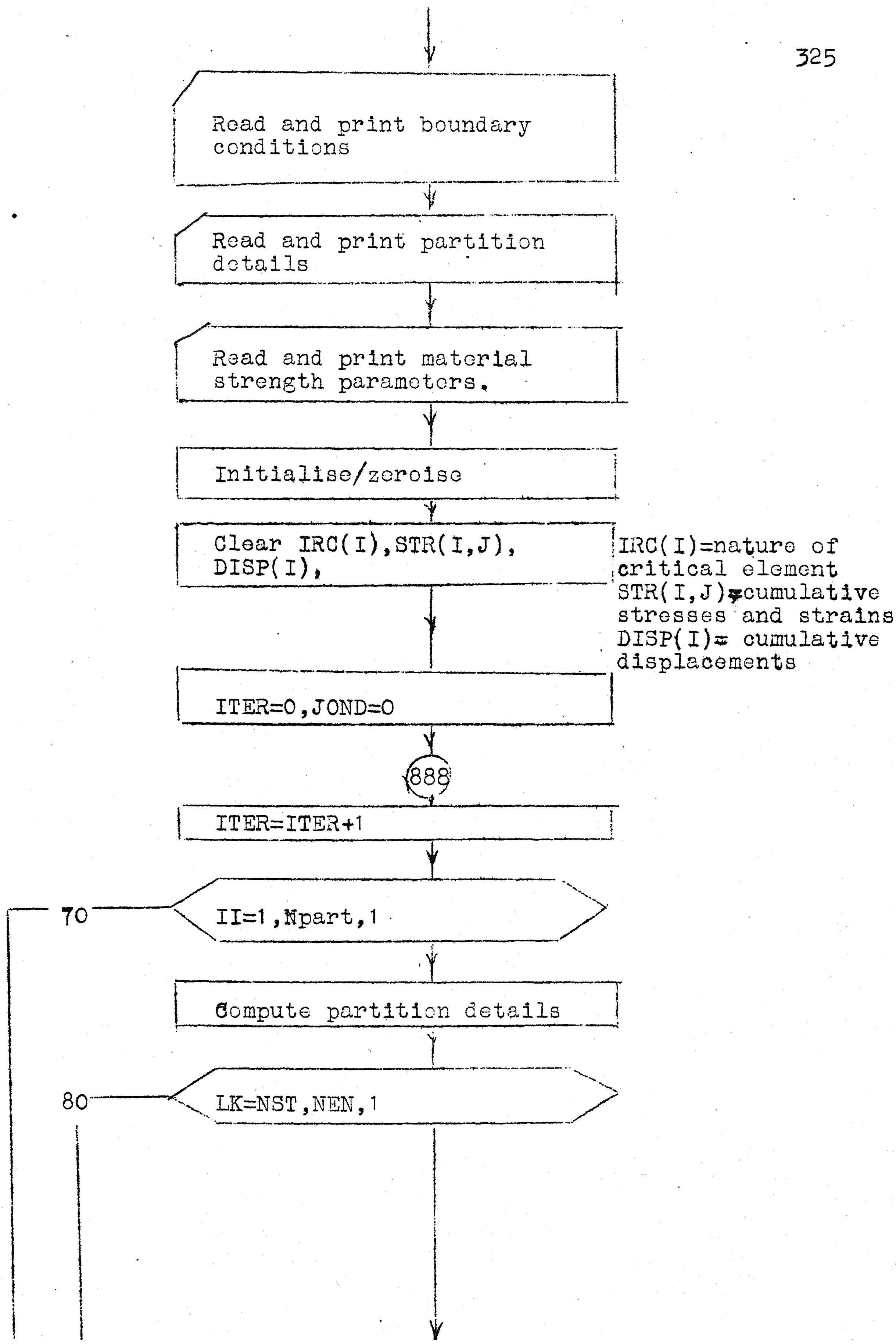
D 22. Cervenka, V., and Gerstle, K.H. 'Inelstic Analysis of Reinforced Concrete Panels ,Part I:Theory'. IABSE Publications,Vol.31-II(1971).

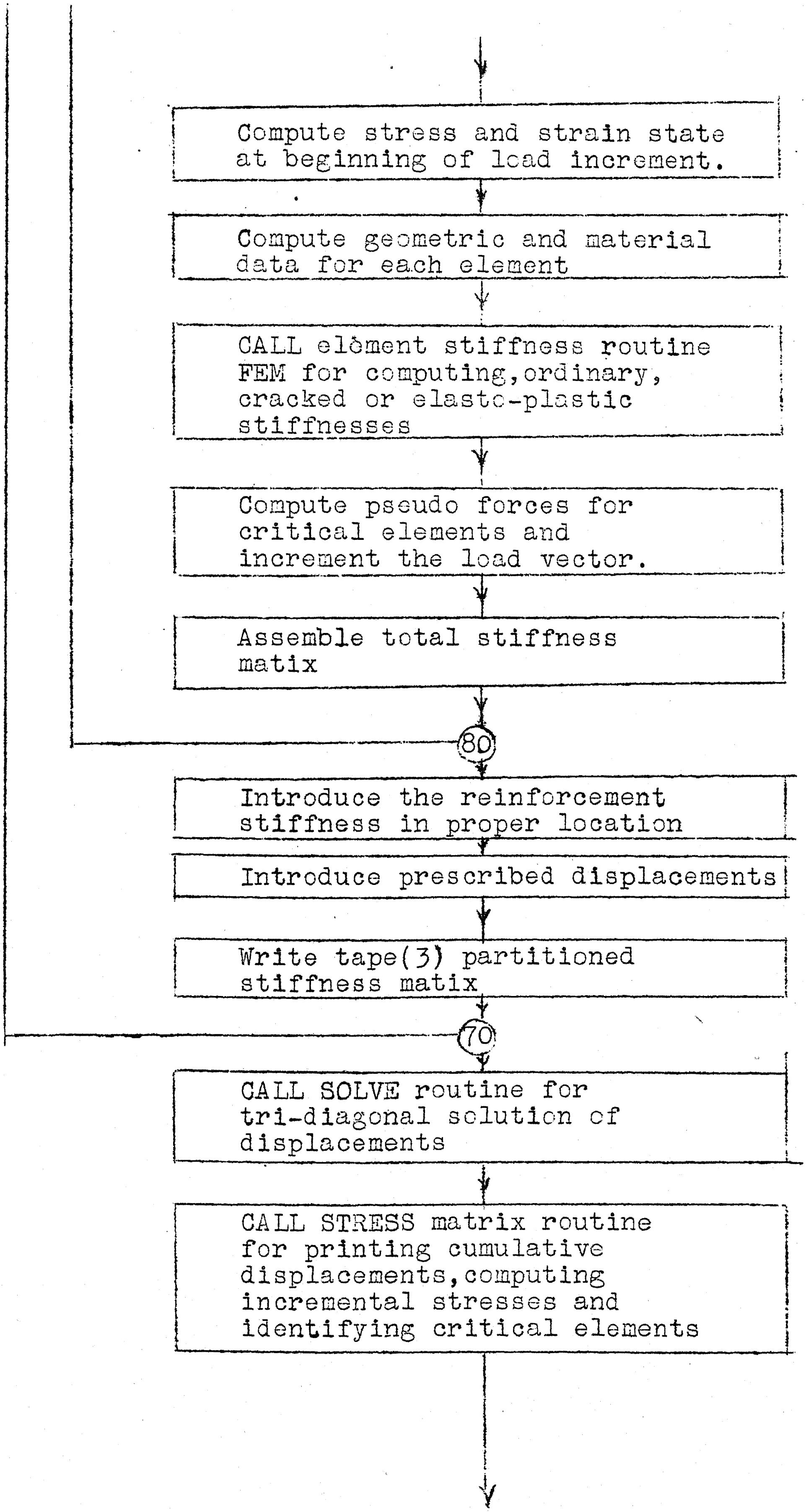
D 23. Cervenka. V., and Gerstle, K.H. 'Inelastic Analysis of Reinforced Concrete Panels: Experimental Verification and Application', IABSE Publications,Vol.32-II(1972).

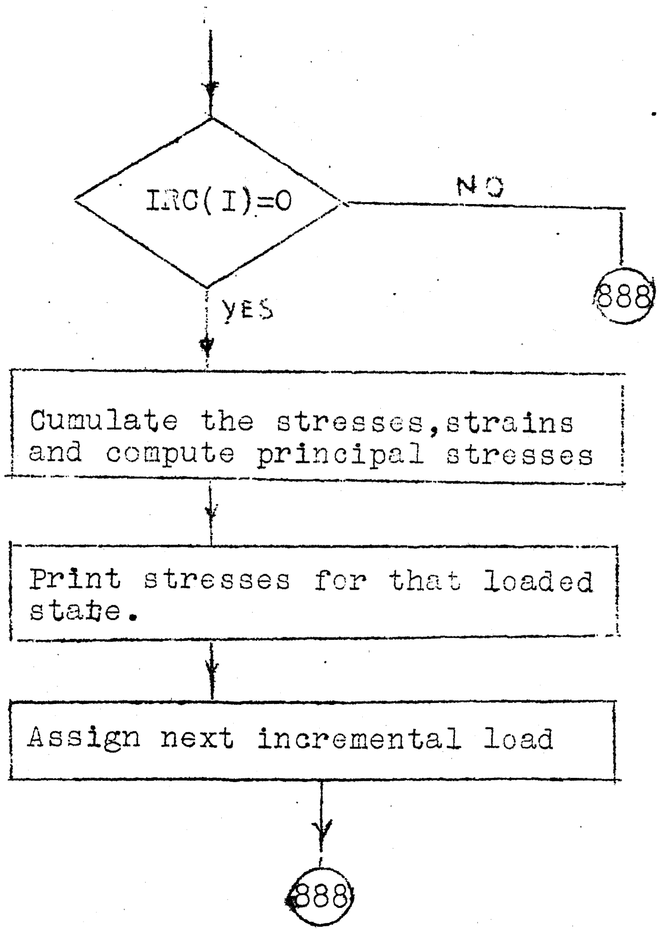
APPENDIX

FLOW CHART FOR FINITE ELEMENT SIMULATION OF THE BEHAVIOUR OF BRICK WALLS ON REINFORCED CONCRETE BEAMS.




Read and print boundary conditions
Read and print partition details
Read and print material strength parameters.
Initialise/zeroise
Clear IRC(I),STR(I,J), DISP(I),
IRC(I)=nature of critical element
STR(I,J)=cumulative stresses and strains
DISP(I)= cumulative displacements
ITER=0,JOND=0
888
ITER=ITER+1
70
II=1,Npart,1
Compute partition details
80
LK=NST,NEN,1


Compute stress and strain state at beginning of load increment.
Compute geometric and material data for each element
CALL elòment stiffness routine FEM for computing,ordinary, cracked or elasto-plastic stiffnesses
Compute pseudo forces for critical elements and increment the load vector.
Assemble total stiffness matix
80
Introduce the reinforcement stiffness in proper location
Introduce prescribed displacements
Write tape(3) partitioned stiffness matix
70
CALL SOLVE routine for tri-diagonal solution of displacements
CALL STRESS matrix routine for printing cumulative displacements,computing incremental stresses and identifying critical elements


IRC(I)=0
NO
888
YES
Cumulate the stresses,strains and compute principal stresses
Print stresses for that loaded state.
Assign next incremental load
888

FORTRAN PROGRAMME

```
£JOB   CER097,TIME008,PAGES020,NAME P.PURUSHOTHAMAN
£IBFTC MAIN     DECK
       DIMENSION XINC(20),YINC(20),D(3,3),BA(3,6),SB(3),PB(6)
       DIMENSION X(120,2),XE(3,2),NF(20),NB(20,2),BV(20,2),NOD(200,3)
       DIMENSION NEP(200),AN(200),THICK(200),E1(8),E2(8),P1(8),P2(8)
       DIMENSION GE(8),NSTART(20),NEND(20),NFIRST(20),NLAST(20)
       DIMENSION ULOAD(240),REIN(6,6),DISP(240),STR(200,6),IRC(200)
       COMMON C(6,6),DBA(3,6),DB(3,6),A(6,6),B(3,6),ST(50,100),U(240,4)
       CALL FLUN(31200)
 31    FORMAT (9I4,2F16.8)
 32    FORMAT(1H ,*NPART*I4,* NPOIN*I4,* NELEM*I4,* NBOUN*I4,* NCOLN*I4
      1* NP*I4,* NFREE*I4,*NCONC*I4,//1X,*COORDINATES*)
 33    FORMAT(I4,2F10.2)
 34    FORMAT(4(4I4,2F5.1,I3,* C *))
 35    FORMAT(7F10.2)
 36    FORMAT(6(I4,2F7.2,* C *))
 37    FORMAT(3I4,3F10.2,2I4)
 38    FORMAT( 5E16.4)
C      ****     ****      ****      ****     READING/GENERATION OF DATA
       DO 20 I=1,3
 20    XINC(I)=7.5
       DO 21 I=4,7
 21    XINC(I)=10.0
       DO 22 I=8,15
 22    XINC(I)=5.0
       DO 23 I=1,20
 23    YINC(I)=4.00
       DO 24 I=8,15
 24    YINC(I)=10.0
       YINC(1)=2.5
       YINC(6)=2.5
       YINC(7)=9.0
       YINC(16)=5.0
       READ 31,NCOLN,NP,NFREE,NBOUN,NCONC,NROW,NODE,NEROW
       WRITE(6,31) NROW,NODE,NEROW
       DO 1099 I=1,8
       READ 35,E1(I),E2(I),P1(I),P2(I),GE(I)
 1099  PRINT 38,E1(I),E2(I),P1(I),P2(I),GE(I)
       AS=0.7854
       ES=2.1E+06
       XXIS=3.14/64.0
       NPART=NROW
       NPOIN=NROW*NODE
       NELEM=(NROW-1)*NEROW
       WRITE(6,32) NPART,NPOIN,NELEM,NBOUN,NCOLN,NP,NFREE,NCONC
```

```
C     ****    ****    ****   AUTOMATIC GENERATION OF NODAL COORDINATES
      DO 28 I=1,NODE
28    X(I,2)=0.0
      DO 30 L=1,NROW
      KK=L-1
      I=KK*NODE+L
      X(I,1)=0.0
      IF(L.EQ.1) GO TO 1234
      IVAL=I-NODE
      X(I,2)=X(IVAL,2)+YINC(KK)
1234  DO 30 M=2,NODE
      MM=M-1
      J=I+M-1
      X(J,1)=X(J-1,1)+XINC(MM)
      IF(L.EQ.1) GO TO 30
      JVAL=J-NODE
      X(J,2)=X(JVAL,2)+YINC(KK)
30    CONTINUE
      WRITE(6,36) (I,(X(I,J),J=1,2),I=1,NPOIN)
C     ****    ****    ****   AUTOMATIC GENERATION OF NODAL CONNECTION
      NROS=NROW-1
      DO 41 M=1,NROS,2
      II=NEROW*(M-1)
      L=0
      DO 41 N=1,NEROW
      I=II+N
      L=L+1
      GO TO(42,43,44,46),L
42    NOD(I,1)=(M-1)*NODE+N/2+1
      NOD(I,2)=(M-1)*NODE+N/2+2
      NOD(I,3)=M*NODE+N/2+1
      GO TO 41
43    NOD(I,1)=(M-1)*NODE+N/2+1
      NOD(I,2)=M*NODE+N/2+1
      NOD(I,3)=M*NODE+N/2
      GO TO 41
44    NOD(I,1)=(M-1)*NODE+N/2+1
      NOD(I,2)=M*NODE+N/2+2
      NOD(I,3)=M*NODE+N/2+1
      GO TO 41
46    L=0
      NOD(I,1)=(M-1)*NODE+N/2
```

```
      NOD(I,3)=M*NODE+N/2+1
 41   CONTINUE
      DO 51 M=2,NROS,2
      II=NEROW*(M-1)
      L=0
      DO 51 N=1,NEROW
      I=II+N
      L=L+1
      GO TO (52,53,54,55),L
 52   NOD(I,1)=(M-1)*NODE+N/2+1
      NOD(I,2)=M*NODE+N/2+2
      NOD(I,3)=M*NODE+N/2+1
      GO TO 51
 53   NOD(I,1)=(M-1)*NODE+N/2
      NOD(I,2)=(M-1)*NODE+N/2+1
      NOD(I,3)=M*NODE+N/2+1
      GO TO 51
 54   NOD(I,1)=(M-1)*NODE+N/2+1
      NOD(I,2)=(M-1)*NODE+N/2+2
      NOD(I,3)=M*NODE+N/2+1
      GO TO 51
 55   L=0
      NOD(I,1)=(M-1)*NODE+N/2+1
      NOD(I,2)=M*NODE+N/2+1
      NOD(I,3)=M*NODE+N/2
 51   CONTINUE
C     ****       ****       ****       PROPERTIES OF CONCRETE BEAM
      DO 400 I=1,192
      AN(I)=0.0
      THICK(I)=15.0
 400  NEP(I)=1
C     ****       ****       ****       RANDOM PROPERTIES OF BRICKWORK
      DO 401 I=193,NELEM,4
      AN(I)=0.0
      THICK(I)=11.0
      XXX=RNDYI(K)
      III=XXX*3.0+2.0
      NEP(I)=III
      NEP(I+1)=III
      NEP(I+2)=III
      NEP(I+3)=III
 401  CONTINUE
      WRITE(6,34)(I,(NOD(I,J),J=1,3),AN(I),THICK(I),NEP(I),I=1,NELEM)
C     ****       ****       ****       BOUNDARY CONDITIONS
      DO 501 I=1,NBOUN
      READ(5,37) NF(I),NB(I,1),NB(I,2),BV(I,1),BV(I,2)
 501  WRITE(6,37) NF(I),NB(I,1),NB(I,2),BV(I,1),BV(I,2)
```

```
     ****      ****      ****      ****     PARTITION DETAILS
      DO 60 I=1,NPART
      READ(5,31) NSTART(I),NEND(I),NFIRST(I),NLAST(I)
0     WRITE(6,31) NSTART(I),NEND(I),NFIRST(I),NLAST(I)
     ****      ****      ****      ****     INITIAL LOADING
      NPOIN2=NPOIN*2
      DO 601 I=1,NPOIN2
01    ULOAD(I)=0.0
      DO 69 J=1,NCOLN
      DO 68 I=1,NPOIN2
8     U(I,J)=0.0
      DO 69 I=1,NCONC
      READ (5,33) K,U(2*K-1,J),U(2*K,J)
      ULOAD(2*K-1)=U(2*K-1,1)
      ULOAD(2*K)=U(2*K,1)
9     WRITE(6,33) K,U(2*K-1,J),U(2*K,J)
     ****      ****      ****     LIMITING STRESSES
      READ 35,CMAXT,CMAXC,BMAXT,BMAXC,SMAXT,SMAXC
      PRINT 35,CMAXT,CMAXC,BMAXT,BMAXC,SMAXT,SMAXC
     ****      ****      ****      ****     INITIALISE
      DPOP=0.2
      REWIND 4
      SIGX=0.0
      SIGY=0.0
      SIGXY=0.0
      EX=0.0
      EY=0.0
      EXY=0.0
      DO 886 I=1,NELEM
      IRC(I)=0.0
      DO 885 J=1,6
885   STR(I,J)=0.0
886   WRITE (4) SIGX,SIGY,SIGXY,EX,EY,EXY
      DO 887 I=1,NPOIN2
887   DISP(I)=0.0
      JOND=0
      ITER=0
     ****      ****      ****     ASSEMBLY OF STIFFNESS MATRIX/PSEUDO LOADS
888   CONTINUE
      ITER=ITER+1
      REWIND 0
      REWIND 3
      INTER=0
      DO 70 II=1,NPART
      REWIND 4
```

```
      CALL FLUN(32000)
      DO 75 I=1,50
      DO 75 J=1,100
75    ST(I,J)=0.0
      NST=NSTART(II)
      NEN=NEND(II)
      K=NFIRST(II)
      L=NLAST(II)
      MINUS=K-1
      IF(II.EQ.1) GO TO 7575
      IREAD=NST-1
      DO 755 I=1,IREAD
755   READ(4) SIGX,SIGY,SIGXY,EX,EY,EXY
7575  CONTINUE
      DO 80 LK=NST,NEN
      READ(4) SIGX,SIGY,SIGXY,EX,EY,EXY
      ST1=STR(I,1)+SIGX
      ST2=STR(I,2)+SIGY
      ST3=STR(I,3)+SIGXY
      ET1=STR(I,4)+EX
      ET2=STR(I,5)+EY
      ET3=STR(I,6)+EXY
      MM=LK-INTER
      DO 85 I=1,3
      JJ=NOD(LK,I)
      XE(I,1)=X(JJ,1)
85    XE(I,2)=X(JJ,2)
      ANG=AN(LK)
      TH=THICK(LK)
      J=NEP(LK)
      LS=IRC(LK)
      YM1=E1(J)
      YM2=E2(J)
      PR1=P1(J)
      PR2=P2(J)
      G=GE(J)
      CALL FEM(XE,YM1,YM2,PR1,PR2,G,ANG,LS,TH,MM,ST1,ST2,ST3,D,BA,Z)
      AREA=Z/2.0
      IF(LS.EQ.0) GO TO 1312
      IF(LS.EQ.1) GO TO 1313
      IF(LS.EQ.13) GO TO 1313
      IF(LS.EQ.14) GO TO 1313
      IF(LS.EQ.2) GO TO 1314
      IF(LS.EQ.23) GO TO 1314
      IF(LS.EQ.24) GO TO 1314
      IF(LS.EQ.3) GO TO 2315
```

```
      IF(LS.EQ.4) GO TO 2316
      GO TO 1312
1313  CONTINUE
      SCR1=0.0
      SCR2=0.0
      SCR3=0.0
      SCR1=SCR1+D(1,1)*EX+D(1,2)*EY+D(1,3)*EXY
      SCR2=SCR2+D(2,1)*EX+D(2,2)*EY+D(2,3)*EXY
      SCR3=SCR3+D(3,1)*EX+D(3,2)*EY+D(3,3)*EXY
      SB(1)=ST1-SCR1
      SB(2)=ST2-SCR2
      SB(3)=ST3-SCR3
      IF(MM)1316,1316,1317
1317  STR(LK,1)=STR(LK,1)+SCR1
      STR(LK,2)=STR(LK,2)+SCR2
      STR(LK,3)=STR(LK,3)+SCR3
      STR(LK,4)=STR(LK,4)+EX
      STR(LK,5)=STR(LK,5)+EY
      STR(LK,6)=STR(LK,6)+EXY
      DO 857 L=1,6
      PB(L)=0.0
      DO 857 K=1,3
857   PB(L)=PB(L)+BA(K,L)*SB(K)*AREA*TH
      DO 858 J=1,3
      JJ=NOD(LK,J)
      U(2*JJ-1,1)=U(2*JJ-1,1)+PB(2*J-1)
858   U(2*JJ,1)=U(2*JJ,1)+PB(2*J)
1316  CONTINUE
      IF(LS.EQ.13) GO TO 2315
      IF(LS.EQ.14) GO TO 2316
      GO TO 1312
1314  CONTINUE
      SCR1=SCR1+D(1,1)*EX+D(1,2)*EY+D(1,3)*EXY
      SCR2=SCR2+D(2,1)*EX+D(2,2)*EY+D(2,3)*EXY
      SCR3=SCR3+D(3,1)*EX+D(3,2)*EY+D(3,3)*EXY
      SB(1)=SIGX-SCR1
      SB(2)=SIGY-SCR2
      SB(3)=SIGXY-SCR3
      IF(MM)1318,1318,1319
1319  STR(LK,1)=STR(LK,1)+SCR1
      STR(LK,2)=STR(LK,2)+SCR2
      STR(LK,3)=STR(LK,3)+SCR3
      STR(LK,4)=STR(LK,4)+EX
      STR(LK,5)=STR(LK,5)+EY
      STR(LK,6)=STR(LK,6)+EXY
      DO 859 L=1,6
```

```
      PB(L)=0.0
      DO 859 K=1,3
859   PB(L)=PB(L)+BA(K,L)*SB(K)*AREA*TH
      DO 860 J=1,3
      JJ=NOD(LK,J)
      U(2*JJ-1,1)=U(2*JJ-1,1)+PB(2*J-1)
860   U(2*JJ,1)=U(2*JJ,1)+PB(2*J)
1318  CONTINUE
      IF(LS.EQ.23) GO TO 2315
      IF(LS.EQ.24) GO TO 2316
      GO TO 1312
2315  CONTINUE
      IF(MM)1312,1312,2317
2317  JJ=NOD(LK,1)
      KK=NOD(LK,2)
      U(2*JJ-1,1)=U(2*JJ-1,1)-2475.0
      U(2*KK-1,1)=U(2*KK-1,1)+2475.0
      GO TO 1312
2316   CONTINUE
      IF(MM)1312,1312,2320
2320  JJ=NOD(LK,1)
      KK=NOD(LK,2)
      U(2*JJ-1,1)=U(2*JJ-1,1)+2475.0
      U(2*KK-1,1)=U(2*KK-1,1)-2475.0
      GO TO 1312
1312  DO 80 LL=1,3
      DO 80 KK=1,3
      IF(NOD(LK,KK)-K)80,131,131
131   IF(NOD(LK,KK)-L)132,132,80
132   M=NFREE*(NOD(LK,KK)-K)
      N=NFREE*(NOD(LK,LL)-K)
      I=NFREE*(KK-1)
      J=NFREE*(LL-1)
      IF(N)80,900,900
900   DO 5 NJ=1,NFREE
      DO 5 MI=1,NFREE
      MMI=M+MI
      NNJ=N+NJ
      IMI=I+MI
      JNJ=J+NJ
5     ST(MMI,NNJ)=ST(MMI,NNJ)+C(IMI,JNJ)
80    CONTINUE
C     ****    ****    ****     INTRODUCING REINFORCEMENT STEEL
8778  IF(II.EQ.2) GO TO 308
      IF(II.EQ.6) GO TO 309
      GO TO 241
```

```
308   JINDEX=1
      INN=34
      IFF=64
      GO TO 300
309   JINDEX=1
      INN=162
      IFF=192
      GO TO 300
300   DO 303 I=INN,IFF,4
      JJS=NOD(I,1)
      KKS=NOD(I,2)
      RLEN=X(KKS,1)-X(JJS,1)
      LS=IRC(I)
       CALL STEEL(RLEN,AS,ES,XXIS,REIN,LS)
      DO 303 LL=1,3
      DO 303 KK=1,3
      IF(NOD(I,KK)-K)303,304,304
304   IF(NOD(I,KK)-L)305,305,303
305   M=NFREE*(NOD(I,KK)-K)
      N=NFREE*(NOD(I,LL)-K)
      IS=NFREE*(KK-1)
      JS=NFREE*(LL-1)
      IF(N)303,306,306
306   DO 307 NJ=1,NFREE
      DO 307 MI=1,NFREE
      MMI=M+MI
      NNJ=N+NJ
      IMI=IS+MI
      JNJ=JS+NJ
307   ST(MMI,NNJ)=ST(MMI,NNJ)+REIN(IMI,JNJ)
303   CONTINUE
      JINDEX=JINDEX+1
       IF(JINDEX.GT.2) GO TO 241
      IF(INN.EQ.34) GO TO 310
      IF(INN.EQ 162) GO TO 311
      GO TO 241
310   INN=35
      IFF=64
      GO TO 300
311   INN=163
      IFF=192
      GO TO 300
241   DO 290 I=1,NBOUN
      M=NF(I)-K
      MM=NF(I)-1
      IF(M)290,242,242
```

```
242   IF(M-24)243,243,290
243   DO 230 J=1,NFREE
      IF(NB(I,J))230,345,230
345   NMI=NFREE*M+J
      ST(NMI,NMI)=ST(NMI,NMI)*.1E+12
      DO 233 JJ=1,NCOLN
      JNJ=NFREE*MM+J
233   U(JNJ,JJ)=ST(NMI,NMI)*BV(I,J)
230   CONTINUE
290   CONTINUE
      INTER=NEN
      MI=NFREE*MINUS+1
      NJ=NFREE*L
      M=NJ-MI+1
      IF(II-NPART)115,116,115
115   NA=NFREE*(NLAST(II+1)-MINUS)
      GO TO 117
116   NA=M+1
117   N=NA-M
      MM=M+1
70    WRITE(3)M,N,((ST(I,J),I=1,M),J=1,M),((ST(I,J),I=1,M),J=MM,NA),
     1((U(I,J),I=MI,NJ),J=1,NCOLN)
      REWIND 0
      REWIND 1
      REWIND 2
      REWIND 3
      CALL SOLVE (NPART,NCOLN)
      REWIND 2
      REWIND 4
      CALL STRESS(NPART,NFIRST,NLAST,NCOLN,NELCM,NOD,NFREE,NPOIN,CMAXT,
     1CMAXC,BMAXT,BMAXC,SMAXT,SMAXC,NEP,AN,DISP,STR,IRC)
      REWIND 4
      DO 6801 I=1,NELEM
      IF(IRC(I).NE.0) JOND=JOND+1
6801  CONTINUE
      IF(JOND.EQ.1) GO TO 6808
      PRINT 31,ITER
      GO TO 888
6808  DO 6809 I=1,NELEM
      READ (4) SIGX,SIGY,SIGXY,EX,EY,EXY
      STR(I,1)=STR(I,1)+SIGX
      STR(I,2)=STR(I,2)+SIGY
      STR(I,3)=STR(I,3)+SIGXY
      STR(I,4)=STR(I,4)+EX
      STR(I,5)=STR(I,5)+EY
      STR(I,6)=STR(I,6)+EXY
```

```
      SX=STR(I,1)
      SY=STR(I,2)
      SXY=STR(I,3)
      CALL PRIN(SX,SY,SXY,PMAX,PMIN,PANG)
6809  WRITE(6,6810)I,(NOD(I,J),J=1,3),SX,SY,SXY,PMAX,PMIN,PANG,IRC(I)
6810  FORMAT(4I4,4X,6F12.2,4X,I8)
      DO 680 J=1,NCOLN
      DO 680 I=1,NPOIN2
680   U(I,J)=DPDP*ULOAD(I)
      ITER=0
      PRINT 8998
8999  FORMAT(1H ,*LOAD INCREMENT CYCLE*)
      GO TO 888
899   STOP
      END
```

```
£IBFTC FEM      DECK
      SUBROUTINE FEM(XE,YM1,YM2,PR1,PR2,G,ANG,LS,TH,MM,SIGX,SIGY,SIGXY
     1D,BA,Z)
      DIMENSION D(3,3),BTDBA(6,6),XE(3,2),R(3,3),ZX(3),ZY(3),R1(3,3)
      DIMENSION PT(3),PTD(3),PPTD(3,3),DPPTD(3,3),BA(3,6)
      COMMON C(6,6),DBA(3,6),DB(3,6),A(6,6),B(3,6),ST(50,100),U(240,
      CALL FLUN(31200)
      DO 20 J=1,6
      DO 21 I=1,3
      B(I,J)=0.0
      BA(I,J)=0.0
      DB(I,J)=0.0
 21   DBA(I,J)=0.0
      DO 20 I=1,6
      A(I,J)=0.0
      BTDBA(I,J)=0.0
 20   C(I,J)=0.0
      DO 22 J=1,3
      DO 22 I=1,3
      R(I,J)=0.0
 22   D(I,J)=0.0
      ORX=(XE(1,1)+XE(2,1)+XE(3,1))*0.3333333
      ORY=(XE(1,2)+XE(2,2)+XE(3,2))*0.3333333
      DO 5 I=1,3
      XE(I,1)=XE(I,1)-ORX
 5    XE(I,2)=XE(I,2)-ORY
      ZX(1)=XE(3,2)-XE(3,2)
      ZX(2)=XE(3,2)-XE(1,2)
      ZX(3)=XE(1,2)-XE(2,2)
      ZY(1)=XE(3,1)-XE(2,1)
      ZY(2)=XE(1,1)-XE(3,1)
      ZY(3)=XE(2,1)-XE(1,1)
      ZK=XE(2,1)*XE(3,2)-XE(3,1)*XE(1,2)
      Z=3.0*ZK
      A(1,1)=ZK/Z
      A(2,1)=ZX(1)/Z
      A(3,1)=ZY(1)/Z
      A(4,2)=A(1,1)
      A(5,2)=A(2,1)
      A(6,2)=A(3,1)
      A(1,3)=ZK/Z
      A(2,3)=ZX(2)/Z
      A(3,3)=ZY(2)/Z
      A(4,4)=A(1,3)
      A(5,4)=A(2,3)
```

```
      A(6,4)=A(3,3)
      A(1,5)=ZK/Z
      A(2,5)=ZX(3)/Z
      A(3,5)=ZY(3)/Z
      A(4,6)=A(1,5)
      A(5,6)=A(2,5)
      A(6,6)=A(3,5)
      B(1,2)=1.0
      B(3,3)=1.0
      B(3,5)=1.0
      B(2,6)=1.0
      DO 23 I=1,3
      DO 23 J=1,6
      BA(I,J)=0.0
      DO 23 K=1,6
23    BA(I,J)=BA(I,J)+B(I,K)*A(K,J)
75    DEN=(1.0-PR1*PR2)
      D(1,1)=YM1/DEN
      D(2,1)=PR1*YM2/DEN
      D(1,2)=PR2*YM1/DEN
      D(2,2)=YM2/DEN
      D(3,3)=G
73    IF(ANG)70,72,70
70    CS=COS(ANG*0.017453)
      SS=SIN(ANG*0.017453)
      R(1,1)=CS**2
      R(2,1)=SS**2
      R(3,1)=SS*CS
      R(1,2)=R(2,1)
      R(2,2)=R(1,1)
      R(3,2)=-R(3,1)
      R(1,3)=2.0*R(3,2)
      R(2,3)=2.0*R(3,1)
      R(3,3)=R(1,1)-R(2,1)
      DO 100 J=1,3
      DO 100 I=1,3
      R1(I,J)=0.0
      DO 100 K=1,3
100   R1(I,J)=R1(I,J)+D(I,K)*R(J,K)
      DO 110 J=1,3
      DO 110 I=1,3
      D(I,J)=0.0
      DO 110 K=1,3
110   D(I,J)=D(I,J)+R(I,K)*R1(K,J)
72    CONTINUE
      IF(LS.NE.2) GO TO 722
      FSIG =SIGX*SIGX-SIGX*SIGY+SIGY*SIGY+3.0*SIGXY*SIGXY-307.1*307.1
```

```
      IF(FSIG.LT.05.0) GO TO 722
C     ELASTO PLASTIC MATRIX DERIVATION
      SIGNOT=307.1
      PT(1)=(SIGX-SIGY/2.0)/SIGNOT
      PT(2)=(SIGY-SIGX/2.0)/SIGNOT
      PT(3)=3.0*SIGXY/SIGNOT
      DO 31 I=1,3
      PTD(I)=0.0
      DO 31 J=1,3
 31   PTD(I)=PTD(I)+PT(J)*D(I,J)
      PTDP=0.0
      DO 32 I=1,3
 32   PTDP=PTDP+PTD(I)*PT(I)
      DO 33 I=1,3
      DO 33 J=1,3
 33   PPTD(I,J)=PT(I)*PTD(J)/PTDP
      DO 34 I=1,3
      DO 34 J=1,3
      DPPTD(I,J)=0.0
      DO 34 K=1,3
 34   DPPTD(I,J)=DPPTD(I,J)+D(I,K)*PPTD(K,J)
       DO 35 I=1,3
      DO 35 J=1,3
 35   D(I,J)=D(I,J)-DPPTD(I,J)
 722  DO 30 J=1,6
      DO 30 I=1,3
      DO 30 K=1,3
 30   DB(I,J)=DB(I,J)+D(I,K)*B(K,J)
      DO 40 J=1,6
      DO 40 I=1,3
      DO 40 K=1,6
 40   DBA(I,J)=DBA(I,J)+DB(I,K)*A(K,J)
      IF(MM)126,126,127
 127  WRITE(O)((DBA(I,J),I=1,3),J=1,6),ORX,ORY,((BA(I,J),I=1,3),J=1,6)
 126  CONTINUE
      VOL=0.5*TH*Z
      DO 50 J=1,6
      DO 50 I=1,6
      DO 50 K=1,3
 50   BTDBA(I,J)=BTDBA(I,J)+B(K,I)*DBA(K,J)*VOL
      DO 60 J=1,6
      DO 60 I=1,6
      DO 60 K=1,6
 60   C(I,J)=C(I,J)+A(K,I)*BTDBA(K,J)
      RETURN
      END
```

```
£IBFTC SOLVE    DECK
      SUBROUTINE SOLVE(NPART,NCOLN)
      DIMENSION AM(50,50),BM(50,50),YM(50,50),TF(50,4),RS(50,4),F(50,4)
      DIMENSION DIS(50,4)
      COMMON C(6,6),DBA(3,6),DB(3,6),A(6,6),B(3,6),ST(50,100),U(240,4)
      EQUIVALENCE (AM(1,1),ST(1,1)),(BM(1,1),ST(1,51)),(TF(1,1),U(1,1))
     1,(DIS(1,1),U(1,2)),(RS(1,1),U(1,3)),(F(1,1),U(1,4))
      CALL FLUN(31100)
      DO 140 I=1,50
      DO 141 J=1,NCOLN
      TF(I,J)=0.0
141   RS(I,J)=0.0
      DO 140 J=1,50
140   YM(I,J)=0.0
      DO 144 LL=1,NPART
      CALL FLUN(32000)
      READ(3) M,N,((AM(I,J),I=1,M),J=1,M),((BM(I,J),I=1,M),J=1,N),
     1((F(I,J),I=1,M),J=1,NCOLN)
150   DO 424 I=1,M
      DO 425 J=1,NCOLN
      F(I,J)=F(I,J)-TF(I,J)
      CALL FLUN(32000)
425   DIS(I,J)=F(I,J)
      DO 424 J=1,M
424   AM(I,J) =AM(I,J)-YM(I,J)
      CALL MATINV(AM,DIS,M,NCOLN )
      WRITE(1) M,N,((AM(I,J), I=1,M),J=1,M),((BM(I,J), I=1,M),J=1,N),
     1((F(I,J),I=1,M),J=1,NCOLN)
      IF(NPART-LL) 437,437,432
432   CALL MATM(AM,F,DIS,M,M,NCOLN)
      CALL MATTM(BM,DIS,TF,N,M,NCOLN)
      DO 110 J=1,N
      DO 110 I=1,M
      YM(I,J)=0.0
      DO 110 K=1,M
110   YM(I,J)=YM(I,J)+AM(I,K)*BM(K,J)
      DO 111 J=1,N
      DO 111 I=1,N
      AM(I,J)=0.0
      DO 111 K=1,M
111   AM(I,J)=AM(I,J)+BM(K,I)*YM(K,J)
      DO 112 I=1,N
      DO 112 J=1,N
112   YM(I,J)=AM(I,J)
144   CONTINUE
```

```
437   REWIND 3
      WRITE(2) ((DIS(I,J),I=1,M),J=1,NCOLN)
      IF(NPART-1) 600,600,601
601   NA=NPART-1
      DO 441 LL=1,NA
      BACKSPACE 1
      BACKSPACE 1
      READ(1) M,N,((AM(I,J),I=1,M),J=1,M),((BM(I,J),I=1,M),J=1,N),
     1((F(I,J),I=1,M),J=1,NCOLN)
      CALL MATM(BM,DIS,TF,M,N,NCOLN)
      DO 444 J=1,NCOLN
      DO 444 I=1,M
444   F(I,J)=F(I,J)-TF(I,J)
      CALL MATM(AM,F,DIS,M,M,NCOLN)
441   WRITE(2) ((DIS(I,J),I=1,M),J=1,NCOLN)
      WRITE(6,515)
515   FORMAT(10H RESIDUALS)
      DO 500 LL=1,NPART
      READ(3) M,N,((AM(I,J),I=1,M),J=1,M),((BM(I,J),I=1,M),J=1,N),
     1((F(I,J),I=1,M),J=1,NCOLN)
      BACKSPACE 2
      READ(2) ((DIS(I,J),I=1,M),J=1,NCOLN)
      IF(LL.NE.NPART) BACKSPACE 2
      BACKSPACE 2
      READ(2)((TF(I,J),I=1,N),J=1,NCOLN)
      DO 510 J=1,NCOLN
      DO 510 I=1,M
      F(I,J)=F(I,J)-RS(I,J)
      DO 512 K=1,M
512   F(I,J)=F(I,J)-AM(I,K)*DIS(K,J)
      DO 510 L=1,N
510   F(I,J)=F(I,J) - BM(I,L)*TF(L,J)
      CALL MATTM(BM,DIS,RS,N,M,NCOLN)
500   WRITE(6,31)((F(I,J),I=1,M),J=1,NCOLN)
31    FORMAT(1H ,12E9.2)
600   CONTINUE
      RETURN
      END
```

```
£IBFTC STRESS  DECK
      SUBROUTINE STRESS(NPART,NFIRST,NLAST,NCOLN,NELEM,NOD,NFREE,NPOIN,
     1CMAXT,CMAXC,BMAXT,BMAXC,SMAXT,SMAXC,NEP,AN,DISP,STR,IRC)
      DIMENSION NFIRST(20),NLAST(20),STR(200,6),IRC(200)
      DIMENSION NOD(200,3),NEP(200),AN(200),DISP(240),BA(3,6),BD(3,6)
      COMMON C(6,6),DBA(3,6),DB(3,6),A(6,6),B(3,6),ST(50,100),U(240,4)
      CALL FLUN(3,200)
      DO 600 II=1,NPART
      JJ=NPART+1-II
      M=NFREE*(NFIRST(JJ)-1)+1
      N=NFREE*NLAST(JJ)
  600 READ (2) ((U(I,J),I=M,N),J=1,NCOLN)
      PRINT 615
  615 FORMAT(1H ,*NODE   X DIS        Y DIS    ETC*)
   32 FORMAT(6(I4,2F8.4))
      NPOIN2=NPOIN*2
      DO 700 I=1,NPOIN2
  700 DISP(I)=DISP(I)+U(I,1)
      PRINT 32,(I,DISP(2*I-1),DISP(2*I),I=1,NPOIN)
      DO 20 LL=1,NELEM
      READ (0) ((DBA(I,J),I=1,3),J=1,6),ORX,ORY,((BA(I,J),I=1,3),J=1,6)
      DO 620 J=1,NCOLN
      DO 620 I=1,3
      JJ=NOD(LL,I)
      C(2*I-1,J)=U(2*JJ-1,J)
  620 C(2*I,J)=U(2*JJ,J)
      DO 630 J=1,NCOLN
      DO 630 I=1,3
      BD(I,J)=0.0
      DB(I,J)=0.0
      DO 630 K=1,6
      BD(I,J)=BD(I,J)+BA(I,K)*C(K,J)
  630 DB(I,J)=DB(I,J)+DBA(I,K)*C(K,J)
      WRITE(4)(DB(I,1),I=1,3),(BD(I,1),I=1,3)
C     ****     ****     ****     CHECK FOR VARIOUS FAILURE MODES
      SX=DB(1,1)
      SY=DB(2,1)
      SXY=DB(3,1)
      SIGX=STR(LL,1)+SX
      EX=ABS(STR(LL,4)+BD(1,1))
      SIGY=STR(LL,2)+SY
      SIGXY=STR(LL,3)+SXY
      CALL PRIN (SIGX,SIGY,SIGXY,PMAX,PMIN,PANG)
      IF(NEP(LL).EQ.1) GO TO 331
      IF(NEP(LL).EQ.5) GO TO 331
      GO TO 332
C     ****     *****     ****     CONCRETE BEAM
  331 IF(PMAX.GE.CMAXT) GO TO 3311
      IF(PMAX.LE.0.0) GO TO 3313
      IF(EX.GE.0.0015) GO TO 3315
```

```
      IRC(LL)= 0
      GO TO 20
3311    NEP(LL)=5
      IRC(LL)=1
      AN(LL)=180.0-PANG
      PRINT 38,LL
      IF(EX.GE.0.0015) GO TO 3315
      GO TO 20
3313  FSIG   =SIGX*SIGX-SIGX*SIGY+3.0 *SIGXY*SIGXY-CMAXC*CMAXC
      IF(FSIG.LT.5.0) GO TO 3315
      IRC(LL)=2
      PRINT 39,LL
      IF(EX.GE.0.0015) GO TO 3315
      GO TO 20
3315  IF(LL.LE.32) GO TO 20
      IF(LL.LE.64) GO TO 3312
      IF(LL.LE.160) GO TO 20
      IF(LL.LE.192) GO TO 3312
      GO TO 20
3312  PRINT 33,LL
      IF(IRC(LL).EQ.1) GO TO 3316
      IF(IRC(LL).EQ.2) GO TO 3317
      IRC(LL)=3
      IF(SIGX.LT.0.0) IRC(LL)=4
      GO TO 20
3316  IRC(LL)=13
      IF(SIGX.LT.0.0) IRC(LL)=14
      GO TO 20
3317  IRC(LL)=23
      IF(SIGX.LT.0.0) IRC(LL)=24
      GO TO 20
332   IF(PMAX.GE.BMAXT) GO TO 3321
      IF(PMAX.LE.0.0) GO TO 3323
      GO TO 20
3323  FSIG=SIGX*SIGX-SIGX*SIGY+SIGY*SIGY+3.0*SIGXY*SIGXY-CMAXC*CMAXC
      IF(FSIG.GE.5.0) GO TO 3322
      GO TO 20
3321  IF(NEP(LL).EQ.2) NEP(LL)=6
      IF(NEP(LL).EQ.3) NEP(LL)=7
      IF(NEP(LL).EQ.4) NEP(LL)=8
      AN(LL)=180.0-A(3,1)
      PRINT 38,LL
      IRC(LL)=1
      GO TO 10
3322  PRINT 39,LL
      IRC(LL)=2
20    CONTINUE
10    FORMAT(1H ,4I4,2F16.8)
31    FORMAT(4I4,2F10.4,4X,6F12.2)
33    FORMAT( 1H ,I4, * STEEL HAS YIELDED*)
38    FORMAT(1H ,I4,* ELEMENT IS CRACKED*)
39    FORMAT(1H ,I4,* ELEMENT HAS VON MISES FAILURE*)
```

```
$IBFTC MATINV  DECK
       SUBROUTINE  MATINV(A,B,N,M)
       DIMENSION A(50,50),B(50,4),INDEX(50,4),PIVOT(50),IPIVOT(50)
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)
C      INITIALIZATION
       CALL FLUN (32000)
   15 DO 20 J=1,N
   20 IPIVOT(J)=0
   30 DO 550 I=1,N
C      SEARCH FOR PIVOT ELEMENT
 40    AMAX=0.0
   45 DO 105 J=1,N
   50 IF (IPIVOT(J)-1) 60, 105, 60
   60 DO 100 K=1,N
 70    IF(IPIVOT(K)-1)80,100,740
 80    IF (ABS (AMAX)-ABS (A(J,K))) 85, 100, 100
 85    IROW=J
 90     ICOLUM=K
   95 AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
  110 IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C      INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
  130  IF(IROW-ICOLUM) 140,260,140
 140  CONTINUE
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  200 A(ICOLUM,L)=SWAP
  205 IF(M) 260, 260, 210
  210 DO 250 L=1, M
  220 SWAP=B(IROW,L)
  250 B(ICOLUM,L)=SWAP
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
  310 PIVOT(I)=A(ICOLUM,ICOLUM)
C      DIVIDE PIVOT ROW BY PIVOT ELEMENT
  330 A(ICOLUM,ICOLUM)=1.0
 340  DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT(I)
  355 IF(M) 380, 380, 360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT(I)
C      REDUCE NON PIVOT ROWS
  380 DO 550 L1=1,N
  390 IF(L1-ICOLUM) 400, 550, 400
```

```
400 T=A(L1,ICOLUM)
420 A(L1,ICOLUM)=0.0
430 DO 450 L=1,N
450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
455 IF(M) 550, 550, 460
460  DO 500 L=1,M
500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
550 CONTINUE
C     INTERCHANGE COLUMNS
600 DO 710 I=1,N
610 L=N+1-I
620 IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
630 JROW=INDEX(L,1)
640 JCOLUM=INDEX(L,2)
650 DO 705 K=1,N
660 SWAP=A(K,JROW)
670 A(K,JROW)=A(K,JCOLUM)
700 A(K,JCOLUM)=SWAP
705  CONTINUE
710 CONTINUE
740 RETURN
    END
```

```
£IBFTC STEEL    DECK
       SUBROUTINE STEEL(RLEN,AS,ES,XXIS,REIN,LS)
        DIMENSION REIN(6,6)
       CALL FLUN(31200)
       DO 10 I=1,6
         DO 10 J=1,6
 10    REIN(I,J)=0.0
       IF(LS.GE.3) GO TO 11
       REIN(1,1)=AS*ES/RLEN
       REIN(1,3)=-REIN(1,1)
       REIN(2,2)=12.0*ES*XXIS/(RLEN*RLEN*RLEN)
       REIN(2,4)=-REIN(2,2)
       REIN(3,1)=-REIN(1,1)
       REIN(3,3)=REIN(1,1)
       REIN(4,2)=-REIN(2,2)
       REIN(4,4)=REIN(2,2)
       GO TO 12
 11    REIN(2,2)=12.0*ES*XXIS/(RLEN*RLEN*RLEN)
       REIN(2,4)=-REIN(2,2)
       REIN(4,2)=-REIN(2,2)
       REIN(4,4)=REIN(2,2)
 12    RETURN
       END
£IBFTC MATM     DECK
      SUBROUTINE MATM(D,B,DB,L,M,N)
      DIMENSION D(50,50),B(50,4),DB(50,4)
       CALL FLUN(31200)
      DO 110 J=1,N
      DO 110 I=1,L
      DB(I,J)= 0.0
      DO 110 K=1,M
 110  DB(I,J)=DB(I,J)+D(I,K)*B(K,J)
      RETURN
       END
£IBFTC MATTM    DECK
      SUBROUTINE MATTM(D,B,DB,L,M,N)
      DIMENSION D(50,50),B(50,4),DB(50,4)
       CALL FLUN(31200)
      DO 110 J=1,N
      DO 110 I=1,L
      DB(I,J)=0.0
      DO 110 K=1,M
 110  DB(I,J)=DB(I,J)+D(K,I)*B(K,J)
      RETURN
      END
£IBFTC PRIN     DECK
       SUBROUTINE PRIN(SX,SY,SXY,PMAX,PMIN,PANG)
       CALL FLUN(31200)
       PMAX=(SX+SY)*0.5+SQRT((SX+SY)**2/4.0+SXY**2)
       PMIN=(SX+SY)*0.5-SQRT((SX+SY)**2/4.0+SXY**2)
       PANG=52.29578*ATAN((PMAX-SY)/SXY)
       RETURN
       END
```